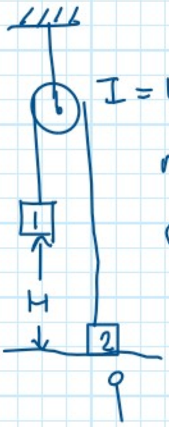
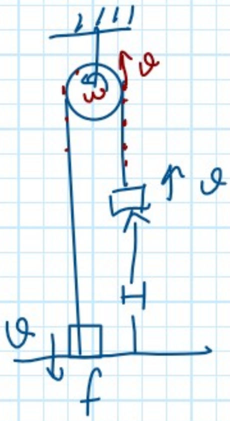


$I = \frac{MR^2}{2}$ $\quad v_f = ?$

$m_1 > m_2$

$v_1 = v_2 = 0$

$w_1 = w_2 = 0$



$E_i = E_f$

$E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}I\omega^2 + m_1 g y_1 + m_2 g y_2$

$E_i = E_f$ $\qquad \boxed{\omega R = v}$

$$m_1 g H + 0 + 0 + 0 = \frac{m_1 v^2}{2} + \frac{m_2 v^2}{2} + m_2 g H + \frac{1}{2} I \omega^2$$

$$+\frac{1}{2} I \frac{v^2}{R^2}$$

$$v^2 = \frac{(m_1 - m_2)gH}{\left(\frac{m_1}{2} + \frac{m_2}{2} + \frac{I}{2R^2}\right)} \Rightarrow v = \sqrt{\frac{2(m_1 - m_2)gH}{m_1 + m_2 + \frac{M}{2}}}$$

$I = \frac{MR^2}{2}$

$\vec{L}$ = angular momentum $\qquad \vec{\tau} = \vec{r} \times \vec{F}$

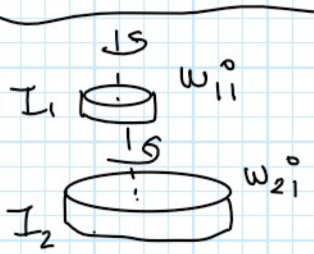
$\vec{P} = m\vec{v}$; $\vec{L} = I\vec{\omega} = \vec{r} \times \vec{P}$ $\qquad \vec{F} = \frac{d\vec{P}}{dt}$

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

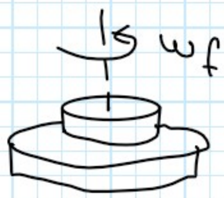
$$\vec{\tau} = \vec{r} \times \frac{d\vec{P}}{dt}$$

$$\vec{\tau}\,dt = \vec{r} \times d\vec{P}$$

$$\vec{L} = \tau t = \vec{r} \times \vec{P}$$



$\Rightarrow$



$\sum L_i = \sum L_f$ angular mom. conservation

$$I_1 w_{1i} + I_2 w_{2i} = I_1 w_f + I_2 w_f \Rightarrow w_f = \frac{I_1 w_{1i} + I_2 w_{2i}}{I_1 + I_2}$$

$I_1 = 0.5\ kgm^2 \quad w_{1i} = 20\ rad/s$

$I_2 = 1.5\ kgm^2 \quad w_{2i} = -5\ rad/s$

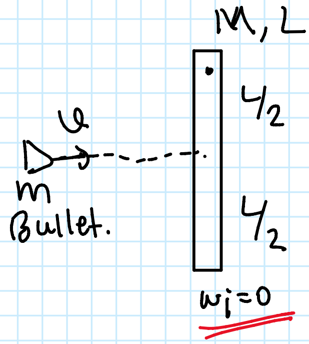
$\sum L_i = \sum L_f$

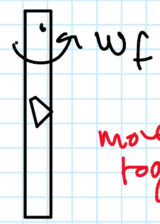
$0.5(20) + 1.5(-5) = (0.5 + 1.5)\,w_f$

M, L

$$= (0.5 + 1.5)\,\omega_f$$

Ex





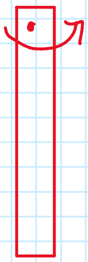
move together

$\omega_f = ?$

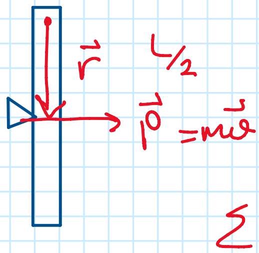
$$\sum \vec{L}_i = \sum \vec{L}_f$$

$$\vec{L} = I\omega$$

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F}$$



$$I_{rod} = \frac{ML^2}{3}$$



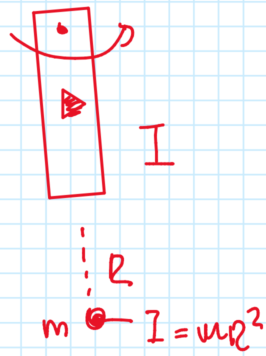
$$L_{BULLET,i} = r p \sin 90 = r m v = m v \frac{L}{2}$$

$$\sum L_i = \sum L_f$$

$$L_{i,rod} + L_{i\ BULLET} = L_{total,\ final}$$

$$I \cancel{\omega_i}^{\,0} + m v \frac{L}{2} = (I_{total})\,\omega_f$$

$$\frac{m v L}{2} = \left[\frac{ML^2}{3} + m\left(\frac{L}{2}\right)^2\right]\omega_f$$

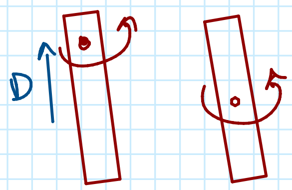


m; M, L, I, v

$$\frac{m v \cancel{L}}{\cancel{2}} = \frac{4ML^{\cancel{2}} + 3mL^{\cancel{2}}}{\cancel{12}\ 6}\,\omega_f$$

$$\frac{6mv}{4ML + 3mL} = \omega_f$$

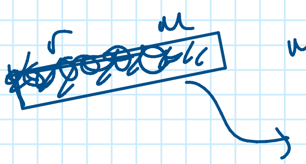
$$MD^2 + \frac{1}{12}ML^2 = \frac{ML^2}{3}$$



$$\frac{1}{12}ML^2$$

$$\int r^2\,dm$$

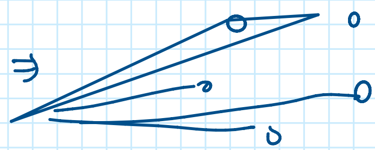
$$D = L/2$$

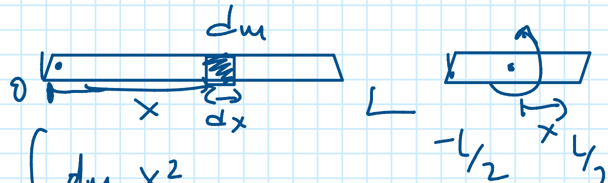


$$m r^2 = I$$

$$\int dm\, r^2$$

$$\sum m_i r_i^2 \Rightarrow$$





$$\rho = \frac{M}{L} = \frac{dm}{dx}$$

$$\int dm\, x^2$$

$$\rho = \frac{M}{L} = \frac{dm}{dx}$$

$$dm = \rho\, dx$$

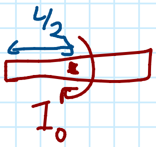
$$\int dm\, x^2$$

$$\int \rho\, dx\, x^2 \qquad \rho \int_0^L x^2 dx \quad -L/2 \to L/2$$

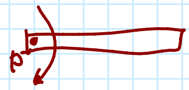
$$\underbrace{\rho \int_0^L x^2 dx}_{\frac{x^3}{3}}$$

$$\frac{ML^2}{3} = \frac{M}{L}\left[\frac{L^3}{3} - 0\right]$$

$$I_0 = I_{rod} = \frac{ML^2}{12}$$



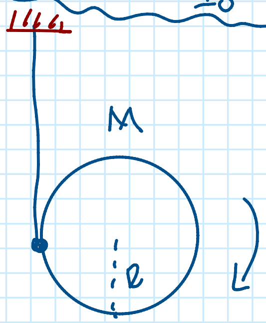
M



Parallel axis theorem

$$I_p = M\left(\frac{L}{2}\right)^2 + \frac{ML^2}{12} = \frac{ML^2}{3}$$

$$I_0 + MD^2 = I_p$$

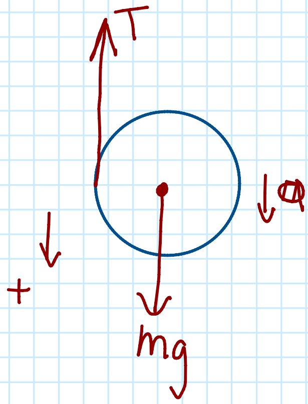


FBD?

$\alpha = ?$

$a_{cm} = ?$

$$I = \frac{MR^2}{2}$$



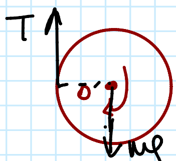
$$\Sigma \vec{F} = m\vec{a}$$

$$\Sigma \tau = I\alpha = Fr\sin\theta$$

T, a, α

$$\boxed{mg - T = ma}$$

$$\boxed{\alpha R = a}$$ rolling w/o slipping

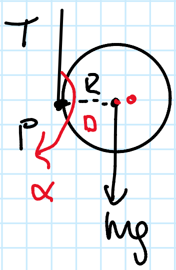


I

$$\Sigma \tau_0 = \cancel{\tau_{mg}} + \tau_T = TR = I\alpha_1 \quad ; \; I_0 = \frac{MR^2}{2} \quad ✓$$

$$\boxed{TR = I\alpha = \frac{MR^2}{2}\alpha}$$

II option



$$\Sigma \tau_p = \cancel{\tau_T} + \tau_{mg} = mgR = I_p \alpha$$

$$I_p = I_0 + MD^2 = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

$$mgR = \frac{3}{2}MR^2\alpha_2$$

$$mg - T = ma$$

$$a = \frac{2g}{3} ✓ \Leftarrow \left\{ mg = \frac{3}{2}MR\alpha = \frac{3}{2}Ma \Rightarrow \right. \quad \boxed{\alpha R = a}$$

$$T = mg/3$$

$$mg - T = ma$$

$$T\cancel{R} = \frac{MR^{\cancel{2}}}{2}\alpha \Rightarrow T = \frac{MR\alpha}{2} = \frac{ma}{2}$$

$$mg - \frac{mg}{2} = ma$$

$$\boxed{a = \frac{2g}{3}}$$

$$\alpha = \frac{a}{R} = \frac{2g}{3R} ✓$$

$T\cancel{R} = \frac{MR^{\cancel{2}}}{2}\alpha \Rightarrow T = \frac{MR\alpha}{2} = \frac{Ma}{2}$

$\boxed{a = \frac{2g}{3}}$

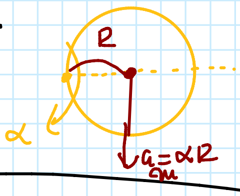
$\alpha = \frac{a}{R} = \frac{2g}{3R}$ ✓

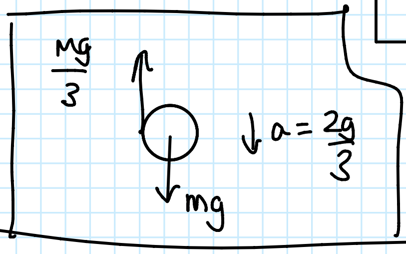
$a = \alpha R$

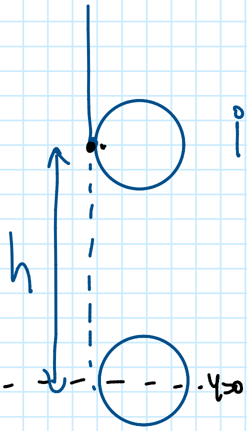
$T = \frac{Mg}{2} = \frac{Mg}{3}$

$\alpha = 10\ rad/s^2$ $R = 1m$

$a_{cm} = 10(1) =$







i $\Rightarrow \omega_i = 0,\ v_i = 0$

$\omega_f = ?$

$v_f = ?$

$a = 2g/3$

$v_f = v_i + at$

$v_f^2 = v_i^2 + 2a\Delta y$

$v_f^2 = 2\left(\frac{2g}{3}\right)h$

$\boxed{v_f = \sqrt{\frac{4g}{3}h}}$

$\left\{\begin{matrix} a = \alpha R \\ v = \omega R \end{matrix}\right\}$

Energy conservation

$I = \frac{MR^2}{2}$

$E_i = E_f$

$U_i + K_i = U_f + K_f$

$mgh + 0 = 0 + \frac{1}{2}I\omega_f^2 + \frac{1}{2}mv_f^2$

$R\omega_f \overset{?}{=} v_f$

$\omega_f = \frac{v_f}{R}$

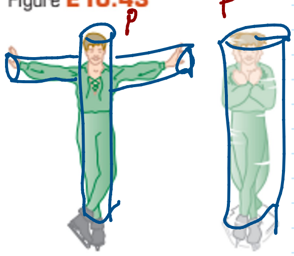
$\cancel{m}gh = \frac{1}{2}\frac{\cancel{m}R^{\cancel{2}}}{2}\frac{v_f^2}{\cancel{R^2}} + \frac{1}{2}\cancel{m}v_f^2$

same

$gh = \frac{3}{4}v_f^2 \Rightarrow v_f = \sqrt{\frac{4gh}{3}}$

10.43 •• **The Spinning Figure Skater.** The outstretched hands and arms of a figure skater preparing for a spin can be considered a slender rod pivoting about an axis through its center (Fig. E10.43). When the skater's hands and arms are brought in and wrapped around his body to execute the spin, the hands and arms can be considered a thin-walled, hollow cylinder. His hands and arms have a combined mass of 8.0 kg. When outstretched, they span 1.8 m; when wrapped, they form a cylinder of radius 25 cm. The moment of inertia about the rotation axis of the remainder of his body is constant and equal to 0.40 kg · m². If his original angular speed is 0.40 rev/s, what is his final angular speed?

Figure **E10.43**



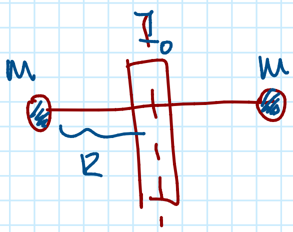
$L_i = L_f$

$I_i\omega_i = I_f\omega_f$

$L = r \times p = rp\sin\theta$

$p = mv$

$L = I\omega$



$\Rightarrow$

$\underbrace{I_i}_{I_0 + 2MR^2}$

$\underbrace{I_f}_{I_0 + \cancel{MO^2} \to 0}$

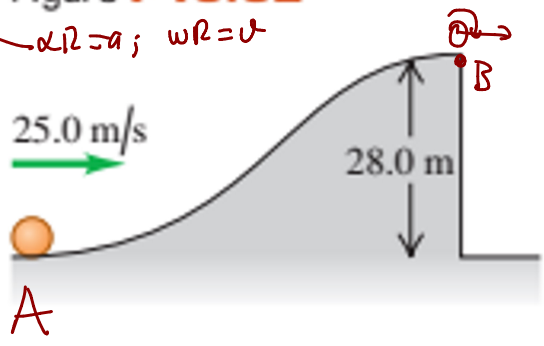
$I_i > I_f$

$\omega_i < \omega_f$

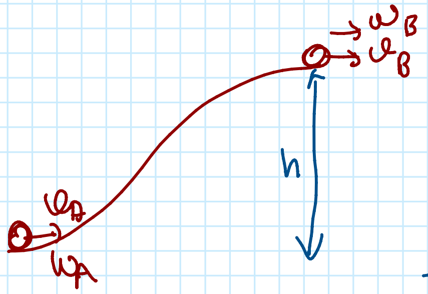
10.82 •• **CP** A solid uniform ball rolls without slipping up a hill, as shown in Fig. P10.82. At the top of the hill, it is moving horizontally, and then it goes over the vertical cliff. (a) How far from the foot of the cliff does the ball land, and how fast is it moving just before it lands? (b) Notice that when the balls lands, it has a greater translational speed than when it was at the bottom of the hill. Does this mean that the ball somehow gained energy? Explain!

$\alpha R = a$; $\omega R = v$

Figure P10.82



$I = \frac{2}{5} m R^2$



$U_i + K_i = U_f + K_f$

$0 + \frac{1}{2} m v_A^2 + \frac{1}{2} I \omega_A^2 = \frac{1}{2} m v_B^2 + \frac{1}{2} I \omega_B^2 + mgh$

$R\omega = v$

$\cancel{\frac{1}{2}} \cancel{m} v_A^2 + \cancel{\frac{1}{2}} \frac{2}{5} \cancel{m} R^2 \omega_A^2 = \cancel{\frac{1}{2}} \cancel{m} v_B^2 + \cancel{\frac{1}{2}} \frac{2}{5} \cancel{m} R^2 \omega_B^2 + \cancel{m} g h \cdot 2$

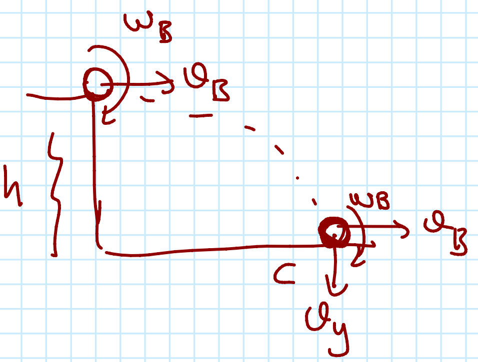
$v_A^2 \qquad v_B^2$

$\frac{7}{5} v_A^2 = \frac{7}{5} v_B^2 + 2gh \Rightarrow v_B^2 = \frac{7 v_A^2 - 10gh}{7}$

$h = 28$ m

$v_A = 25$ m/s

$v_B = \sqrt{\frac{7(25)^2 - 10(9.8)28}{7}}$



$E_B = E_C$

$U_B + \cancel{\frac{1}{2} I \omega_B^2} + \frac{1}{2} m v_B^2 = U_C + \cancel{\frac{1}{2} I \omega_C^2} + \frac{1}{2} m v_C^2$

ω_B^2

$v_y = \sqrt{2gh}$

$v_C^2 = v_y^2 + v_B^2 \Rightarrow \sqrt{2gh + v_B^2} = v_C$

$v_B^2 = \frac{7 v_A^2 - 10gh}{7}$

$v_C^2 = v_B^2 + 2gh$

$v_C^2 = \frac{7 v_A^2 - 10gh}{7} + 2gh = \frac{7 v_A^2 + 4gh}{7}$

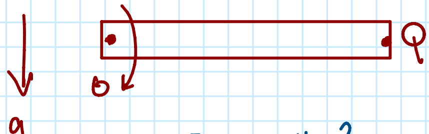
$v_C > v_A$

$\frac{v_C}{R} > \omega_A = \frac{v_A}{R}$

$v_C = R\omega_C$ rolls w/o slipping

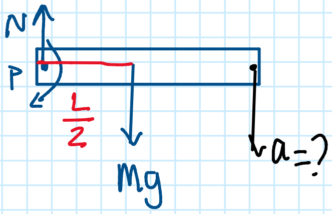
It will slip !!

M, L



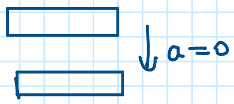
Rod starts to rotate ; $\omega_i = 0$
what's the acceleration at the edge of the rod?
at the moment it starts to rotate?

$I_0 = I_{cm} = \frac{mL^2}{12}$

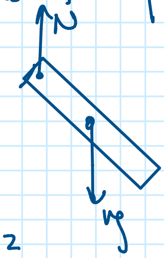


$\sum \vec{F} = 0$ rod is not falling !

$a = 0$



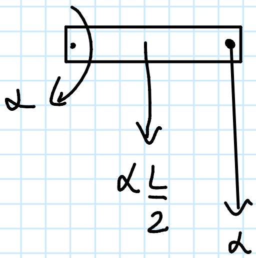
$\sum \vec{F} = 0$



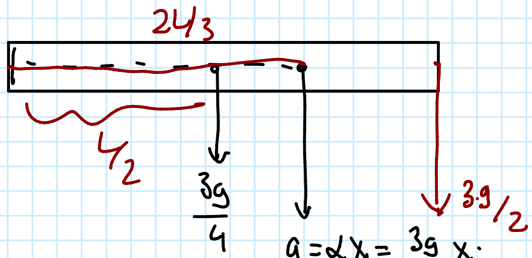
$\sum \tau = I\alpha$

$\tau_{mg} + \cancel{\tau_N}^{0} = I_p \alpha$; $I_p = I_0 + MD^2 = \frac{mL^2}{3}$

$\cancel{M}g \frac{\cancel{L}}{2} = \frac{\cancel{M}L^{\cancel{2}}}{3} \alpha \Rightarrow \alpha = \frac{3g}{2L}$



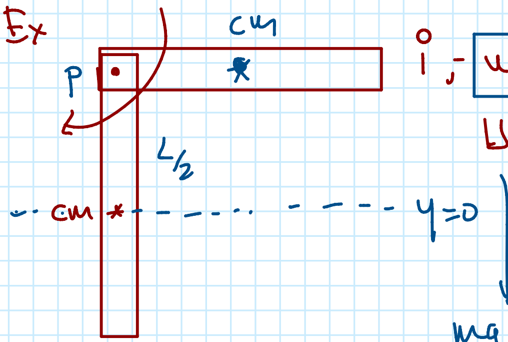
$\alpha L = \frac{3g}{2L} \cdot L = \underline{\underline{\frac{3g}{2}}} > g$!!



$g = \alpha x = \frac{3g}{2L} x$

$x = 2L/3$

Ex



i ; $\boxed{\omega_i = 0}$

$E_i = E_f$

$U_i^{\,0} + \frac{1}{2} I \omega_i^2 + \frac{1}{2} m \cancel{v_i^2}^{0} = U_f + \frac{1}{2} I \omega_f^2 + \frac{1}{2} m \cancel{v_f^2}^{0}$

$mgy_f = 0$

$v_i = 0$
$v_f = 0$ } translational KE
pivot (cylinder)

$\omega_f = ?$

$v_{f,cm} = ?$

$\alpha \Rightarrow \{ \cancel{\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta} \}$ not const

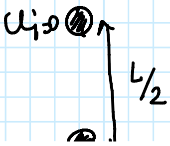
$mg \frac{L}{2} = \frac{1}{2} I_p \omega_f^2$

$\cancel{m}g \frac{\cancel{L}}{\cancel{2}} = \frac{1}{\cancel{2}} \left(\frac{\cancel{m}L^{\cancel{2}}}{3} \right) \omega_f^2$

$\sqrt{\frac{3g}{L}} = \omega_f$

$v_{f,cm} = ?$ $\omega_f \frac{L}{2} = v_{f,cm}$

$\sqrt{\frac{3g}{L}} \frac{L}{2} = \frac{\sqrt{3gL}}{2} = v_{f,cm}$



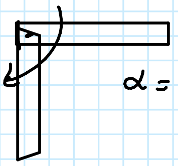
$= v_{f,cm}$

$L/2$

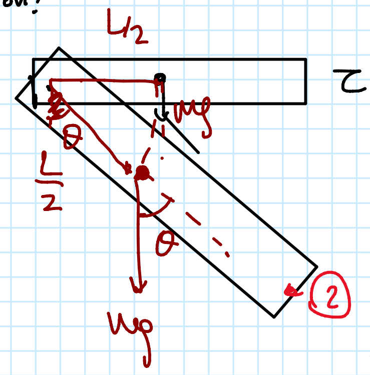
$v_f = \sqrt{2gh} = \sqrt{2g\frac{L}{2}} = \sqrt{gL}$

$\sqrt{gL} > \frac{\sqrt{3gL}}{2}$

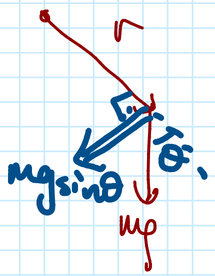
Is α constant in this motion?



α = sabit mi?



$\tau_1 = mg\frac{L}{2}$

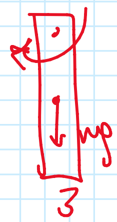


$\tau_2 = mg\frac{L}{2}\sin\theta$

$\tau_1 > \tau_2$

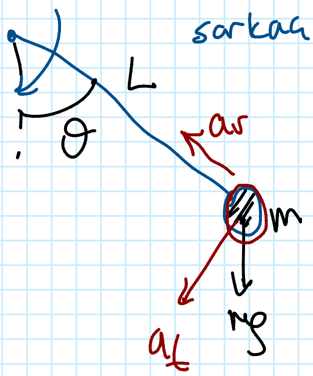
$\alpha_1 > \alpha_2$

$\tau = I\alpha$



$\tau_3 = 0 = mg\frac{L}{2}\sin 0° = 0$

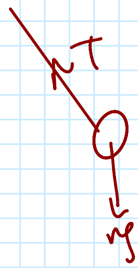
sarkaç pendulum



$\tau = mgL\sin\theta = I\alpha \qquad I = mL^2$

$\cancel{m}g\cancel{L}\sin\theta = \cancel{m}L^{\cancel{2}}\alpha$

$\alpha = \frac{g\sin\theta}{L}$



$a_t = \alpha L$

$a_t = \frac{g\sin\theta}{L} L = g\sin\theta$

$$\left\{ \begin{aligned} a_r &= \frac{v^2}{L} \\ a_t &= g\sin\theta \end{aligned} \right\}$$

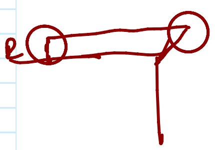
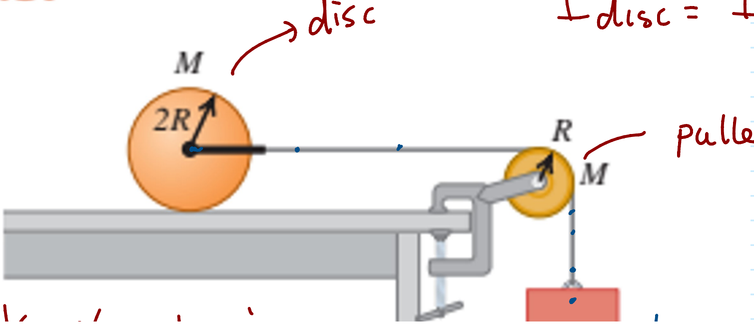


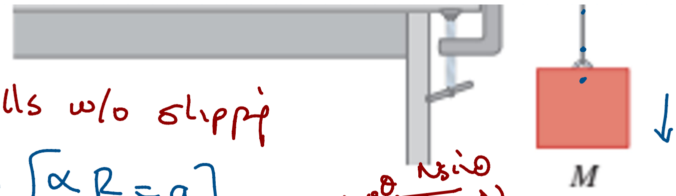
Figure **P10.87**



$I_{disc} = I_{pulley} = \frac{MR^2}{2}$

FBD?

$\sum \vec{F} = m\vec{a}$



$\Sigma \vec{F} = m\vec{a}$

$\Sigma \vec{\tau} = I\vec{\alpha}$

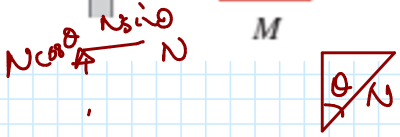
$a = ?$

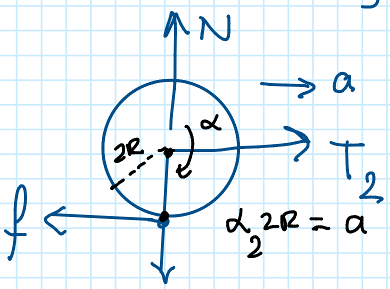
$T = ?$

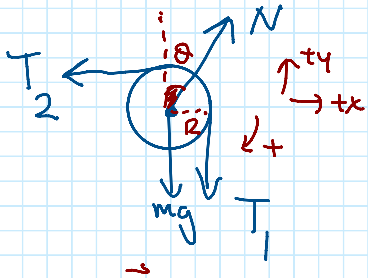
$\alpha = ?$

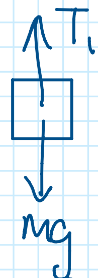
rolls w/o slipping → $[\alpha R = a]$

$\downarrow a = ?$









$\Sigma \vec{F} = m\vec{a}$ → $+x$

$T_2 - f = ma$

$f = \mu N = \mu mg$ ($\Sigma F_y = 0$)

$\boxed{T_2 - \mu mg = ma}$ (3)

$\Sigma \tau = I\alpha$

$\cancel{\tau_N} + \cancel{\tau_{mg}} + \cancel{\tau_{T_2}} + \tau_f = I_0 \alpha_2$ (0, 0, 0)

$f(2R) = I_0 \alpha_2$

$f\,2R = \frac{m(2R)^2}{2}\alpha_2$

$\boxed{\alpha_2\, 2R = a}$

$\boxed{\mu mg = f = \frac{m(2R)\alpha_2}{2} = \frac{ma}{2}}$

(4) $\boxed{f = \frac{ma}{2}}$

$\Sigma \vec{F} = 0$

$\Sigma F_x = 0$ $\quad \Sigma F_y = 0$

$\{N\sin\theta - T_2 = 0\}$ $\quad \{N\cos\theta - mg - T_1 = 0\}$

$\Sigma \tau_0 = I_0 \alpha$

$T_1 R - T_2 R = I\alpha_1$

$(T_1 - T_2)R = \frac{mR^2}{2}\alpha_1$

$\boxed{T_1 - T_2 = \frac{mR\alpha_1}{2} = \frac{ma}{2}}$ (2)

$\boxed{\alpha_1 R = a}$

(1) $\boxed{mg - T_1 = ma}$

$mg - \cancel{T_1} = ma$

$\cancel{T_1} - \cancel{T_2} = \frac{ma}{2}$

$\cancel{T_2} - \cancel{f} = ma$

$+ \quad \cancel{f} = \frac{ma}{2}$

$mg = 3ma$

$\boxed{a = g/3}$

$\alpha_1 R = a = \alpha_2\, 2R$

$\alpha_1 = \frac{g}{3R}$; $\quad \alpha_2 = \frac{g}{6R}$

$f = \frac{ma}{2} = \frac{mg}{6}$; $\quad T_2 - f = ma \quad T_2 = \frac{3}{2}ma = \frac{mg}{2}$

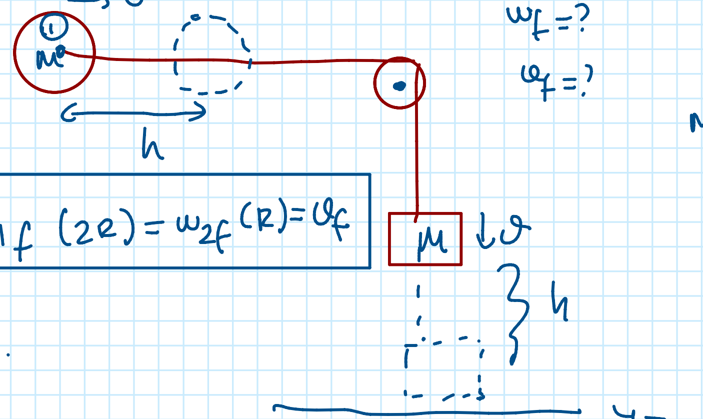
$f = \mu mg = \frac{mg}{6} \quad mg - T_1 = ma \quad T_1 = mg - ma = \frac{2mg}{3}$

$\mu = 1/6$

$w_i = v_i = 0$

$w_f = ?$

$v_f = ?$



$$\boxed{w_{1f}(2R) = w_{2f}(R) = v_f}$$

$$\Sigma E_i = \Sigma E_f$$

$$U_i + K_i = U_f + K_f$$

$$mgh + 0 + 0 + 0 + 0 =$$

$$= 0 + \frac{1}{2}mv_f^2 + \frac{1}{2}mv_f^2 + \frac{1}{2}I_1 w_{1f}^2 + \frac{1}{2}I_2 w_{2f}^2$$

$$I_1 = \frac{m(2R)^2}{2}$$

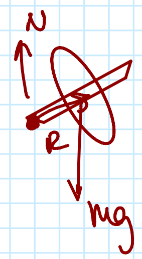
$$I_2 = \frac{mR^2}{2}$$

$$\cancel{m}gh = 2\left(\frac{1}{2}\cancel{m}v_f^2\right) + \frac{1}{2}\frac{\cancel{m}\cancel{(2R)^2}}{2}\frac{v_f^2}{\cancel{(2R)^2}} + \frac{1}{2}\cancel{m}\cancel{R^2}\frac{v_f^2}{\cancel{R^2}}$$

$$gh = v_f^2\left(1 + \frac{1}{2} + \frac{1}{4}\right) = \frac{7v_f^2}{4} \Rightarrow v_f = \sqrt{\frac{4gh}{7}}$$

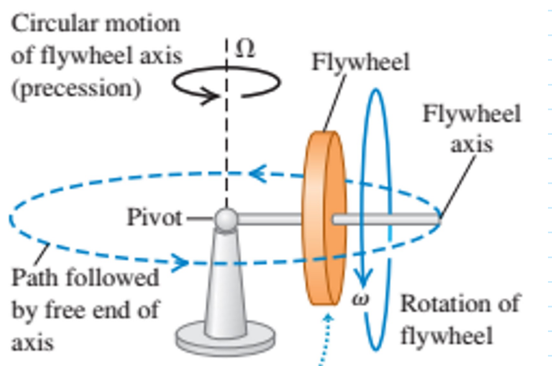
FLYWHEEL (TOPAG)

PRECESSION



$$\vec{\tau} = \vec{r} \times m\vec{g}$$

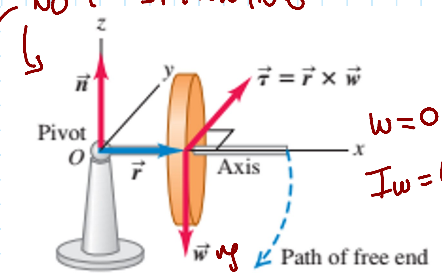
not spinning = fall down



When the flywheel and its axis are stationary, they will fall to the table surface. When the flywheel spins, it and its axis "float" in the air while moving in a circle about the pivot.

SPINS

NOT SPINNING



When the flywheel is not rotating, its weight creates a torque around the pivot, causing it to fall along a circular path until its axis rests on the table surface.

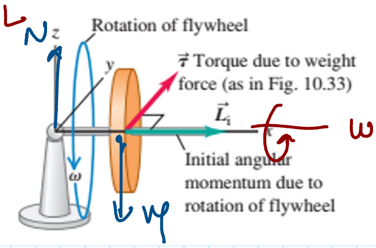
$w = 0$

$I w = 0 = L$

(a) Rotating flywheel

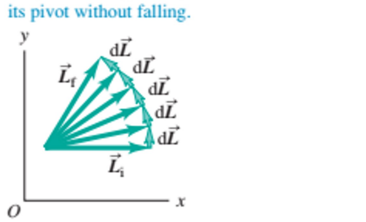
When the flywheel is rotating, the system starts with an angular momentum $\vec{L}_i$ parallel to the flywheel's axis of rotation.

= A



(b) View from above

Now the effect of the torque is to cause the angular momentum to precess around the pivot. The gyroscope circles around its pivot without falling.



$\Sigma F = 0$

$N - mg = 0$

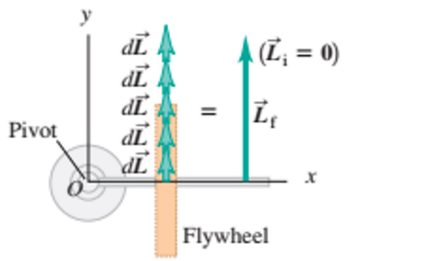
$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

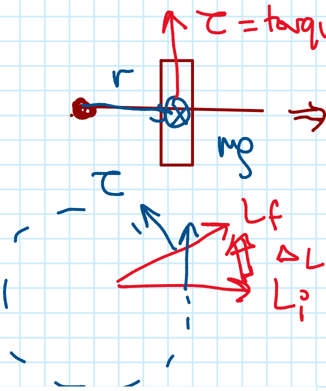
(b) View from above as flywheel falls

TOP VIEW

τ = torque



In falling, the flywheel rotates about the pivot and thus acquires an angular momentum $\vec L$. The *direction* of $\vec L$ stays constant.



$\vec r \times m\vec g = \vec\tau = \vec r \times \vec f$

precission

$L = r \times p$

$\vec\tau = \vec r \times \vec F$

$\frac{dL}{dt} = \tau$ $\frac{dP}{dt} = F$

$$\Omega = \frac{d\phi}{dt} = \frac{|d\vec L|/|\vec L|}{dt} = \frac{\tau_z}{L_z} = \frac{wr}{I\omega}$$

≡ precession angular speed

omega

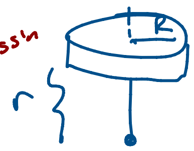
$\Omega = \frac{mgr}{\frac{mR^2}{2}\omega} = \frac{2gr}{R^2\omega}$?

$\Omega = \frac{\frac{dL}{L}}{dt} = \left[\frac{rad}{s}\right]$ = Torque $\left(\frac{dL}{dt}\right)\frac{1}{L} = \frac{mgr}{I\omega} = \Omega$

$[\Omega = 1.57\ rad/s \quad r = 0.02\ m \quad R = 0.03\ m]$

$$\omega = \frac{wr}{I\Omega} = \frac{\cancel{m}gr}{(\cancel{m}R^2/2)\Omega} = \frac{2gr}{R^2\Omega}$$

→ precession



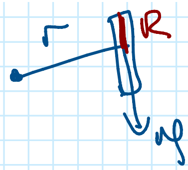
$\Rightarrow I = \frac{mR^2}{2}$

Ω ✓ r ✓ R ✓

$$\omega = \frac{2(9.8\ m/s^2)(2.0\times10^{-2}\ m)}{(3.0\times10^{-2}\ m)^2(1.57\ rad/s)} = 280\ rad/s = 2600\ rev/min$$

Ω = 1 rot in 4s ⇒ rad/s

$\frac{2\pi}{4s}$ rad $= \frac{6.28}{4} = 1.57$ rad/s

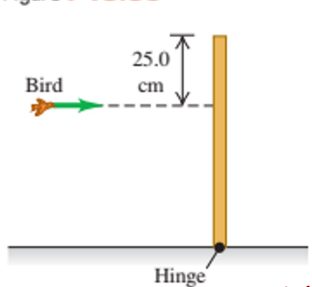


τ = mgr

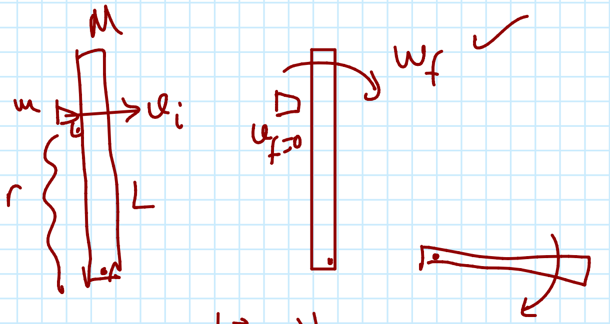
10.95 ••• A 500.0-g bird is flying horizontally at 2.25 m/s, not paying much attention, when it suddenly flies into a stationary vertical bar, hitting it 25.0 cm below the top (Fig. P10.95). The bar is uniform, 0.750 m long, has a mass of 1.50 kg, and is hinged at its base. The collision stuns the bird so that it just drops to the ground afterward (but soon recovers to fly happily away). What is the angular velocity of the bar (a) just after it is hit by the bird and (b) just as it reaches the ground?

→ $w_f = ?$

Figure **P10.95**



$L_i = L_f$



$L = I\omega = |\vec r \times \vec p|$

rotating objects

point mass

çarptıktan sonra

objects

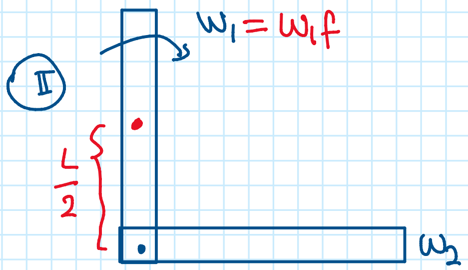
$$L_i = L_f$$

$$m v_i r + I\omega_i^{\to 0} = I\omega_f + m v_f^{\to 0} r$$

çarptıktan sonra kuş yere düşüyor, onun için $v_f = 0$

$$m v_i r = \frac{mL^2}{3} \omega_{1f} \quad \text{(I)}$$

after collision



Energy is conserved. $E_1 = E_2$

$$U_1 + K_1 = U_2 + K_2$$

$$mg\frac{L}{2} + \frac{1}{2} I \omega_1^2 = 0 + \frac{1}{2} I \omega_2^2$$