Phys2-02 Electric Potential

Monday, April 10, 2023 2:40 PM

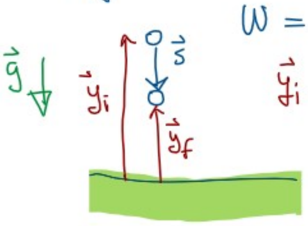
— Chapter 23 —

Electric Potential $\equiv V \equiv$ [Volt]

El. Pot $\neq$ ENERGY

Phys I $\quad W = \Delta K$; $K = \frac{1}{2}mv^2$

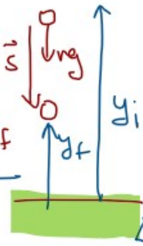
$W = -\Delta U \qquad U = mgy$



$\vec{y}_i - \vec{y}_f = -\vec{s} \Rightarrow \vec{y}_f - \vec{y}_i = \vec{s}$

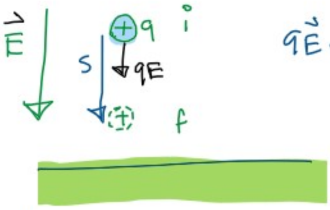
$W_{mg} = m\vec{g}\cdot\vec{s} > 0$

$= \underbrace{mgs}_{+} = \underbrace{mgy_i - mgy_f}_{+}$



Δ = final − initial

$W_{mg} = mg(-\Delta y) = -\Delta mgy = -\Delta U$



$q\vec{E}\cdot\vec{s} = W_E > 0$

$\underset{\sim\sim\sim}{W_E = -\Delta U_E}$; U_E = Electric Potential Energy $= U$

W_E = work done by El. field Force $= W$

$U_E = U \equiv qV$

pot. Energy = charge (El pot.)

$V = \frac{U}{q} = \left[\frac{J}{C} = \text{Volt}\right]$

$W = -\Delta U = -\Delta(qV) = -q\Delta V = W = q\vec{E}\cdot\vec{s} \quad \Rightarrow \quad -\Delta V = \vec{E}\cdot\vec{s}$ ↳ path

$\boxed{\Delta V = -\vec{E}\cdot\vec{s}} \quad \Rightarrow \vec{E}$ is uniform (constant)

E, i, s, f, E

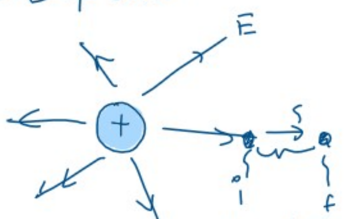
(change in El. pot) = (− El field)(displacement)

$\left[V = \frac{N}{C}m = \frac{J}{C}\right]$ *

{ s ⟷ x ⟷ l ⟷ r location }

$$\boxed{\Delta V = -\int \vec{E}\cdot d\vec{s} = -\int \vec{E}\cdot d\vec{x} = -\int \vec{E}\cdot d\vec{l} = -\int \vec{E}\cdot d\vec{r}}$$

If $\vec{E} \neq$ const.



$E = \frac{kq}{r^2} \qquad E_i \neq E_f$

$$\boxed{\Delta V = -\int_i^f \vec{E}\cdot d\vec{r}} \quad ***$$

i $d\vec{r}$ f, E

$\underset{\sim\sim}{\vec{E}\cdot d\vec{r}} > 0 \Rightarrow \Delta V < 0$

$V_f - V_i < 0$

$\underset{\sim\sim\sim}{V_f < V_i}$

If the ⊕ charge goes ALONG $\vec{E}$ direction its electric potential will DECREASE!!

$d\vec{r}$, i, E, f

$\vec{E}\cdot d\vec{r} < 0$

$\Delta V > 0$

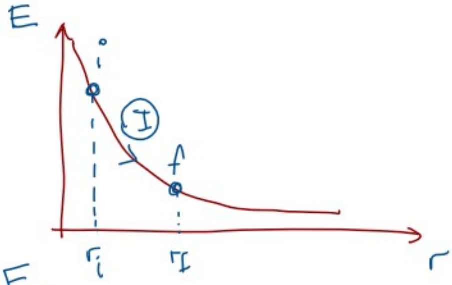
$V_f > V_i$

If ⊕ charge goes AGAINST $\vec{E}$ its potential WILL INCREASE!!

⊕ - - r - - → E $\frac{q}{4\pi\varepsilon_0 r^2} = E$

its potential WILL INCREASE!!

$\oplus_q \; - - r - - \rightarrow E \quad \frac{q}{4\pi\varepsilon_0 r^2} = E$

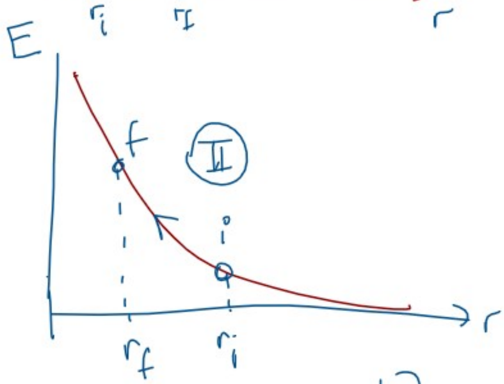


$$\Delta V = -\int_i^f \vec{E}\cdot d\vec{r} = -\int_i^f \frac{q}{4\pi\varepsilon_0 r^2}\,\underbrace{\hat{r}\cdot d\vec{r}}_{dr}$$

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\hat{r}\cdot d\vec{r} = |\hat{r}|\,|d\vec{r}|\cos 0 = dr$

$$\Delta V = -\frac{q}{4\pi\varepsilon_0}\int_{r_i}^{r_f}\frac{dr}{r^2} = -\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r_i}^{r_f} = \underbrace{\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_f}-\frac{1}{r_i}\right]}\ ***$$



I $\quad \Delta V = \frac{q}{4\pi\varepsilon_0}\Big[\underbrace{\frac{1}{r_f}-\frac{1}{r_i}}_{(-)}\Big]$

$r_i < r_f$

$\boxed{V = \frac{q}{4\pi\varepsilon_0 r}}$ POINT CHARGE $\quad \Leftarrow \quad V_f - V_i = \frac{q}{4\pi\varepsilon_0 r_f} - \frac{q}{4\pi\varepsilon_0 r_i}$

$r_f > r_i$

$V_f - V_i < 0$

$V_f < V_i$

$\Delta V = \frac{q}{4\pi\varepsilon_0}\Big[\underbrace{\frac{1}{r_f}-\frac{1}{r_i}}_{+}\Big]$

$r_i > r_f$

$V_f - V_i > 0\ ;\ V_f > V_i$

$\oplus - - - - \overset{i}{\circ} \longrightarrow \overset{f}{\circ}$

$V_i > V_f$

$V_f > V_i$

point charge

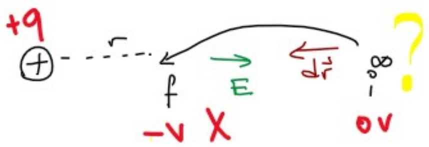
$$\Delta V = -\int_{r_i=\infty}^{r_f=r}\vec{E}\cdot d\vec{r} \Rightarrow -\int_\infty^r \frac{kq}{r^2}\,\underbrace{\hat{r}\cdot d\vec{r}}_{(-)} = -\int_i^f \frac{kq}{r^2}(-dr) = \int_\infty^r \frac{kq}{r^2}dr \overset{?}{=} -\left[\frac{kq}{r}\right]_\infty^r$$

???!!

LET'S DISCUSS it LATER!!

$\Delta V = V_f - V_i = -\frac{kq}{r} - \left[-\frac{kq}{\infty}\right]$

$V_f \neq -\frac{kq}{r}$

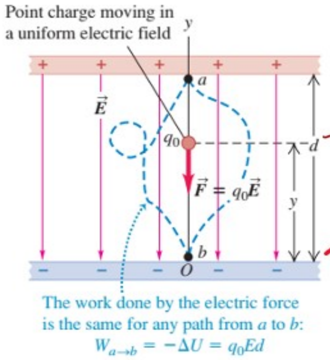


$$\Delta V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_f}-\frac{1}{r_i}\right] = V_f - V_i$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{1}{r_f} - \cancel{\frac{q}{4\pi\varepsilon_0 r_{i\to\infty}}} = V_f - V_i$$

$\cancel{\frac{q}{4\pi\varepsilon_0 r_{i\to\infty}}} \to 0, \quad V_i \to 0$

$$V_f = \frac{q}{4\pi\varepsilon_0 r_f} \Rightarrow \boxed{V(r) = \frac{q}{4\pi\varepsilon_0 r}}$$



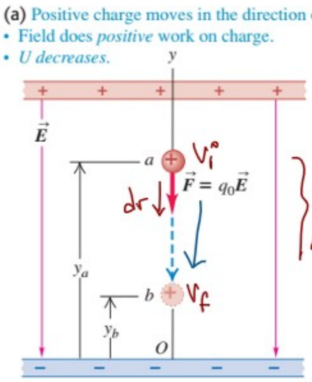
$U_i = qV_i$

$W > 0$ $\quad W = -\Delta U$

$V_i > V_F$

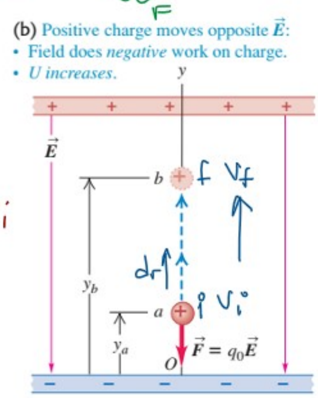
$U_f = qV_F$

$$q\Delta V = -q\int \underbrace{\vec{E}}_{F} \cdot d\vec{r} = -\int \vec{F} \cdot d\vec{r}$$



$\Delta V = V_f - V_i$

$\Delta V < 0$



$$W = -\Delta U = -q\Delta V$$

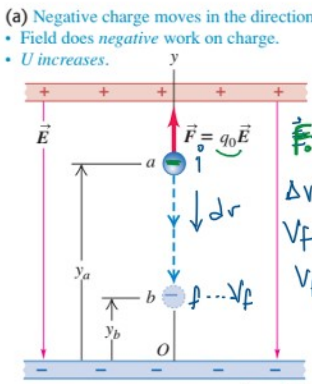
$\underbrace{\vec{E} \cdot d\vec{r}}_{(-)}$

$$\Delta V = -\underbrace{\int \vec{E}\, d\vec{l}}_{(-)} = (+)$$

$V_f > V_i$

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

$$W = \int \vec{F} \cdot d\vec{r} = \vec{F} \cdot \Delta\vec{r}$$



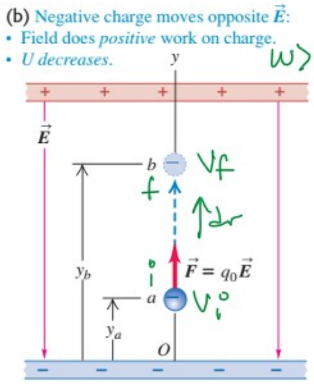
$W < 0$

$\vec{F} \cdot d\vec{r} < 0$

$\Delta V = -(-) = +$

$V_f - V_i > 0$

$V_f > V_i$



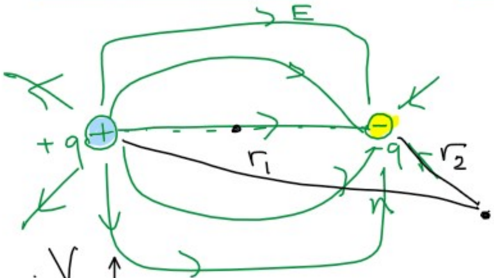
$W > 0$

$\underbrace{\int \vec{F} \cdot d\vec{r}}_{+}$

$\Delta V = -(+) < 0$

$V_f - V_i < 0$

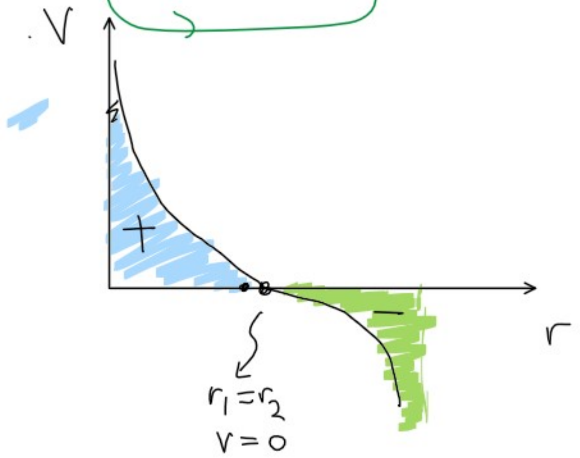
$V_f < V_i$

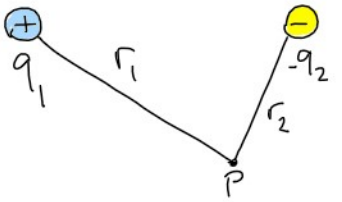


$$V(r) = \frac{kq}{r} = \frac{q}{4\pi\varepsilon_0 r}$$

$q \rightarrow +$, $q \rightarrow -$

$$V = V_+ + V_- = \frac{kq}{r_1} + \frac{k(-q)}{r_2} = kq\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

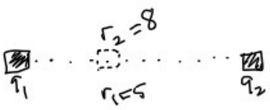




$$V_p = \frac{kq_1}{r_1} + \frac{k(-q_2)}{r_2} = k\left(\frac{q_1}{r_1} - \frac{q_2}{r_2}\right)$$

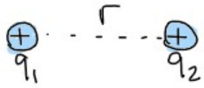
If $V_p = 0$ $\quad \frac{q_1}{r_1} = \frac{q_2}{r_2}$ $\quad \underbrace{r_1 = r_2 \frac{q_1}{q_2}}_{V=0}$ $\quad q_1 \to +$, $q_2 \to -$

grid= izgara ...



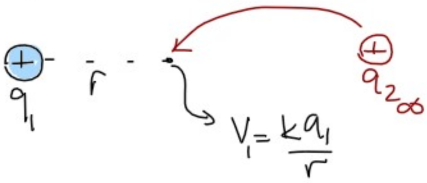
----> maybe some HW ?!

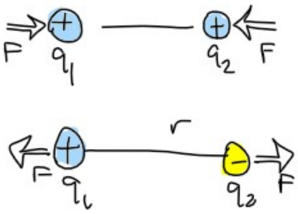
$qV = U$



what is the potential energy two put q_1 & q_2 r distance away from each other?

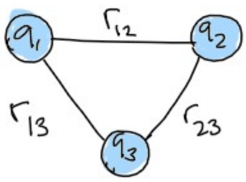
$q_2V_1 = U$ → energy



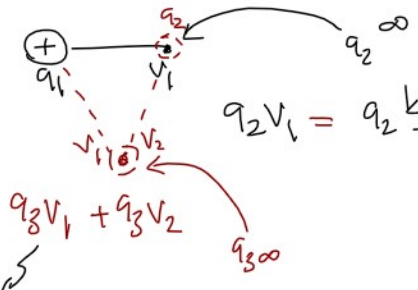


$U > 0$

$U < 0$



U = energy for this setup?



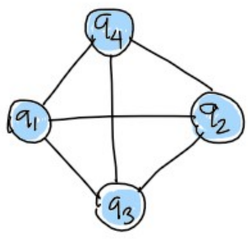
$$q_2V_1 = q_2\frac{kq_1}{r_{12}}$$

$$q_3V_1 + q_3V_2$$

$$q_3\frac{kq_1}{r_{13}} + q_3\frac{kq_2}{r_{23}}$$

$$\underbrace{U = k\frac{q_1q_2}{r_{12}} + k\frac{q_1q_3}{r_{13}} + k\frac{q_2q_3}{r_{23}}}_{\text{3 terms.}}$$

3 terms for 3 charge



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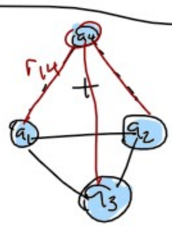


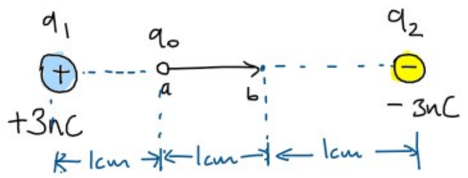
$$\underbrace{k\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right) + k\left(\frac{q_1q_4}{r_{14}} + \frac{q_2q_4}{r_{24}} + \frac{q_3q_4}{r_{34}}\right)}_{\text{6 terms for 4 charges}}$$

$$U = \sum_{j=1}^{N}\sum_{i=1}^{N}\frac{1}{2}\frac{q_iq_j}{r_{ij}} \quad \{i \neq j\} \qquad \binom{4}{2} = \frac{4(3)}{2} = 6 \text{ terms}$$

$$\left\{ U = \sum^{N}\sum^{N} \frac{kq_iq_j}{\ } \right\} \qquad \binom{10}{2} = \frac{10(9)}{2} = \underline{\underline{45 \text{ terms}}}$$

$$\left\{ U = \sum_{i=1}^{N} \sum_{j>i}^{N} \frac{k q_i q_j}{r_{ij}} \right\} \qquad \binom{10}{2} = \frac{10(9)}{2} = \underline{\underline{45 \text{ terms}}}$$

(ex)



q_0 charge ($q_0 > 0$) starts from $\underline{\underline{a}}$ ends up at $\underline{\underline{b}}$

$q_0 = +2nC$

a) $\Delta V = ?$ for this motion

$$\Delta V = V_f - V_i = V_b - V_a$$

$$V_b = V_+ + V_-$$

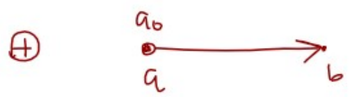
$$= \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = k\left(\frac{3nC}{2\times10^{-2}} + \frac{-3nC}{1\times10^{-2}}\right) = 9\times10^9 \times 3\times10^{-9}\left(\frac{100}{2} - \frac{100}{1}\right)$$

$$= 27(-50) = \underline{-1350\,V}$$

$$V_a = V_+ + V_- = \frac{kq_1}{r_1} + k\frac{q_2}{r_2} = 9\times10^9 \times 3\times10^{-9}\underbrace{\left(\frac{1}{10^{-2}} - \frac{1}{2\times10^{-2}}\right)}_{+50} = +1350\,V$$

$$\Delta V = V_b - V_a = -1350 - (+1350) = \underline{\underline{-2700\,V}}$$

b) $\Delta U = ?$ for this motion

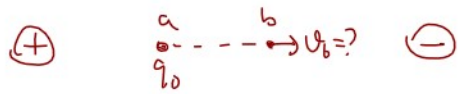


$$\Delta U = q\,\Delta V = (2\times10^{-9}C)(-2700\,V)$$

$$= -5400\times10^{-9}\,CV$$

$$\Delta U = -0.54\times10^{-5}\,J$$

c) m of q_0 $1ng = 10^{-9}kg$ $\quad v_a = 0 \quad v_b = ?$



$$W = \underline{-\Delta U} = \Delta K = K_b - K_a = \frac{1}{2}mv_b^2 - \frac{1}{2}mv_a^2 \rightarrow 0$$

$$\Delta U = -0.54\times10^{-5}\,J$$

$$W = -\Delta U = 0.54\times10^{-5} = \frac{1}{2}(10^{-9})\,v_b^2$$

$$1.08\times10^4 = v_b^2 \Rightarrow v_b = \sqrt{1.08}\times10^2\,m/s \simeq \underline{\underline{10\ \ ls}}$$

$$\left[V \rightarrow \underbrace{U}_{\text{pot. E}} = qV\right] \Rightarrow \left[\text{Volt} = \frac{J}{C}\right]$$

$$\underbrace{20\,eV}_{1.6\times10^{-19}C} \Rightarrow \text{Energy} = 20\,(1.6\times10^{-19})\,J \quad ; \quad 20\,\overset{10^{-3}}{m}eV; \quad 20\,\underset{10^9}{G}eV$$

$20\,eV \Rightarrow$ Energy $= \underbrace{20\,(1.6\times10^{-19})\,J}$; $20\,meV$; $20\,GeV$

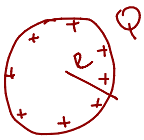
$\hookrightarrow 1.6\times10^{-19}\,C$ $\qquad \hookrightarrow 10^9$

$e = 1.6\times10^{-19}\,C$

Phys I $\Rightarrow \left\{ \begin{array}{l} W = -\Delta U \\ qV = U \;;\; F = qE \\ W = \int \vec{F}\cdot d\vec{r} \end{array} \right\} -\Delta(qV) = W = \int \vec{F}\cdot d\vec{r}$

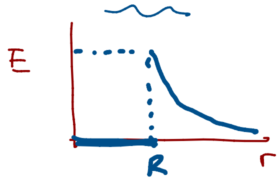
$-\cancel{q}\,\Delta V = \int \cancel{q}\vec{E}\cdot d\vec{r} \Rightarrow \boxed{\Delta V = -\int \vec{E}\cdot d\vec{r}}$

V for charged metal sphere (inside, outside)?



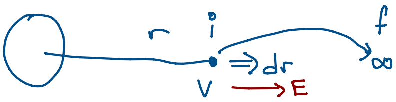
$\Delta V = -\int \vec{E}\cdot d\vec{r}$; $E\,(r>R) = \dfrac{Q}{4\pi\varepsilon_0 r^2}$; $E(r<R) = 0$

? Gauss Law



$r>R$

$\Delta V = -\int \vec{E}\cdot d\vec{r}$



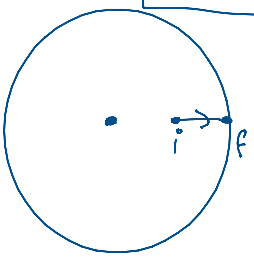
$\vec{E}\cdot d\vec{r} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\,\hat{r}\cdot d\vec{r} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\,dr$

$|\hat{r}||d\vec{r}|\cos 0$: 1, dr, 1

$V_f - V_i = \Delta V = -\int_r^\infty \dfrac{Q}{4\pi\varepsilon_0 r^2}dr = -\dfrac{Q}{4\pi\varepsilon_0}\int_r^\infty \dfrac{dr}{r^2} = -\dfrac{Q}{4\pi\varepsilon_0}\left[-\dfrac{1}{r}\right]_r^\infty = \dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{\infty} - \dfrac{1}{r}\right]$

$V(\infty) - V(r)$

$V(\infty) - V(r) = \dfrac{Q}{4\pi\varepsilon_0}\left[0 - \dfrac{1}{r}\right] \Rightarrow V(r>R) = \dfrac{Q}{4\pi\varepsilon_0 r} = \underline{k\dfrac{Q}{r}}$ $(r>R)$



$\Delta V = -\int_{r_i}^{r_f} \vec{E}\cdot d\vec{r}$; $E = 0$ (inside)

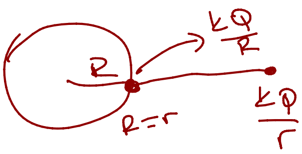
$\Delta V = 0 = V_f - V_i = 0$

$V_f = V_i$ (every point within the sphere has the same potential)

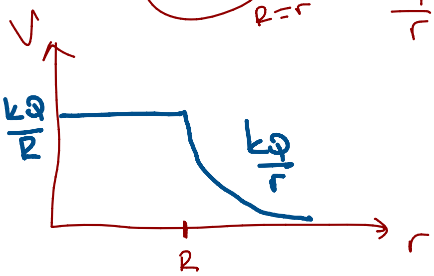
$\cancel{V(r<R) = 0}$

$V(r_f) = V(r_i)$

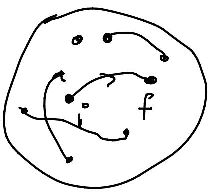
$\dfrac{kQ}{R} = V(R) = V(r)$



$V(r=R) = \dfrac{kQ}{R}$



within the sphere $V(r<R) = \dfrac{kQ}{R}$



$\Delta V = 0$

$\underline{q\,\Delta V = \Delta U} =$ no pot. energy difference within the sphere

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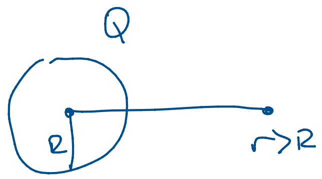
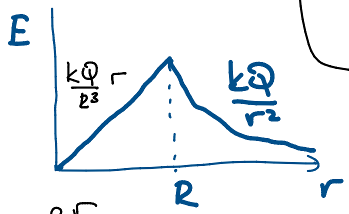
V of a uniformly charged insulating

q_0 = test charge

V of a uniformly charged insulating sphere



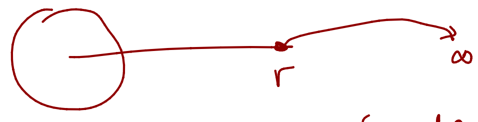


q_0 = test charge within the sphere
If you move q_0, you don't pay any energy for this motion

difference ΔV within the sphere

$$E(r<R) = \frac{kQ}{R^3}r = \frac{Q\,r}{4\pi\varepsilon_0 R^3} = \frac{\rho r}{3\varepsilon_0}$$

$$\left\{ \rho = \text{charge density} = \frac{Q}{\frac{4}{3}\pi R^3} \right\}$$



$$\Delta V = -\int \vec{E}\cdot d\vec{r}$$

$$V(\infty) - V(r) = -\int \frac{kQ}{r^2}\,dr$$

$$\left\{ V(r) = \frac{kQ}{r} \right\}$$

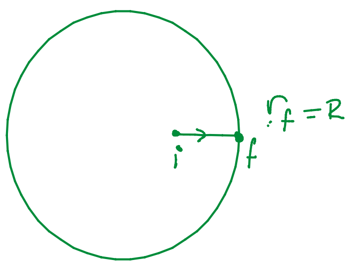
$$V(r=R) = \frac{kQ}{R} = \frac{Q}{4\pi\varepsilon_0 R}$$

r<R within insulating sphere

$$\Delta V = -\int \vec{E}\cdot d\vec{r} = -\frac{\rho}{3\varepsilon_0}\int_{r_i}^{r_f} \underbrace{\vec{r}\cdot d\vec{r}}_{\int r\,dr} = \quad \vec{dr} \;\; \vec{r}$$

$$V(r_f) - V(r_i) = -\frac{\rho}{3\varepsilon_0}\left[\frac{r^2}{2}\right]_{r_i = r}^{r_f = R}$$

$$r_f = R$$



$$V(R) - \underbrace{V(r)} = -\frac{\rho}{3\varepsilon_0}\left[\frac{R^2}{2} - \frac{r^2}{2}\right]$$

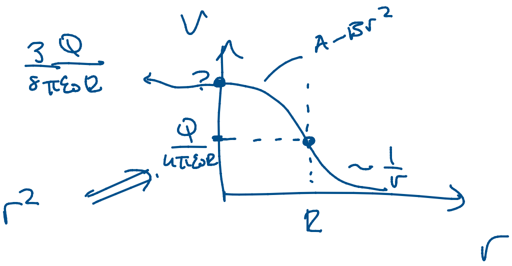
$$\rho = \frac{Q}{\frac{4}{3}\pi R^3} \Rightarrow \frac{\rho}{3\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 R^3}$$

$$V(R) + \frac{\rho}{3\varepsilon_0}\frac{R^2}{2} - \frac{\rho}{3\varepsilon_0}\frac{r^2}{2} = V(r)$$

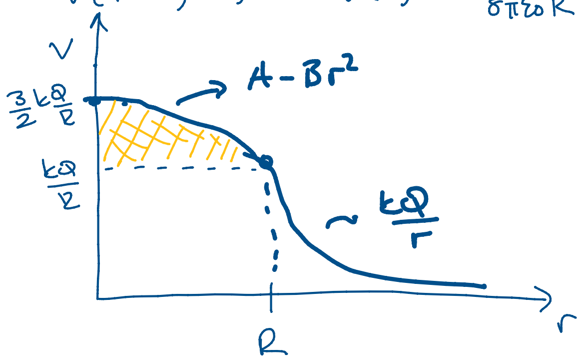
$$\frac{Q}{4\pi\varepsilon_0 R} + \frac{Q}{4\pi\varepsilon_0 \cancel{R^3}}\frac{\cancel{R^2}}{2} - \frac{Q}{4\pi\varepsilon_0 R^3}\frac{r^2}{2} = V(r)$$

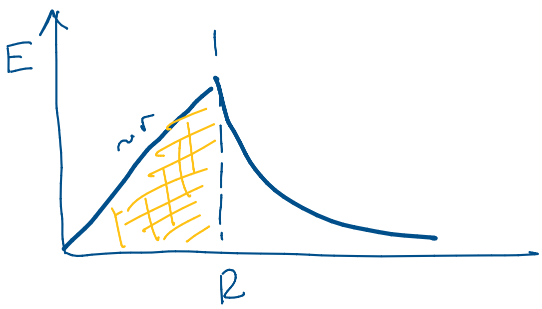
$$\frac{Q}{4\pi\varepsilon_0 R} + \frac{Q}{8\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0 R^3}\frac{r^2}{2} = V(r)$$

$$\frac{3Q}{8\pi\varepsilon_0 R} - \frac{Q}{8\pi\varepsilon_0 R^3}r^2 = V(r) = A - Br^2$$



$$V(r=0) \Rightarrow V(0) = \frac{3Q}{8\pi\varepsilon_0 R}$$





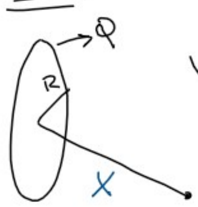
$\Delta V = -\int \vec{E}\cdot d\vec{r} \Rightarrow$ if we know $\vec{E}$: $\underline{\underline{\Delta V}}$ ✓

$V(r) = \frac{kq}{r}$

dq - - - ; $k\frac{dq}{r}$

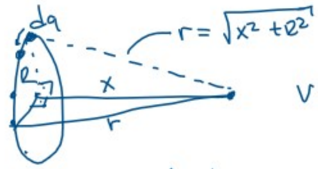
$\Rightarrow$

$\Rightarrow$



$V(r) = ?$

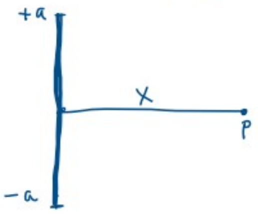
$V(r) = \int k\frac{dq}{r}$



$V = \int k\frac{dq}{\sqrt{x^2+R^2}} = \frac{k}{\sqrt{x^2+R^2}}\underbrace{\int dq}_{Q} = \underline{\underline{\frac{kQ}{\sqrt{x^2+R^2}} = V(x)}}$

move dq to another point on the RING
r = does NOT CHANGE

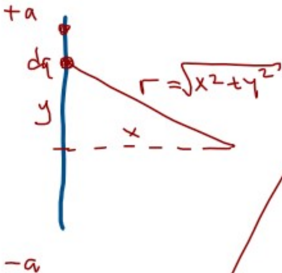
ROD, Q , $L=2a$



$V_p = ?$

$V = \int \frac{k\,dq}{r}$

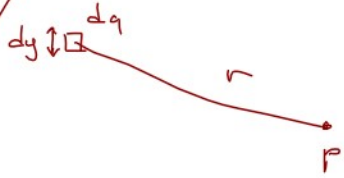
$\underbrace{d\vec{E} = \int \frac{k\,dq}{r^2}\,dr\,\hat{r}}$ vector, - tedious



$\int \frac{k\,dq}{\sqrt{x^2+y^2}}$ $\Rightarrow$ charge $\leftrightarrow$ length $\Rightarrow$ charge density

$dq \leftrightarrow dy$

$\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{2a} = \frac{dq}{dy}$



$\lambda = \frac{dq}{dy}$

$dq = \lambda\,dy$

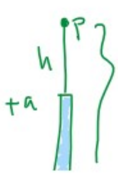
$\int k\frac{\lambda\,dy}{\sqrt{x^2+y^2}} = k\lambda\int_{-a}^{+a}\frac{dy}{(x^2+y^2)^{1/2}} \rightarrow$ integral table $\Rightarrow \underline{k\lambda \ln\left(\frac{\sqrt{a^2+x^2}+a}{\sqrt{a^2+x^2}-a}\right)}$

$\lambda \rightarrow \frac{Q}{2a}$

$\frac{dy}{y} \sim \ln y$

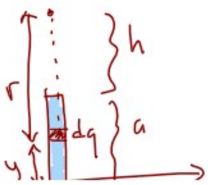
$\underline{V_p = \frac{kQ}{2a}\ln\left[\frac{(a^2+x^2)^{1/2}+a}{(a^2+x^2)^{1/2}-a}\right]}$

$x \gg 2a \Rightarrow V_p \rightarrow \frac{kQ}{x}$



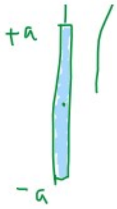
$V_p = ?$

$V = \int \frac{k\,dq}{\underline{\underline{r}}}$

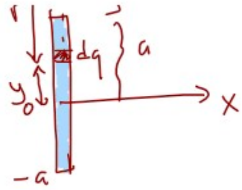


$a + h = r + y$

$r = a + h - y$

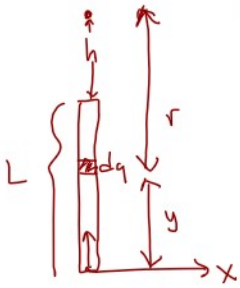


$$k\int_{-a}^{+a}\frac{\lambda\,dy}{a+h-y} \longrightarrow \ln(\ldots)$$



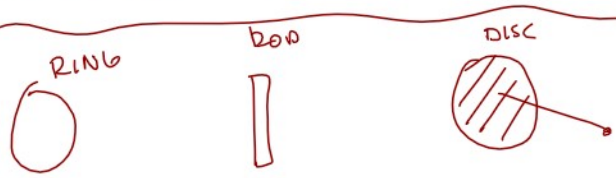
$r = a+h-y$

$dq = \lambda\,dy$



$L+h = r+y$

$$k\int_{0}^{L}\frac{\lambda\,dy}{L+h-y} = V_p$$

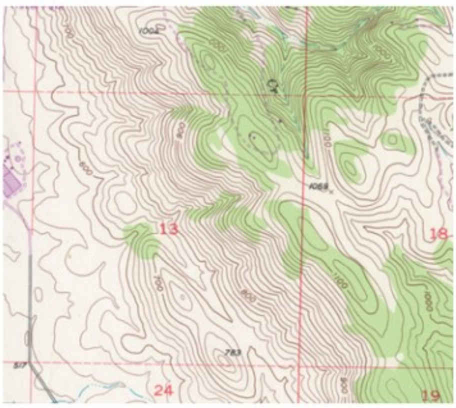


$$V = \int \frac{k\,dq}{r}$$

EQUIPOTENTIAL SURFACES (EŞ POTENSIYEL YÜZEYLERİ)

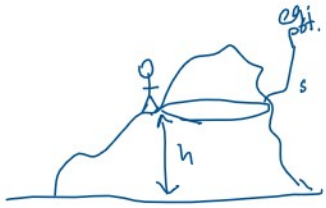
↳ surface, place that has the same potential value on it.

23.22 Contour lines on a topographic map are curves of constant elevation and hence of constant gravitational potential energy.

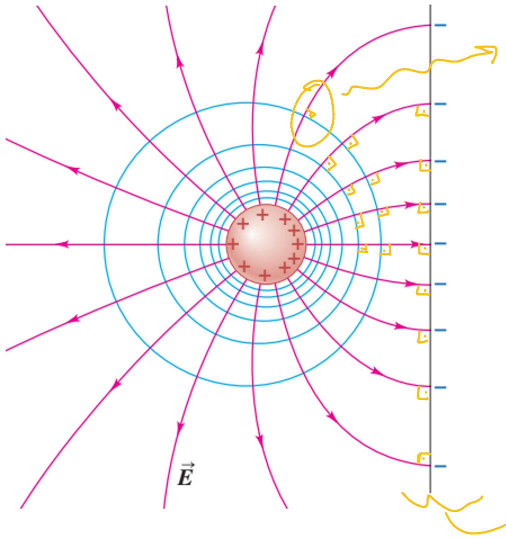


→ lines are drawn such that they have the same potential energy $mgh = U$

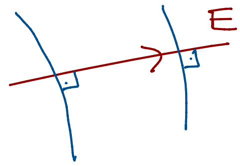
we can use this analogy to understand eq. pot. surfaces.



23.24 When charges are at rest, a conducting surface is always an equipotential surface. Field lines are perpendicular to a conducting surface.

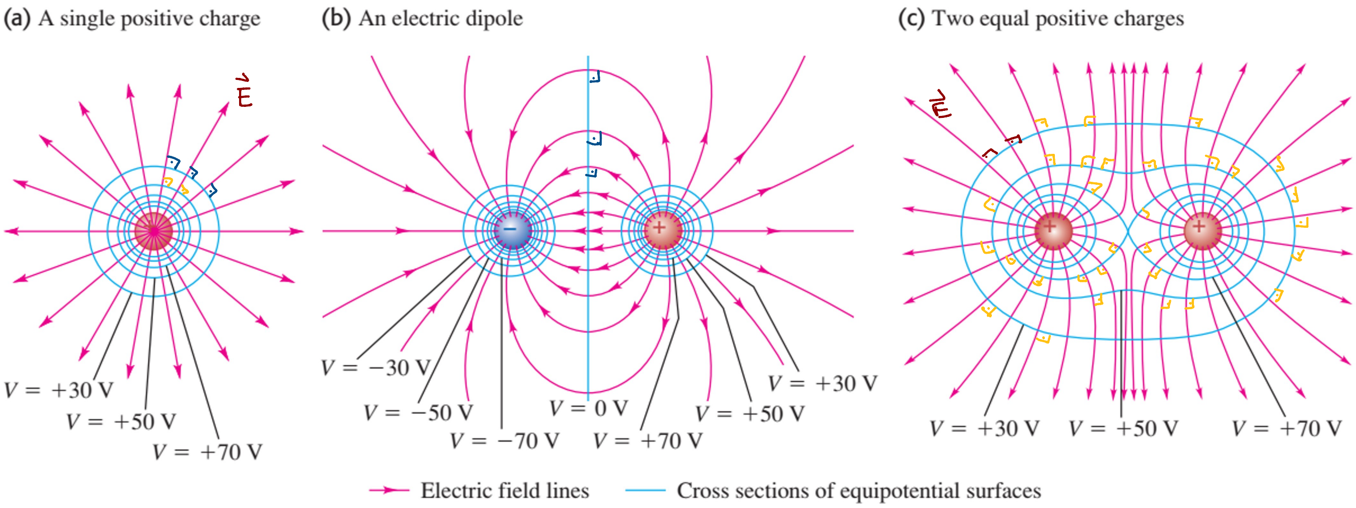


$\vec{E}$ lines are $\perp$ perpendicular to each eq. pot. surface



metal surfaces are eq-pot. surfaces

metal



$$\Delta V = -\int_i^f \vec{E}\cdot d\vec{r} = V_f - V_i \Rightarrow V_i - V_f = \int_i^f \vec{E}\cdot d\vec{r}$$

$$\Delta V = \int_i^f dV = -\int_i^f \vec{E}\cdot d\vec{r} \Rightarrow dV = -\vec{E}\cdot d\vec{r} = -\vec{E}\cdot d\vec{l}$$

$$-\vec{E}\cdot d\vec{r} = -(E_x\hat{i} + E_y\hat{j} + E_z\hat{k})\cdot(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}) = dV$$

$$E_x\,dx + E_y\,dy + E_z\,dz = -dV \qquad \underbrace{V(\underline{x}, \underline{y}, \underline{z})}$$

$\vec{E}$ from V ?? $\qquad V \longrightarrow E = ?$

$$E_x = -\frac{\partial V}{\partial x} \; ; \; E_y = -\frac{\partial}{\partial y}V \; ; \; E_z = -\frac{\partial}{\partial z}V \; ;$$

$\frac{\partial}{\partial x}$ } partial derivative kısmî türev

$$\vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right)$$

$$\vec{\nabla} \equiv \text{gradient} \equiv \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$$

$$\vec\nabla \equiv \text{gradient} \equiv \left(\frac{\partial}{\partial x}\hat\imath + \frac{\partial}{\partial y}\hat\jmath + \frac{\partial}{\partial z}\hat k\right)$$

"del"

$$\boxed{\vec E = -\vec\nabla V}$$

ex)

$q \;\text{- - - } r \text{ - - -}\; p$

$r = \sqrt{x^2+y^2+z^2}$

$V(r) = \frac{kq}{r}$ $\quad E = ?$

$$E = -\frac{\partial}{\partial r}V(r) = -kq\frac{d}{dr}\left(\frac{1}{r}\right) = -kq(-1)r^{-2} = \frac{kq}{r^2} = E$$

ex

Q RING

$V_p = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2+R^2}}$ $\quad E = ?$

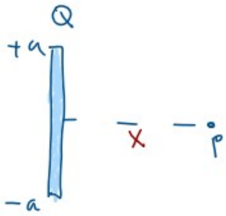
R = const

x = variable

$$E = -\hat\imath\frac{\partial V}{\partial x} = -\hat\imath\frac{Q}{4\pi\varepsilon_0}\frac{d}{dx}(x^2+R^2)^{-1/2}$$

$$\vec E_p = \frac{Q}{4\pi\varepsilon_0}\frac{x}{(x^2+R^2)^{3/2}}\hat\imath = -\hat\imath\frac{Q}{4\pi\varepsilon_0}(x^2+R^2)^{-3/2}\left(-\frac{1}{2}\,2x\right.$$

ch 22 c



$$V_p = \frac{kQ}{2a}\ln\left(\frac{\sqrt{a^2+x^2}+a}{\sqrt{a^2+x^2}-a}\right)$$

$$E = -\frac{\partial V}{\partial x} = \ldots$$

ex) $V = Ax^2y$ $\quad \vec E = ?$

$$\vec E = -\hat\imath\frac{\partial V}{\partial x} - \hat\jmath\frac{\partial V}{\partial y} - \hat k\frac{\partial V}{\partial z}$$

$$\vec E = -\hat\imath(A2xy) - \hat\jmath(Ax^2) - k\,0$$

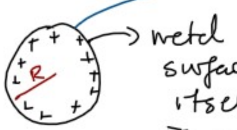
$W = -\Delta U$; $U = qV$ $\Rightarrow$ $\Delta V = -\int \vec E\cdot d\vec r$ ⁂

$\vec E = -\vec\nabla V$

$\int k\frac{dq}{dr} = V_p$

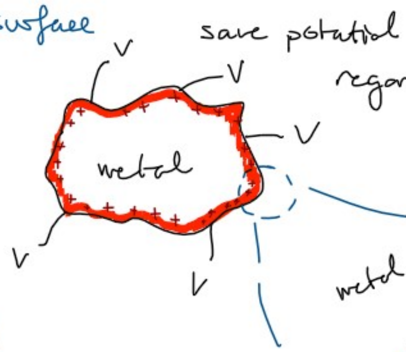
$$\boxed{\text{Equipotential surfaces are perpendicular to } \vec E \text{ field lines}}$$

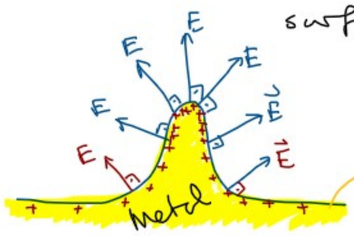
V = constant at the surface



surface itself is an equipotential surface.

same potential regardless of the shape

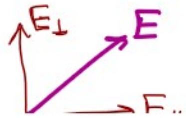


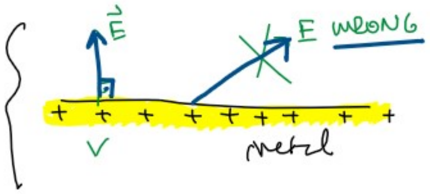


metal surface is Equipotential surface !!

electrostatic situation

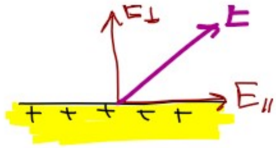
what if we have





electrostatic situation

$E \perp$ to surface $\Longrightarrow$ what if we have non perp. E field



if $E_{\parallel}$ exists then q charges will flow in $E_{\parallel}$ direction

It's NOT ELECTROSTATIC $\Longleftarrow$ if q flow