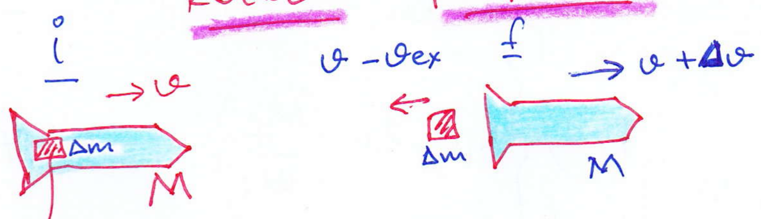
Momentum ; Center of Mass

Rocket Propulsion



v_{ex} = exhaust speed of fuel.

push out the fuel at the same speed $= v_{ex}$

as the fuel is exhausted (pushed) the rocket will have new speed of $v + \Delta v$

v_{ex} with respect to the Rocket

M = mass of rocket

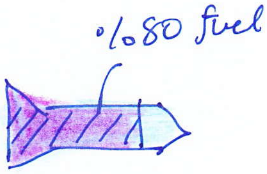
Δm = " of fuel exected!

$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$(M + \Delta m)\vec{v} = \Delta m(\vec{v} - \vec{v}_{ex}) + M(\vec{v} + \Delta\vec{v})$$

$$Mv + \Delta m v = \Delta m v - \Delta m v_{ex} + Mv + M\Delta v$$

$$M\Delta v = \Delta m v_{ex}$$



Technically total mass rocket $= M + \Delta m = M_T$

$M_T = M_B + M_{fuel}$

Total mass of rocket

t	M_T	M_B	Δm	M_{fuel}
0	10000	1000	0	9000
1	9900	1000	100	8900
2	9800	1000	$\boxed{100}$	8800

100

100

$\boxed{\Delta m = -\Delta M \quad M_B = \text{Bare weight of rocket w/o fuel}}$

$M_T = M_B + M_{fuel}$ Δm $\Delta m = -\Delta M_T$

M : total mass of rocket.

$$\boxed{M \Delta v = -\Delta M v_{ex}}$$

$$\boxed{M\, dv = -dM\, v_{ex}}$$

as the fuel mass ejected; total mass change is negative

$\hookrightarrow dm = -dM$

$$\boxed{M\,dv = -dM\,v_{ex}} \quad ; \quad v_{ex} = \text{const.}$$

$$\int_i^f -\frac{dv}{v_{ex}} = \int_i^f \frac{dM}{M}$$

$$-\frac{\Delta v}{v_{ex}}\Big|_i^f = \ln M \Big]_i^f$$

$$\ln M_f - \ln M_i = \frac{-(v_f - v_i)}{v_{ex}}$$

$$-v_{ex} \ln\left(\frac{M_f}{M_i}\right) = v_f - v_i$$

$$\boxed{v_f - v_i = v_{ex} \ln \frac{M_i}{M_f}}$$

$M_i > M_f$; fuel is exhausted.

$$\frac{d}{dt}(M\,dv) = -\frac{d}{dt}(dM\,v_{ex})$$

$$\frac{d\vec{P}}{dt} = -\frac{d\vec{P}}{dt}$$

$$\overset{\vec{a}}{\left| M \frac{dv}{dt} \right|} = \left| v_{ex} \frac{dm}{dt} \right|$$

$$\vec{F} = \frac{dM}{dt} v_{ex} \quad ; \quad a = \frac{dv}{dt} = \frac{v_{ex}}{M}\left(\frac{dm}{dt}\right)$$

force on rocket

---

ex.) A rocket ejects fuel in the 1^{st} second; it eject $\frac{1}{120}$ of its initial mass at a speed of 2400 m/s.

$\vec{a} = ?$ acc. of rocket.

$$\frac{dM}{dt} = \frac{\left(\frac{M}{120}\right)}{1\,\text{second}}$$

$$\boxed{M\,dv = -dM\,v_{ex}}$$

$$M\,a = -v_{ex} \frac{dM}{dt}$$

$$M\,a = (2400)\frac{M}{120} \Rightarrow \underline{a = 20\ m/s^2} \simeq 2g$$

②

b) If $v_i = 0$ m/s and the $\frac{3}{4}$ of the mass of rocket is fuel; fuel is consumed at const rate in 90 seconds.

$v_f = ?$ of the rocket.

$$M\,dv = -dM\,v_{ex} \Rightarrow v_f - v_i = v_{ex} \ln \frac{M_i}{M_f}$$

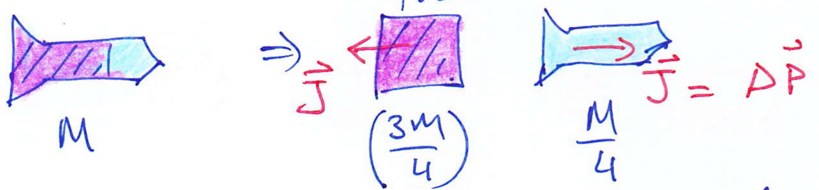
$$m_f = m_i - M_{fuel} = M - \frac{3}{4}M = \frac{M}{4}$$

$$v_f = 2400 \ln \frac{M}{\left(\frac{M}{4}\right)} = 2400 \ln 4$$

$$v_f = 3327.\ \text{m/s} \quad @\ 90\text{s}$$

c) If $M = 1000$ kg fuel is ejected for 90s;

what's the propulsion force on the rocket?



$$\frac{\vec{J}}{\Delta t} = \vec{F} = \frac{\Delta \vec{P}}{\Delta t}$$

$$F = M\frac{dv}{dt} = v_{ex}\frac{dM}{dt} \Rightarrow F = (2400)\frac{\left(\frac{3M}{4}\right)}{90\text{s}} =$$

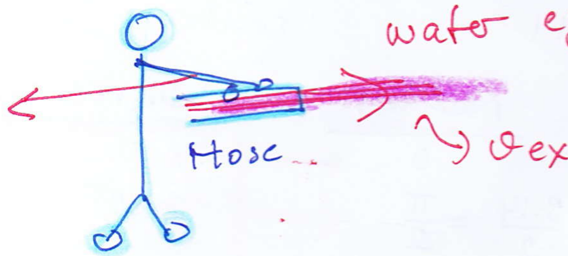
$$= 2400\,\frac{\frac{3000}{4}}{90}$$

$$= 20\,000\ \text{N}$$

$$= 20\ \text{kN}$$

$$M\frac{dv}{dt} = F = v_{ex}\frac{dM}{dt}$$; Fire fighters (itfayei)

$$\frac{dM}{dt} = \left[\frac{kg}{s}\right] \cdot \left[\frac{m}{s}\right] = v_{ex}\ F$$



ex) if water is exhausted at $\frac{3600L}{\text{minute}}$ from a fire hose; the force applied on the hose is 600 N; whats v_{ex} =?

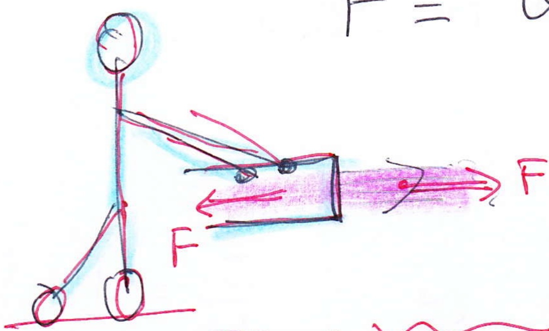
$$\frac{L}{min} \sim \frac{kg}{s}$$

1 L ≃ 1 kg of water
1 min = 60 s

$$3600 \frac{L}{min} = \frac{dM}{dt} = \frac{3600\,kg}{60\,s} = 60 \frac{kg}{s}$$

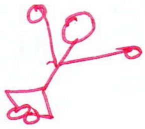
$$F = v_{ex} \frac{dM}{dt} \Rightarrow 600\,N = v_{ex}\, 60 \frac{kg}{s}$$

$$v_{ex} = 10\ m/s$$



600 N   600 N ⇒ 10 m/s = v_{ex}

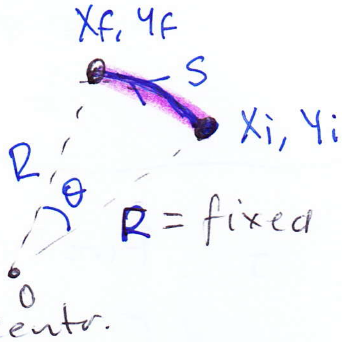
Ch8 Finished



Chapter 9 Rotation of Rigid Objects

Rotation

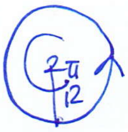
2 dimentional motion



arclength of path

$$\boxed{S = R\theta}$$

θ ≡ in terms of radians.



$R(2\theta)$ = circumfere of circle

π = 3.14

radians	degree
0	0
1.67 = $\frac{3.14}{2}$ = $\frac{\pi}{2}$	90°
π (3.14)	180
$\frac{3\pi}{2}$ ($\frac{3}{2}$ 3.14)	270
2π (2 3.14)	360°

$$180° = \pi = 3.14\ rad.$$

$$1° = \frac{3.14\ rad}{180}$$

$$\frac{180°}{3.14} = 57.3° = 1\ rad$$

rad ⇒ unitless number

$$\boxed{1\ \text{rad} = \frac{180}{\pi} = 57.3^\circ}$$

angular displacements will be interms of radians.

Linear motion		Rotational motion	
X	⟷	θ	= angular displacement
v	⟷	ω	= angular velocity
a	⟷	α	= angular acceleration

$$X \xrightarrow{\frac{d}{dt}} v \xrightarrow{\frac{d}{dt}} a \qquad \theta \xrightarrow{\frac{d}{dt}} \omega \xrightarrow{\frac{d}{dt}} \alpha$$

α is related to a_t = tangential acceleration.

$\theta \Rightarrow \omega$ (omega) $= \dfrac{d\theta}{dt}$ (instantaneous angular velocity)

$\bar{\omega} = \omega_{av} = \dfrac{\Delta\theta}{\Delta t}$ (average ang. velocity)

$\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$ (instantaneous angular acc.)

$\bar{\alpha} = \alpha_{av} = \dfrac{\Delta\omega}{\Delta t}$ (average ang. accelration)

(alpha)

$\theta \Rightarrow [\text{rad}]$; $\omega = \left[\dfrac{\text{rad}}{s}\right]$; $\alpha = \left[\dfrac{\text{rad}}{s^2}\right]$

⇒ 2π

θ

1 rotation
1 revolution } 6.28 rad

10 rpm = 10 rotation per minute = ω

10 rpm = $\dfrac{10 \times 2\pi}{60\,s}$ rad

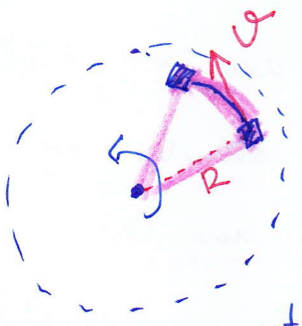
$\boxed{10\ \text{rpm} = 1.05\ \dfrac{\text{rad}}{s}}$

when $a = \text{const}$ rotation $\alpha = \text{const}$

$$x_f = x_i + v_i t + \frac{1}{2} a t^2 \longrightarrow \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$v_f = v_i + a t \longrightarrow \omega_f = \omega_i + \alpha t$$

$$v_f^2 = v_i^2 + 2a\,\Delta x \longrightarrow \omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta \checkmark$$



$R = \text{const}$

$$x \simeq s = R\theta$$

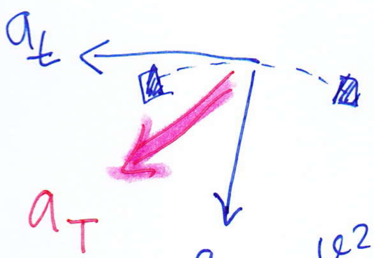
$$\frac{dx}{dt} = \cancel{\frac{dR}{dt}}^{\,0}\,\theta + R\frac{d\theta}{dt}$$

$$\boxed{v = R\omega} \; ; \; \boxed{x = R\theta}$$

$$\frac{d}{dt}$$

$$\frac{dv}{dt} = R\frac{d\omega}{dt}$$

$$\boxed{a_t = R\,\alpha}$$ (tangential acceleration)



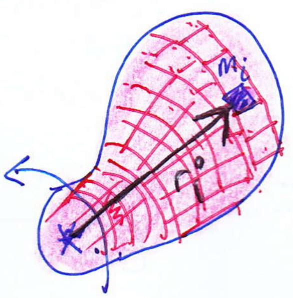
$$a_r = \frac{v^2}{R}$$ (radial acc.)

$$a_t \neq a_r = \frac{v^2}{R}$$

$$\boxed{\vec{a}_T = \vec{a}_r + \vec{a}_t}$$

non uniform circular motion

ENERGY in ROTATIONAL MOTION



KE = ? of rotating object. $\frac{1}{2} m v^2 = K$

$$K_i = \frac{1}{2} m_i v_i^2 \; ; \; \sum K_i = K \checkmark$$

every little piece will have K_i kinetic energy.

~~it will~~ all K_i will be different.

$\frac{1}{2} m_i v_i^2 = K_i$; $v_i = r_i \omega = \omega r_i$

every m_i will have SAME ω; different r_i value.

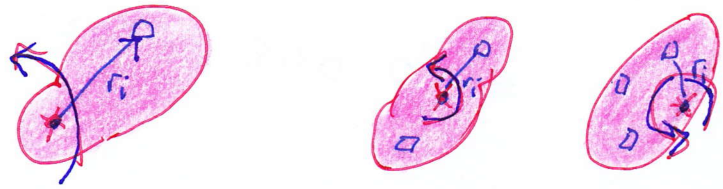
$K_i = \frac{1}{2} m_i (\omega r_i)^2$; $K = \sum K_i$

$$K \equiv \frac{1}{2} I \omega^2 \Longleftarrow = \sum_i \frac{1}{2} m_i r_i^2 \omega^2$$

I (moment of inertia) (eylemsizlik momenti)

$$\boxed{I = \sum m_i r_i^2}$$

moment of inertia.



moment of inertia depends on the rotation axis

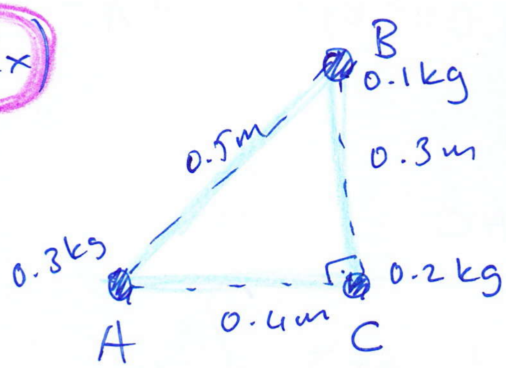
$K = \frac{1}{2} I \omega^2 \;\langle \simeq \rangle\; K = \frac{1}{2} m v^2$

m (linear) $\rightsquigarrow$ I (rotation)

$\perp$ perpendicular

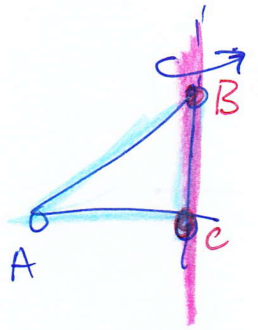
r is the $\perp$ distance to rotation axis.

Ex



system is composed of 3 masses and distributed as in the figure.

$I_{BC} = ?$ when the system is rotated along BC Line?



$$I_{BC} = \sum m_i r_i^2 = m_A r_A^2 + m_B r_B^2 + m_C r_C^2$$

$$= 0.3 (0.4)^2 + (0.1)\, 0^2 + (0.2)\, 0^2$$

$$\boxed{I_{BC} = 0.012 \text{ kgm}^2}$$

Rotational Motion Cnt'd

11.01.21

1

$\theta = [\text{rad}]$ (ang. displacement) $\Leftrightarrow$ x

$\omega = \left[\frac{\text{rad}}{s}\right]$ ang. inst. velocity $\Leftrightarrow$ v

$\alpha = \left[\frac{\text{rad}}{s^2}\right]$ ang. inst. acce. $\Leftrightarrow$ a_t (tangential acc.)

$x = S = R\theta$

$v = R\omega$

$a_t = R\alpha$

$a_t \neq \frac{v^2}{R} = a_r$

$a_t = \frac{d|\vec{v}|}{dt}$

$I \equiv$ moment of inertia $= [\text{kg}\,m^2]$

$I = \sum m_i r_i^2$ (point like masses)

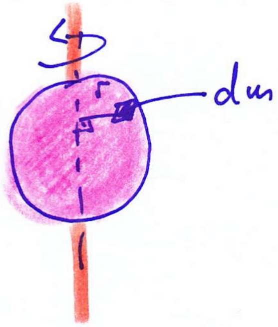
$I = m_A r_A^2 + m_B r_B^2 + m_C r_C^2$

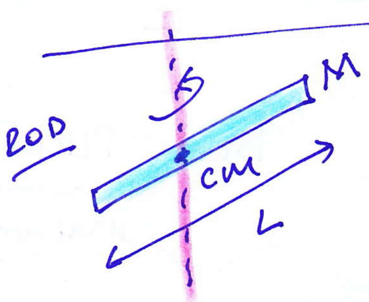
I is a measure of "inertial mass to rotate"

if $I \uparrow$, it's harder to rotate.

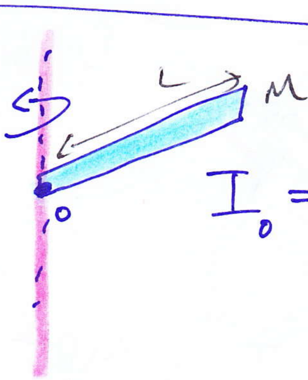
if masses are not point like $\Rightarrow$ rod, rectangle, sphere, cylinder $\Big\}$ $I = \int dm\, r^2$

we will not ask you to derive I from integral.





$I_{cm} = \frac{ML^2}{12}$ $(\Leftarrow I = \int r^2 dm)$



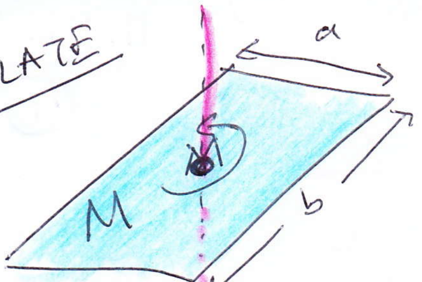
$I_0 = \frac{ML^2}{3}$

$I_{cm} < I_0$

easier to rotate.

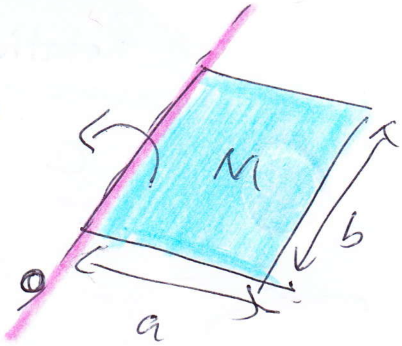
parallel axis theorem

PLATE



axis thru center

$$I_{cm} = \frac{M(a^2+b^2)}{12}$$

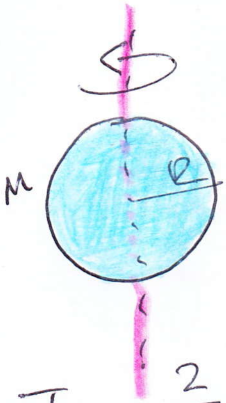


$$I_o = \frac{Ma^2}{3}$$

axis along one edge.

sphere

HOLLOW SPHERE (shell)
(İçi boş)



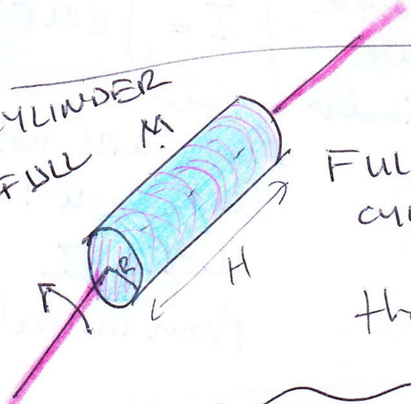
$$I = \frac{2}{3} MR^2$$

$$I = \int r^2 dm$$

FULL SPHERE
(İçi dolu)

$$I = \frac{2}{5} MR^2$$

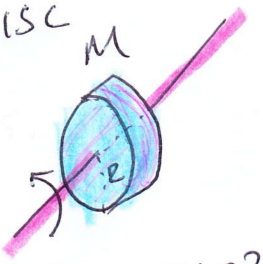
CYLINDER FULL



FULL CYLINDER

$$I = \frac{MR^2}{2} \Leftrightarrow$$

DISC



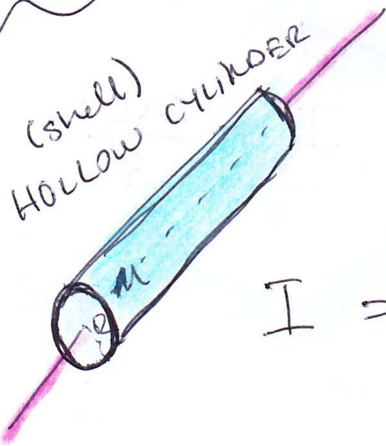
$$I = \frac{MR^2}{2}$$

the height, (length) does not matter
M rotation cylinder

$$I_{cylinder} = I_{disc}$$

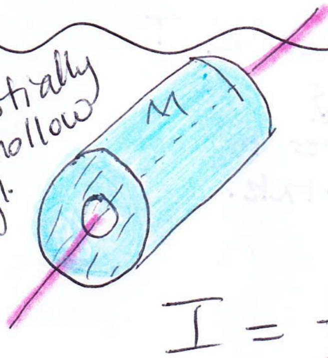
DISC ≡ PULLEY
makara

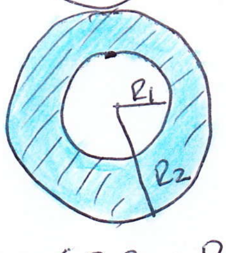
(shell) HOLLOW CYLINDER



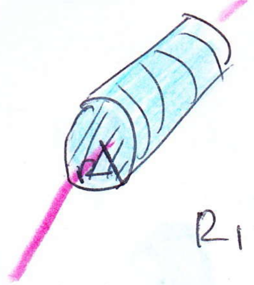
$$I = MR^2$$

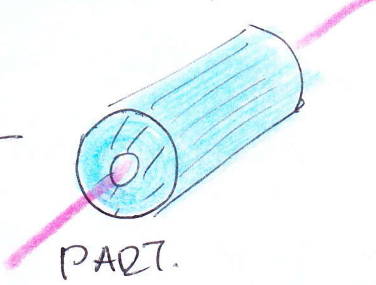
partially hollow cyl.

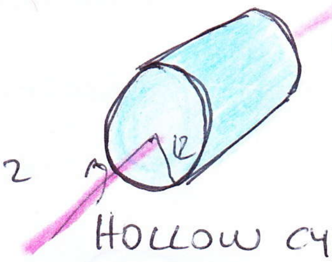




$$I = \frac{1}{2} M (R_1^2 + R_2^2)$$







$\leftarrow$ $R_1 = 0$ $\rightarrow$ $R_1 = R_2$

FULL CYL.   PART. HOLLOW CYL.   HOLLOW CYL

$R_1 = 0$
$R_2 = R$

$$I = \frac{1}{2} M(R_1^2 + R_2^2) \Rightarrow \frac{M(R^2+R^2)}{2} = MR^2 \quad \checkmark$$

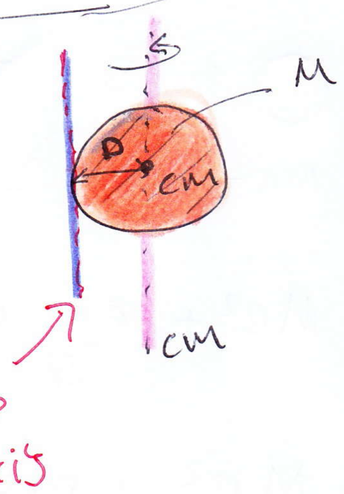
$R_1 = R_2 = R$

$$\Leftarrow \quad I = \frac{M(0^2+R^2)}{2}$$

$$I = \frac{MR^2}{2} \quad \checkmark$$

PARALLEL AXIS THEOREM

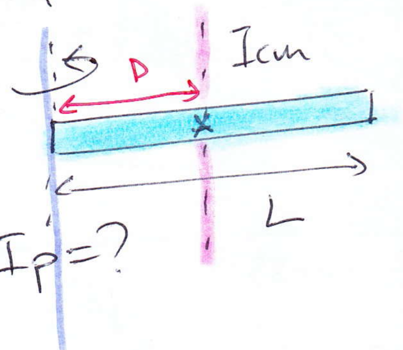
If you know I_{cm} moment of inertia on an axis going thru the center of mass object.



$$\boxed{I_p = I_{cm} + MD^2}$$

(ex) If I_{cm} of a rod is given $I_{cm} = \frac{ML^2}{12}$

I of the rod rotating at the edge of the rod.



$$I_p = I_{cm} + MD^2$$

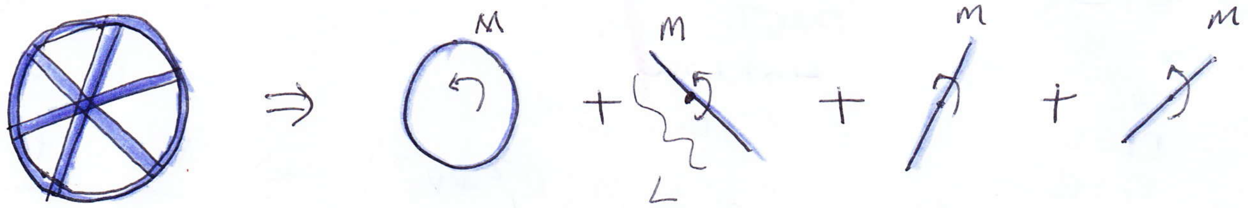
$$D = \frac{L}{2}$$

$$= \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2$$

$$= ML^2\left(\frac{1}{12} + \frac{1}{4}\right) = ML^2\frac{4}{12}$$

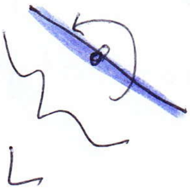
$$\boxed{I_p = \frac{ML^2}{3} \quad (\text{edge})}$$

(9.33) wagon wheel; find the moment of inertia when the wagon wheel rotating from its CM?



= ring + 3 rods.

$I_{Ring} = MR^2$ ($\simeq$ hollow cylinder)



$\frac{ML^2}{12} = I_{rod}$ $\Rightarrow$  $\Rightarrow$  + 3 

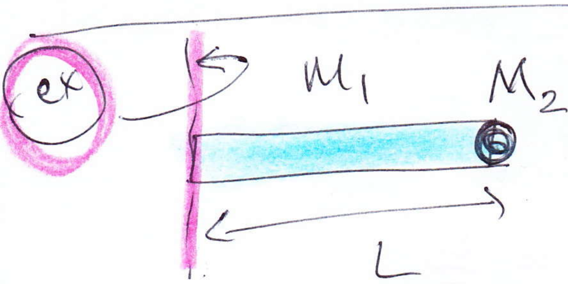
$MR^2 + 3\frac{mL^2}{12}$

$L = 2R$

$MR^2 + \cancel{3}\frac{m}{4}4R^2$

$\boxed{(M+m)R^2} \Leftarrow \boxed{I_{wagon\ wheel} = MR^2 + mR^2}$

ex



rod + point mass ; $I_{total} = \frac{M_1L^2}{3} + M_2L^2$

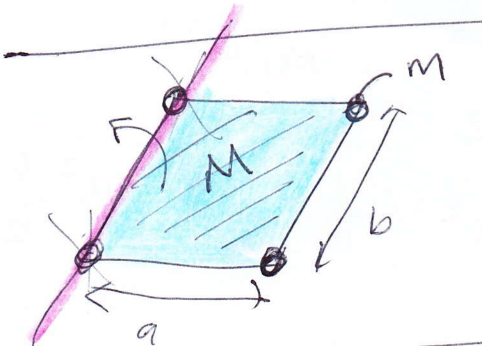
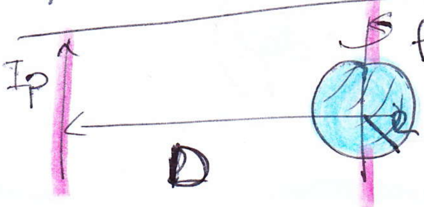


plate + 4 point mass

$I_{total} = \frac{Ma^2}{3} + \cancel{2m0^2} + 2ma^2$ ✓



full sphere

$I_p = I_{cm} + MD^2$

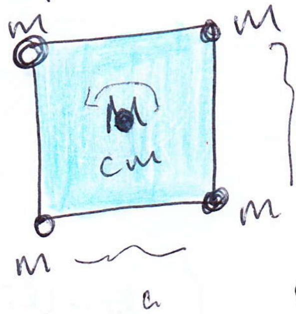
$= MR^2\frac{2}{5} + MD^2$

parallel axis theorem.

Becareful; allways take $\underline{\underline{r}}$ to be perpendicular distance betwen rotation axis & the object. ③

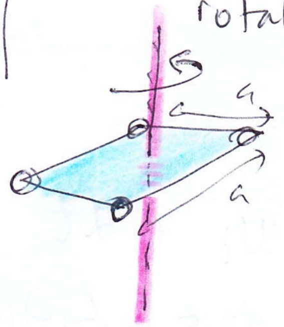
square plate

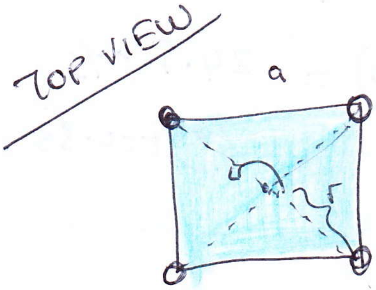
ex



4 point mass + square.

rotating it from cm thru the square plate

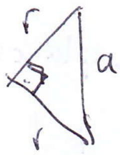




square plate

$$I_{sq} = \frac{M}{12}\left(a^2 + a^2\right) = \frac{Ma^2}{6}$$

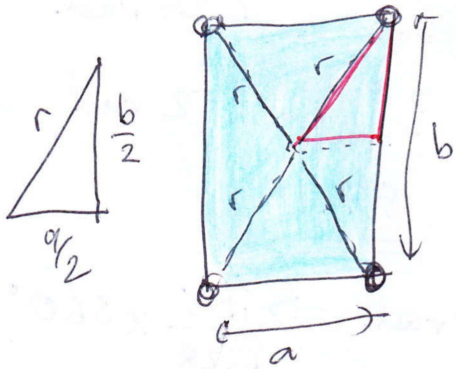
each mass distance to rotation axis is r



$$r = \frac{a}{\sqrt{2}}$$

$$I_{total} = \frac{Ma^2}{6} + 4m\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{Ma^2}{6} + 2ma^2$$

ex If it was rectangular plate.



$$I_{total} = I_{rectangle} + 4 \text{ point mass } I$$

$$= \frac{M}{12}(a^2+b^2) + 4mr^2$$

$$= \frac{M}{12}(a^2+b^2) + 4m\left(\frac{b^2+a^2}{4}\right)$$

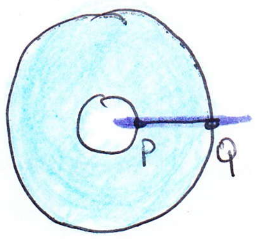
$$r^2 = \frac{b^2}{4} + \frac{a^2}{4}$$

$$I_{total} = \left(\frac{M}{12} + m\right)\left(a^2+b^2\right)$$

ex CD Disc is slowing down to stop.

at 0s $\omega = 27.5$ rad/s ; $\alpha = -10$ rad/s²

PQ line lies along X axis at $t = 0s$.



at $t = 0s$

a) $\omega_f = ?$ (final angular velocity) at $t = 0.3s$.

$$\omega_f = \omega_i + \alpha t \quad (v_f = v_i + at)$$

$$\omega_f = 27.5 + (-10)(0.3) = 24.5 \text{ m/s}$$

@ $t = 0.3s$

b) what is the angle of PQ line at $t = 0.3s$

- asking about the displacement.

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\theta_f = 0 + (27.5)(0.3) + \frac{1}{2}(-10)(0.3)^2 = 7.8 \text{ rad}$$

* remember 1 full circle ; rotation 2π rad. $= 2 \times 3.14$ rad $= 6.28$ rad

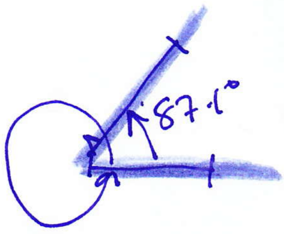
$7.8 - 6.28 = 1.52$ rad

angle?

$360° = 6.28$ rad.

$?° = 1.52$ rad

$87.1°$

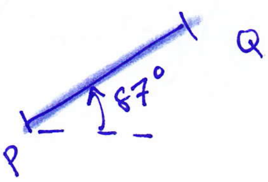


also total rad. displacement is 7.8 rad. $\Rightarrow \frac{7.8}{6.28} \times 360°$

$\Rightarrow 447°$

$- 360$

$87°$

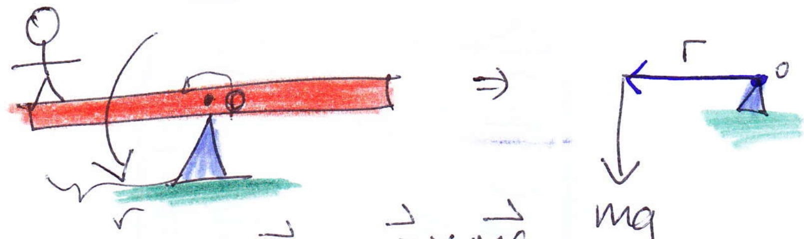


End of Ch9

Dynamics of Rotational Motion Ch 10

TORQUE (tork) (τ = tau)

rotates due to tork.

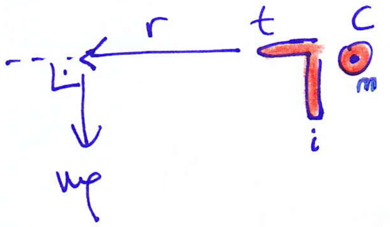


$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

cross multiplication
vector "
(right hand rule)

$$|\vec{\tau}| = |rF\sin\theta|$$

Torque $= \vec{\tau} = \vec{r} \times m\vec{g}$ mg.

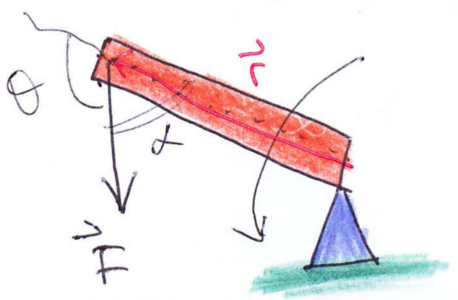


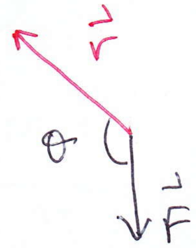
$\vec{A} \times \vec{B} = \vec{C}$

thumb, index, middle

$\vec{\tau} = \vec{r} \times m\vec{g}$ (out of page towards us)

$|\vec{\tau}| = rmg\sin 90 = rmg = mgr$

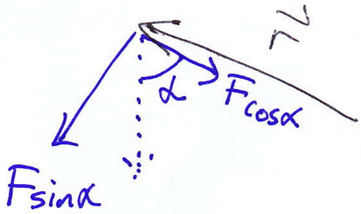




$\vec{\tau}$ towards us

$|\vec{\tau}| = |Fr\sin\theta|$

$\boxed{\sin\alpha = \sin\theta}$



$\vec{\tau} = \vec{r} \times (\vec{F}\cos\alpha + \vec{F}\sin\alpha)$

$\vec{r} \times \vec{F}\cos\alpha = 0$
parallel.

$$\boxed{|\vec{\tau}| = |rF\sin\alpha|}$$

$\vec{\tau} = \vec{r} \times \vec{F}$
$= [m\,N] = [Nm]$
unit.

Nm $\neq$ joule

$W = \vec{F} \cdot \vec{S} = [Nm] = [\text{Joule}]$

unit for torque is Nm ; NOT Joule

Tork is not an energy concept.

θ	$\longleftrightarrow$	x
ω	$\longleftrightarrow$	v
α	$\longleftrightarrow$	a
I	$\longleftrightarrow$	m
τ	$\longleftrightarrow$	F

✓

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

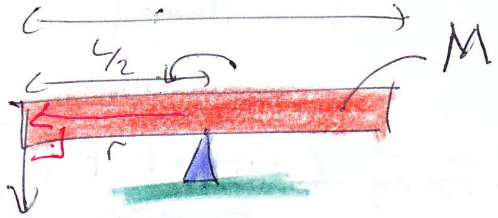
$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

$$F = ma \Longleftrightarrow \tau = ?$$

$$\boxed{\tau = I\alpha} \quad ***$$

rotational motion

ex



mg=F

$r = \frac{L}{2}$

$$\tau = I\alpha$$

$$|\vec{r}\times\vec{F}|$$

$$mgr = I_{cm}\alpha$$

$$mg\frac{L}{2} = \left(\frac{ML^2}{12}\right)\alpha$$

$$\frac{6mg}{ML} = \alpha$$

$$\boxed{\tau = I\alpha} \quad ***$$

I = will be given in the problem.

$$\boxed{F = ma = m(\alpha r)}$$

$$rF = rm\alpha r$$

$$rF = mr^2\alpha$$

$$\boxed{\tau = I\alpha}$$

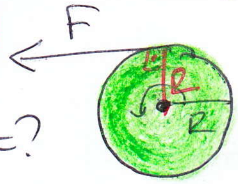
$$Nm \qquad kg\,m^2\,\frac{rad}{s^2}$$

$$kg\frac{m}{s^2}m = kg\frac{m^2}{s^2}(rad)$$

$\hookrightarrow$ number

rad is not an actual unit.

ex



$\alpha = ?$

M, R, F given

$$\tau = I\alpha$$

$$FR\sin 90 = (MR^2)\alpha$$

$$\boxed{\alpha = \frac{F}{MR} = \left[\frac{rad}{s^2}\right]}$$

* now our pulley will rotate it will affect the acceleration of the system!!

$\theta, \omega, \alpha \;;\; \tau \;,\; I$

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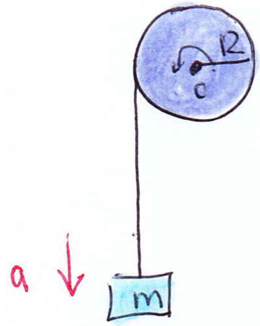
1

$$\tau = I\alpha = |\vec{r} \times \vec{F}|$$

rotation axis is important

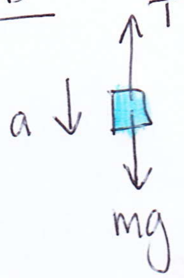
---

ex) M disk



$I = \frac{MR^2}{2}$ } m going down; $\vec{a}$ of m?

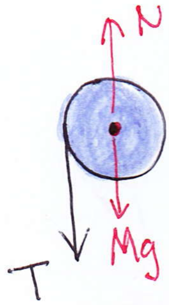
FBD



$\Sigma F = ma$

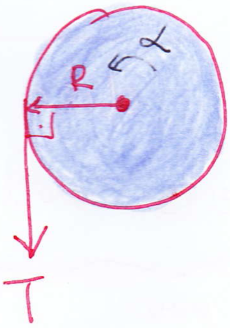
$mg - T = ma$

? ?



rotating

ΣF on $\underset{\text{disc}}{M}$ $= 0$ b/c it's not going up / down



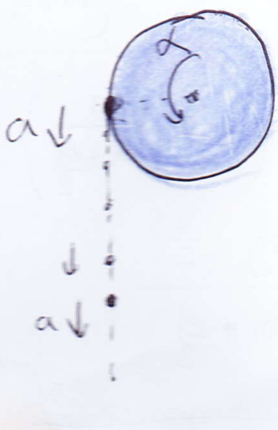
$I\alpha = \tau = |\vec{r} \times \vec{F}| = FR$

$TR = I\alpha$

$T = \frac{I\alpha}{R}$

$mg - T = ma \Rightarrow mg - \frac{I\alpha}{R} = ma$?

There is a relation between α ; a ?



rope has acc. $\underline{a}$; at the rim / edge of disc.

$\boxed{\alpha R = a}$

no skipping !!

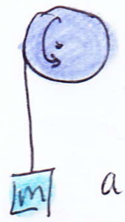
b/c rope is not sliding / skipping

rope is always in contact w/ disc.

$$mg - \frac{I\alpha}{R} = ma \quad ; \quad \alpha R = a \quad ; \quad \alpha = \frac{a}{R}$$

$$mg - \frac{I\frac{a}{R}}{R} = mg - \frac{Ia}{R^2} = ma \quad ; \quad a = \frac{mg}{\left(m + \frac{I}{R^2}\right)}$$

$$I_{disk} = \frac{MR^2}{2} \quad ; \quad a = \frac{mg}{m + \frac{MR^2}{2R^2}} \rightarrow \boxed{\frac{m}{\left(m + \frac{M}{2}\right)} g = a}$$



$$a = \frac{mg}{\left(m + \frac{M}{2}\right)} < \underline{g}$$

If Mass of disc $= 0 \Rightarrow a = \frac{mg}{m+0} = \underline{g}$ (free fall)

Rotating disc will be IMPORTANT.

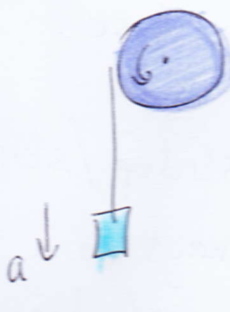
$\Rightarrow$ PULLEYs will be rotating ; pulley = disc

$\hookrightarrow \alpha, I \dots$

a will be different from Ch 5 !!

$\Rightarrow$ till now we assumed pulleys are not rotating or have $\underline{0}$ mass

$$\boxed{\alpha = \frac{a}{R} = \frac{mg}{\left(m + \frac{M}{2}\right)R}}$$



$T = ?$

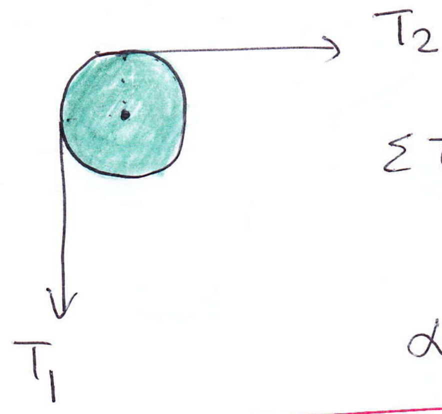
$$mg - T = ma$$

$$T = m(g-a) = mg\left(1 - \frac{m}{m + \frac{M}{2}}\right)$$

$$a = \frac{m}{\left(m + \frac{M}{2}\right)} g$$

$$\boxed{T = \frac{mg\left(\frac{M}{2}\right)}{\left(m + \frac{M}{2}\right)}}$$

How pulleys rotate? (there is friction between the ropes and pulley)



$\Sigma\tau$ on pulley?

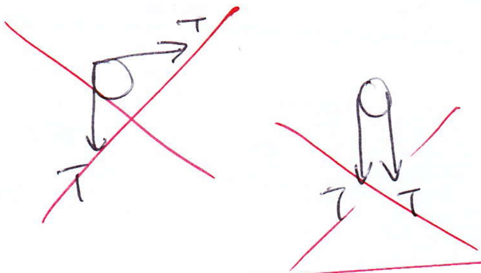
$$\Sigma\tau = T_1R - T_2R = I\alpha$$

$\alpha > 0$; $T_1 > T_2$; $T_1 \neq T_2$

$\alpha = 0$ $T_1 = T_2$ $\Rightarrow$ no rotation!!

---

before Ch5 tension on each side of a pulley will be same if its the same rope

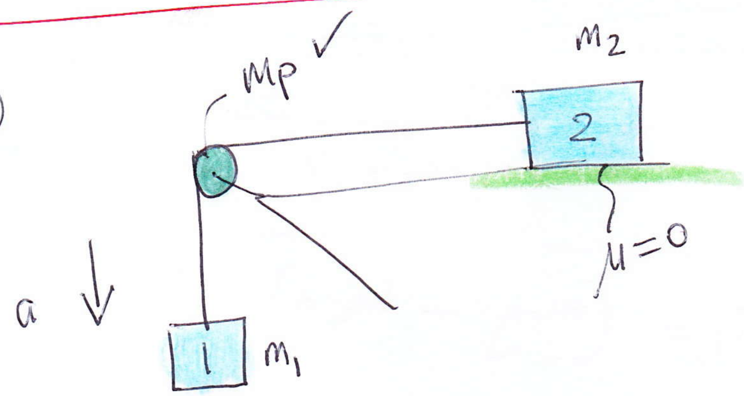


because we assumed pulleys are NOT rotating

Will not be SAME from now on!!

---

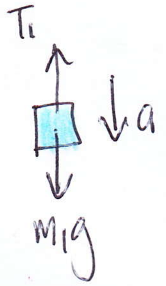
ex

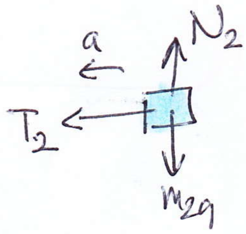


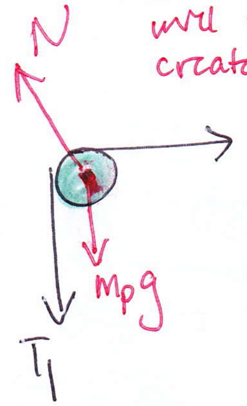
$a = ?$ $T = ?$

$\alpha = ?$

FBD







N & mg will not create torque!!

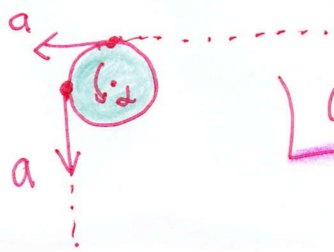
$\Sigma\vec{F} = 0$

$$m_1g - T_1 = m_1a$$

$$T_2 = m_2a$$

$$\Sigma\tau = I\alpha$$

$$T_1R - T_2R + m_pg(0) + N(0)$$



$$a = \alpha R$$

$$(T_1 - T_2)R = I\alpha$$

$$(T_1 - T_2)R = \frac{Ia}{R}$$

$$m_1 g - T_1 = m_1 a \quad (1)$$
$$T_2 = m_2 a \quad (2)$$
$$(T_1 - T_2) = \frac{I a}{R^2}$$

$(1) + (2)$

$$m_1 g - T_1 + T_2 = (m_1 + m_2) a$$
$$m_1 g - \frac{I a}{R^2} = (m_1 + m_2) a$$

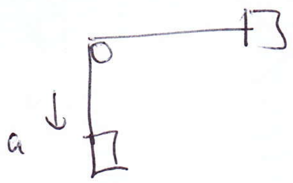
$$I = \left(\frac{m_p R^2}{2}\right)$$

$$m_1 g = \left(m_1 + m_2 + \frac{I}{R^2}\right) a$$

$$\boxed{a = \frac{m_1}{\left(m_1 + m_2 + \frac{I}{R^2}\right)} g} = \frac{m_1}{\left(m_1 + m_2 + \frac{m_p}{2}\right)} g$$

if $m_p = 0$; not rotating

ChT



$$a = \frac{m_1 g}{m_1 + m_2}$$

$T_2 = m_2 a$; $\quad T_2 = m_2 \dfrac{m_1 g}{\left(m_1 + m_2 + \frac{m_p}{2}\right)}$

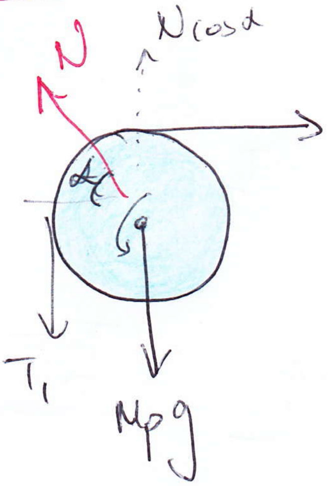
$$\alpha = \frac{a}{R} = \frac{m_1 g}{\left(m_1 + m_2 + \frac{m_p}{2}\right) R}$$

$$T_1 = (m_1 g - m_1 a) = (\cdot \quad \cdot \quad \cdot \cdot)$$

FBD of pulley

$\sum F = 0$

$$\vec{T_1} + \vec{T_2} + \vec{N} + \vec{m_p g} = 0$$



$$T_2 = \frac{m_1 m_2 g}{\left(m_1 + m_2 + \frac{m_p}{2}\right)}$$

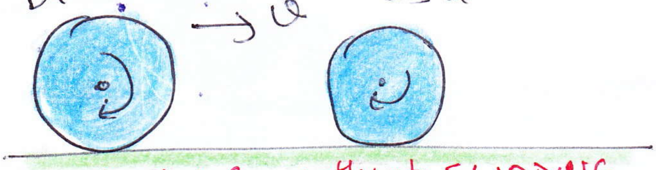
$$\tan\alpha = \frac{T_2}{T_1 + m_p g}$$

$$\boxed{N \cos\alpha = T_1 + m_p g}$$
$$\boxed{N \sin\alpha = T_2}$$

can find out what N?
α?

disc rotating ; Roll a wheel / disc on flat surface

Disc = wheel

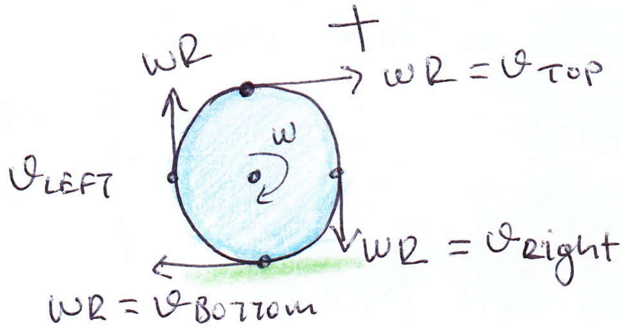


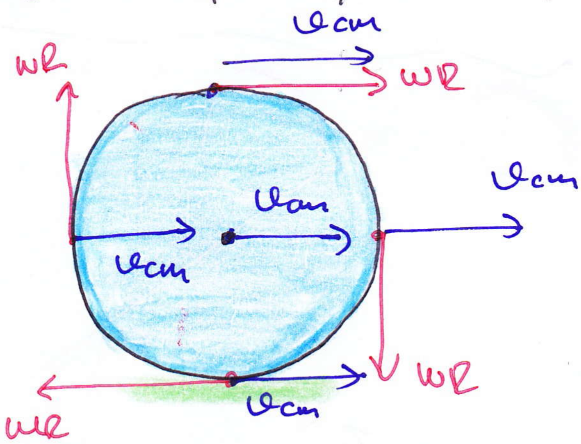
ROLLING without SLIPPING

{ TRANSLATION + ROTATION

Translation all points have v_{cm} speed / velocity

+

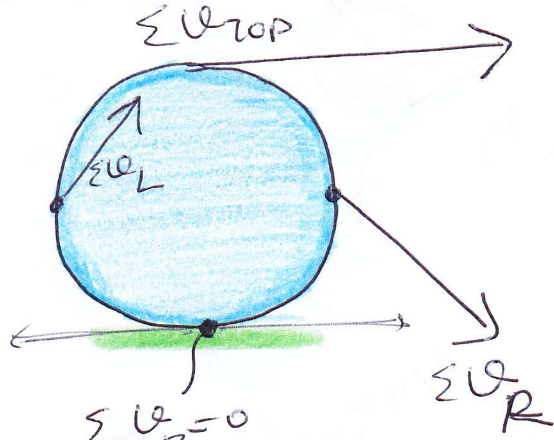




CONTACT POINT
Bottom is important

Rolling without slipping
skidding
(kaymadan dönme)

→ means at the bottom



v rotation = v translation

$v_{cm} = wR \Rightarrow$ no slipping

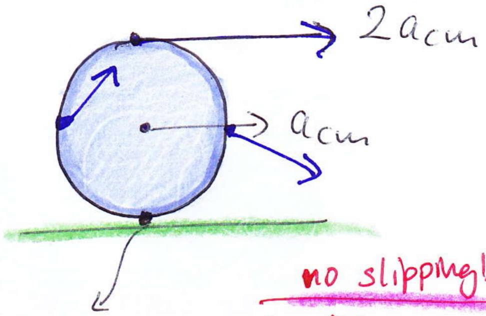
$wR > v_{cm}$ patinaj çekme !!

$v_{cm} > wR$ fren yapıp tekerin iz bırakması

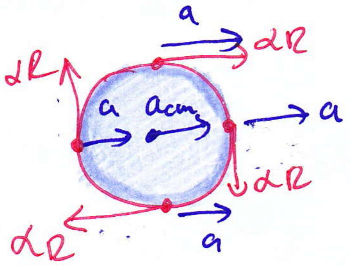
Condition for rolling w/o slipping

$$v_{cm} = \omega R$$

$$\boxed{a_{cm} = \alpha R}$$



$\Sigma a = 0$



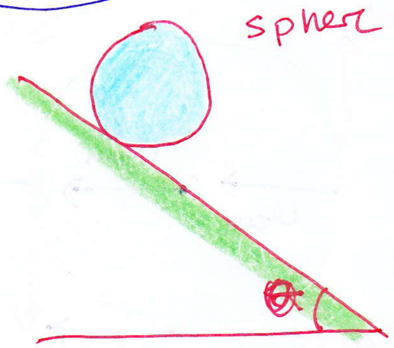
@ Bottom contact point $\Sigma a = 0$!

no slipping!!

$$\boxed{\alpha R = a}$$
$$\boxed{\omega R = v}$$
$$\boxed{\theta R = s}$$

---

ex



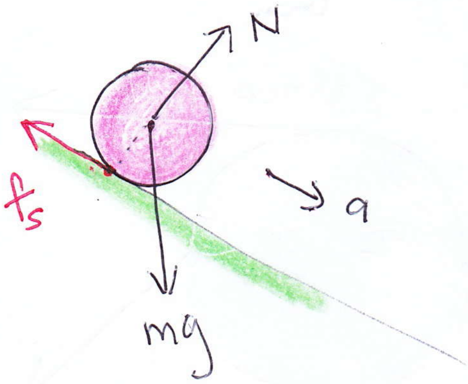
sphere rolls without slipping

FBD?

$I = \frac{2}{5} mR^2$

a=?

frictional force?



there must be friction for the sphere to roll w/o slip.

if no friction it will slide, no rolling

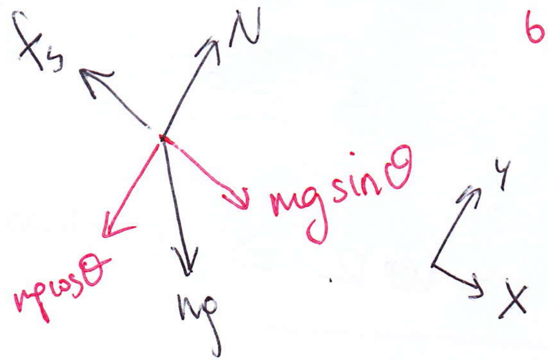
friction is the force makes it roll!!

f force ≡ STATIC fric. force!

b/c it's not slipping

the contact point between sphere and the plane ~~DOF~~ IS STATIONARY IS NOT MOVING!

Static fric. force



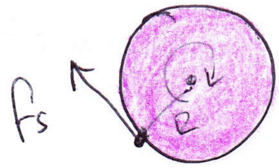
$\Sigma F_y = 0 \quad \boxed{N = mg\cos\theta}$

$\Sigma F_x = \quad \boxed{mg\sin\theta - f_s = ma} \Rightarrow \boxed{mg\sin\theta - mg\mu_s\cos\theta = ma}$

$f_s = \mu_s N = \mu_s mg\cos\theta$

Rotation

$\tau_{cm} = I_{cm}\alpha = f_s R$



$\frac{2}{5} MR^2 \alpha = (\mu_s mg\cos\theta) R$

no slippy $R\alpha = a$

$\frac{2}{5} \cancel{M}\cancel{R^2} \frac{a}{\cancel{R}} = \mu_s \cancel{m}g\cos\theta \cancel{R}$

Linear motion

$a = \frac{5}{2}\mu_s g\cos\theta$

~~mg − μs~~

$mg\sin\theta - \mu_s mg\cos\theta = ma$

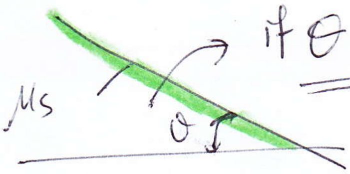
$(\sin\theta - \mu_s\cos\theta) g = a$

$(\sin\theta - \mu_s\cos\theta)\cancel{g} = \cancel{g}\frac{5}{2}\mu_s\cos\theta$

$\sin\theta = \left(\frac{5}{2}+1\right)\mu_s\cos\theta$

condition for sphere to roll w/o slippy

$\boxed{\frac{2}{7}\tan\theta = \mu_s}$ * * * ⇔ If θ is such that value; noslippy



if $\theta\uparrow$; slip!! slip occurs!!

$\boxed{(\text{const})\tan\theta = \mu_s}$

↳ depends on $I = \frac{2}{5}mR^2$;

$= \frac{mR^2}{2}$; $\frac{2}{3}mR^2$

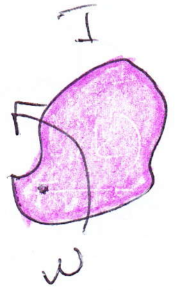
from translational motion $a = g(\sin\theta - \mu_s\cos\theta)$

rotational motion $\tau = I\alpha = \alpha MR^2\alpha = \cancel{MR}a\alpha = \mu_s \cancel{m}g\cos\theta\cancel{R}$

$a = \cancel{g}\frac{\mu_s\cos\theta}{\alpha} = \cancel{g}(\sin\theta - \mu_s\cos\theta)$

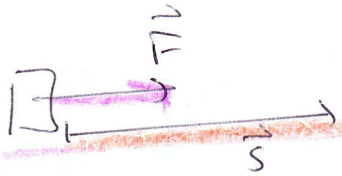
$\left(\frac{1}{\alpha}+1\right)\mu_s\cos\theta = \sin\theta \Rightarrow \tan\theta = \mu_s\left(\frac{\alpha+1}{\alpha}\right)$

Rotation

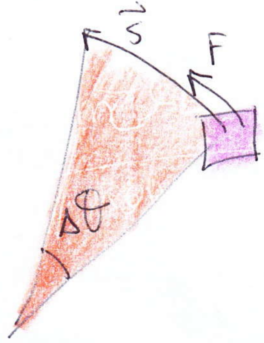


$$\frac{1}{2} I \omega^2 = K \qquad I = \int dm\, r^2$$

$$\text{Work} = \vec{F} \cdot \vec{S} \rightarrow$$



$$= \underbrace{F\,R}_{\text{Tork}}\,\Delta\theta$$



$$\text{Work} = \text{Tork}\ \Delta\theta \Rightarrow W = \tau\,\Delta\theta$$

(Nm) (rad)

$$W = \tau\,\theta$$

joule = Nm

in cars engines horsepower ~ joule / Nm

↳ tork = Nm

work - KE theorem → $\sum W = W_{tot} = \Delta K$

$$\boxed{\tau\theta = K_f - K_i = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2}$$

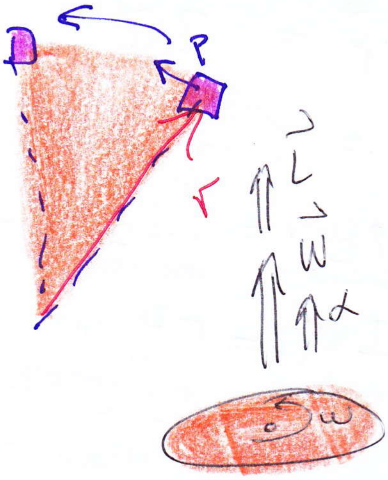
$$\text{power} = \text{Power} = \frac{W}{\text{time}} = \frac{\tau\,\theta}{t} = \tau\omega = \text{power}$$

P = momentum = $m\vec{v}$ linear motion

$$\frac{d\vec{p}}{dt} = \vec{F} \qquad \vec{r} \times \vec{F} = \vec{\tau}$$

$$\vec{L} = \text{angular momentum} = \vec{r} \times \vec{p} \qquad \frac{d\vec{L}}{dt} = \vec{\tau}$$

$$\cancel{\frac{d\vec{r}}{dt}}_{\,0} \times \vec{P} + \vec{r} \times \frac{d\vec{P}}{dt} = \vec{\tau}$$



$\vec{r} \times \vec{p} = \vec{L}$ (angular momentum)

$\vec{p} = m\vec{v}$;

$\vec{L} = I\vec{\omega} = \vec{r} \times \vec{p}$

$\vec{\tau} = I\vec{\alpha} = \vec{r} \times \vec{F}$

Direction of $\vec{\omega}, \vec{L}, \vec{\tau}, \vec{\alpha}$ vectors are perpendicular to rotation surface

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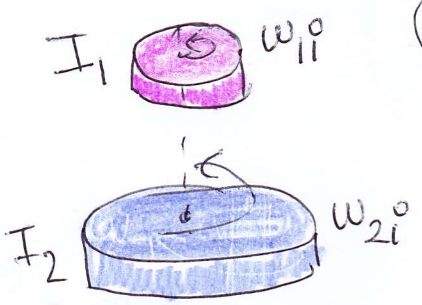
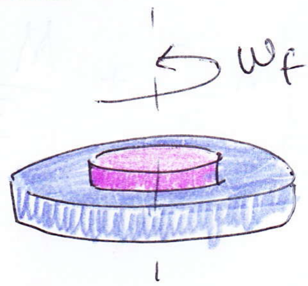
$\sum \vec{P}_f = \sum \vec{P}_i$ mom. conservation collision

$\sum \vec{L}_f = \sum \vec{L}_i$ collisions.





$I_1 + I_2 =$

$\sum L_i = \sum L_f$

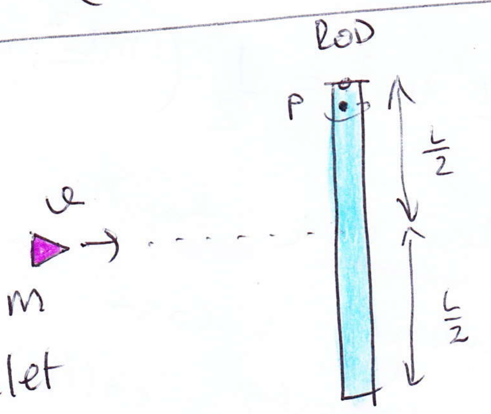
$$\boxed{I_1\omega_{1i} + I_2\omega_{2i} = (I_1 + I_2)\omega_f}$$

collis.

how gear works in gearbox car (vites kutusu)

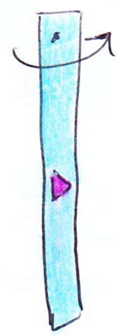
they are rotating in same direction so add up L

Ex)



$I_p = \frac{1}{3}ML^2$

ω_i rod $= 0$



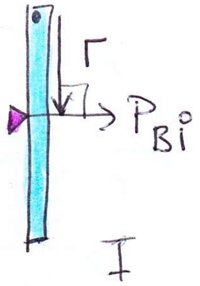
move together

$\omega_f = ?$

final energy?

initial energy?

$$\sum \vec{L}_i = \sum \vec{L}_f \qquad \vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$



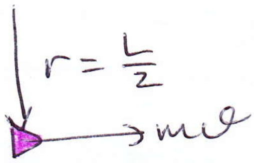
$$\sum \vec{L}_i = \sum \vec{L}_f$$

$$\vec{L}_{iB} + \vec{L}_{iR} = \vec{L}_{FR} + \vec{L}_{FB}$$

$$r P_i \sin 90 + I\omega_i = r P_f \sin 90 + I\omega_f$$

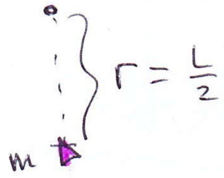
$\omega_i \Rightarrow 0$

move together



$$\frac{L}{2} m v + 0 = \left(I_B + I_{ROD}\right)\omega_f$$

$I_{ROD} = \frac{1}{3}ML^2$

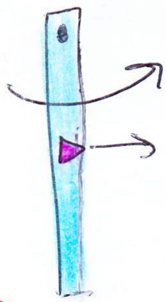


$$I_{Bullet} = \sum m_i r_i^2 \equiv \text{point mass}$$

$$= m r^2 = m\frac{L^2}{4}$$

$$\frac{m v \cancel{L}}{2} = \left(m\frac{L^{\cancel{2}}}{4} + \frac{ML^{\cancel{2}}}{3}\right)\omega_f$$

$$\boxed{\omega_f = \frac{\frac{mv}{2}}{\left(\frac{mL}{4} + \frac{ML}{3}\right)}}$$



$$\omega_f = \frac{v_f}{r} = \frac{v_f}{(L/2)}$$

$$E_i = K_i = \frac{1}{2} m v^2 + \frac{1}{2} I \omega_i^2 \rightarrow 0$$

$$K_f = \frac{1}{2} I' \omega_f^2 = \frac{1}{2}\left(\frac{mL^2}{4} + \frac{ML^2}{3}\right)\omega_f^2$$

$E_f < E_i$; they move together

energy is lost

this inelastic collision !!