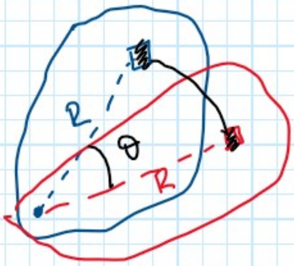
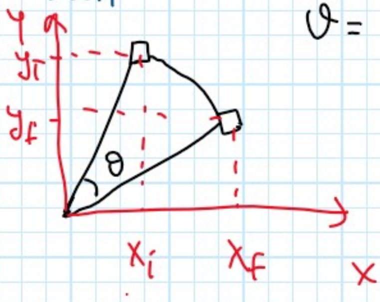
Phys1- 05

Wednesday, December 14, 2022 11:11 AM

ROTATION of RIGID BODIES

Katı





$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} = \bar{v}_x$$

$$\frac{\Delta y}{\Delta t} = \frac{y_f - y_i}{\Delta t} = \bar{v}_y$$

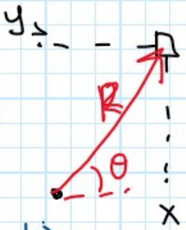
$$\boxed{\vec{v} = \bar{v}_x \hat{i} + \bar{v}_y \hat{j}}$$

polar coord. Change the coordinate sys.

$$(R, \theta) \leftarrow (x, y)$$

R const when it rotates!!

just measure θ to describe the motion



θ? theta ≡ radians

2π rad = 6.28 rad = 360°

π rad = 180°

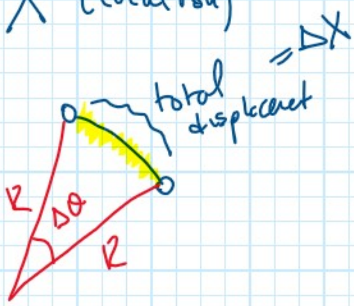
1 rad = $\frac{180°}{3.14}$ = 57.3°

1 dim motion

X (location)

rotational motion

θ (rad) location θ ≡ unitless → rad → degrees



$$\Delta X = R \, \Delta\theta$$

$$X = R\theta$$

circumference = $R 2\pi$ = $R \Delta\theta$

θ ≡ location $\Delta\theta = \theta_f - \theta_i$ = angular displacement.

$x \longleftrightarrow \theta$ (theta)

$\bar{v} = \frac{\Delta x}{\Delta t}$ (omega) $\bar{\omega} = \frac{\Delta\theta}{\Delta t}$ (average angular velocity)

$v = \frac{dx}{dt}$ $\omega = \frac{d\theta}{dt}$ (instantaneous " ")

$\bar{a} = \frac{\Delta v}{\quad}$

$$\bar{a} = \frac{\Delta v}{\Delta t} \qquad (\text{alpha}) \quad \bar{\alpha} = \frac{\Delta w}{\Delta t} \quad (\text{average angular acceleration})$$

$$a = \frac{dv}{dt} \qquad \alpha = \frac{dw}{dt} \quad (\text{instantaneous} \;\;" \;\; \text{acc.})$$

$$\theta = [\text{rad}] \qquad w = \left[\frac{\text{rad}}{s}\right] \qquad \alpha = \left[\frac{\text{rad}}{s^2}\right] \qquad \boxed{\pi \text{ rad} = 180^\circ}$$

$$\theta \equiv \text{rotation} \qquad w = \left[\frac{\text{rot}}{s}\right]$$

Hard disks 7200 rpm

$$\left[w \equiv \text{rpm} = \text{rotation per minute} \right]$$

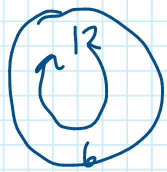
$$w = 7200 \text{ rpm} = \; ? \;\; \text{rad}/s$$

$$\left[7200 \text{ rpm} = 7200 \times 0.104 \text{ rad}/s \right]$$

1 rot per minute

$$1 \text{ rpm} = \frac{1 \text{ rotation}}{60 s} = \frac{2\pi \text{ rad}}{60 s} = \frac{6.28}{60} \text{ rad}/s = 0.104 \text{ rad}/s$$

find w of second-hand, minute-hand, hour-hand of a clock?



$$\text{second-hand (saniye ibresi)} = \frac{1 \text{rot}}{60 s} = \frac{6.28 \text{ rad}}{60 s} = 0.104 \text{ rad}/s$$

$$\text{minute} \;\; " \;\; (\text{yelkovan}) = \frac{1 \text{rot}}{1 \text{hr}} = \frac{6.28 \text{ rad}}{60 \times 60} = \frac{0.104}{60} = 1.73 \times 10^{-3} \text{ rad}/s$$

$$\text{hour} \;\; " \;\; (\text{akrep}) = \frac{1 \text{rot}}{12 \text{ sa}} = \frac{6.28 \text{ rad}}{12 \times 60 \times 60} = \frac{1.73 \times 10^{-3}}{12} = 1.44 \times 10^{-4} \text{ rad}/s$$

⇒ w of earth rotation on its axis (kendi etrafında dönüş hızı)

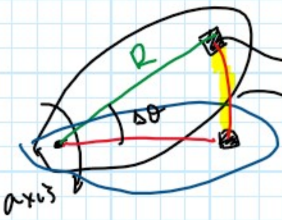
$$\frac{1 \text{rot}}{24 \text{hr}} = \frac{6.28}{24 \times 60 \times 60} = 7.22 \times 10^{-5} \text{ rad}/s$$

$$\frac{1\,rot}{24\,hr} = \frac{6.28}{24\times60\times60} = 7.22\times10^{-5}\ rad/s$$

earth around the sun: $\frac{1\,rot}{365\times24\times60\times60\,s} = 2\times10^{-7}\ rad/s$

$\theta \qquad \omega \qquad \alpha$

$$x \xleftrightarrow{R} \theta \qquad v \quad \omega \qquad a \quad \alpha$$



p (θ, ω, α)

length $\quad R\,\Delta\theta = \text{arc length} = \Delta x$

$a = a_{tangential} \neq a_{radial}$

$$\boxed{x = R\theta \quad ; \quad v = R\omega \qquad a = R\alpha}$$

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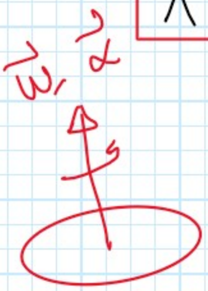
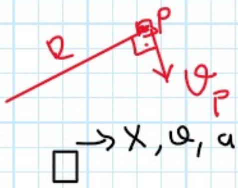
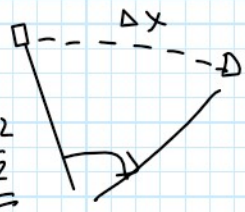


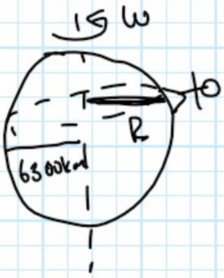


$a = \alpha R \neq \frac{v^2}{R}$



$\frac{\Delta x}{\Delta t} = \bar{\theta} = \omega$

what's your velocity due to earth rot. about its axis?



$v = ?$ $\qquad v = R\omega$

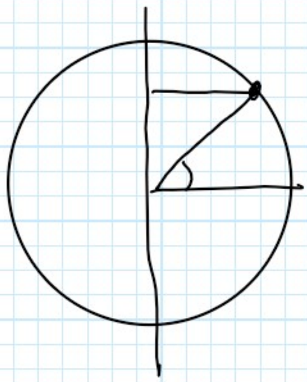
$R \simeq 6000\ km$

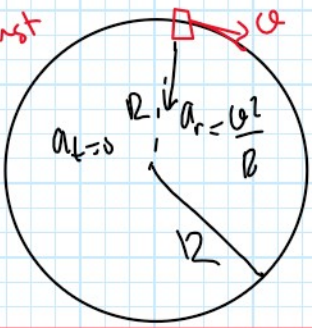
$\omega = 7.2\times10^{-5}$

$v = 7.2\times10^{-5} \times 6\times10^{6}\ m/s$

$= 432\ m/s = 432\times3.6\ km/hr = 1555\ km/hr$

$\simeq 1500\ km/s$





$v = const$

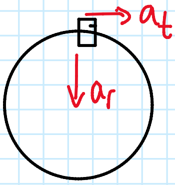
$R\omega = v$

$\underbrace{\omega = const}$

$\alpha = 0 \qquad a_t = \alpha R = 0$

$\alpha = \frac{d\omega}{dt} = 0$

$a_t \neq a_r$



$\omega \neq$ const

$a_r = \frac{v^2}{R}$; $a_t = \alpha R$

$\alpha = \frac{d\omega}{dt}$

$$\begin{cases} \theta = 3t^2 - 5t \\ \omega = \frac{d\theta}{dt} = 6t - 5 \\ \alpha = \frac{d\omega}{dt} = 6 \text{ rad/s}^2 \end{cases}$$

$$\boxed{\begin{aligned} x &= R\theta \\ v &= R\omega \\ a &= R\alpha \end{aligned}}$$

1 dim Linear motion	Rotation motion
a = const	α = const

- $x_f = x_i + v_i t + \frac{1}{2} a t^2 \longleftrightarrow \theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$

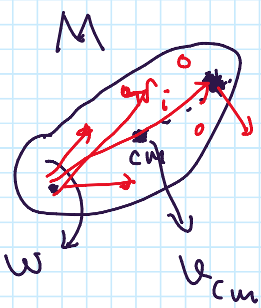
  $\cancel{R}\theta_f = \cancel{R}\theta_i + \cancel{R}\omega_i t + \frac{1}{2} \cancel{R}\alpha t^2$

- $v_f = v_i + at \longleftrightarrow \omega_f = \omega_i + \alpha t$

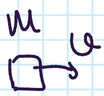
- $v_f^2 = v_i^2 + 2a(x_f - x_i) \longleftrightarrow \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$

ENERGY in ROTATIONAL MOTION

Kinetic energy of this object?



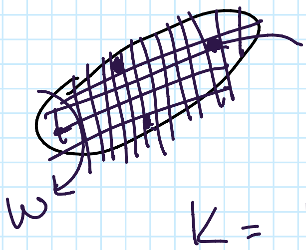
$v_i = \omega r_i$



$\frac{1}{2} m v^2 = K$

v^2

$\frac{1}{2} \, ? \, (\omega^2 R^2) = \frac{1}{2} \left(?? \right) \omega^2$



$\frac{m_i v_i^2}{2} = \frac{m_i \omega_i^2 r_i^2}{2}$

$K = \sum \frac{1}{2} m_i v_i^2$ $\quad \boxed{\omega_i = \omega}$

$K = \sum_{i=1}^{N} \frac{1}{2} m_i r_i^2 \omega_i^2$

$K = \frac{1}{2} \left(\sum m_i r_i^2 \right) \omega^2$

I = moment of inertia (eylemsizlik momenti)

$$\boxed{I = \sum m_i r_i^2 = \int r^2 \, dm}$$

I depends on how mass distributed onto the shape.

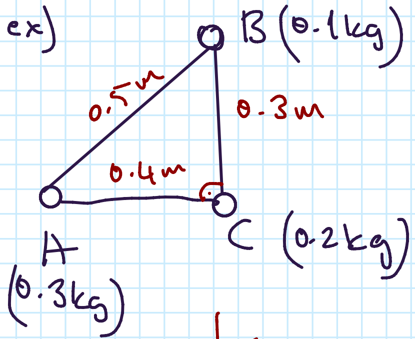
onto the shape.

$I \propto$ geometry

$I = \underbrace{\sum m_i r_i^2}$

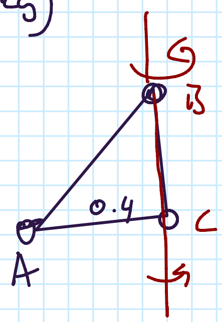
applicable to point masses.

ex)



a system is composed of 3 point masses.

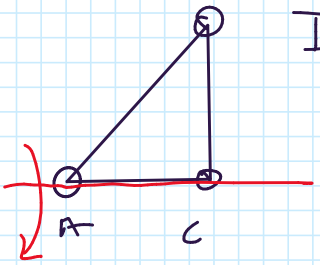
$I_{BC} = ?$ mom. inertia with respect to the axis goes through BC line



$I = m_A r_A^2 + m_B r_B^2 + m_C r_C^2$

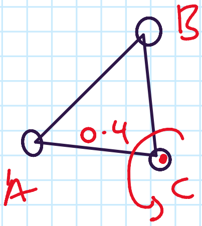
$r \equiv$ distance which's perpendicular rot. axis

I_{BC}; $r_B = r_C = 0$ $\quad I_{BC} = (0.3)(0.4)^2 = 0.048\ kg\,m^2$



$I_{AC} = (0.1)(0.3)^2 = 0.009\ kg\,m^2$

$r_A = r_C = 0$



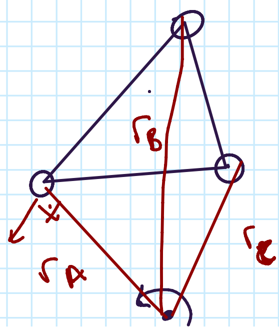
$I_C = m_A r_A^2 + m_B r_B^2 + 0 = (0.3)(0.4)^2 + (0.1)(0.3)^2$

$= 0.057\ kg\,m^2$

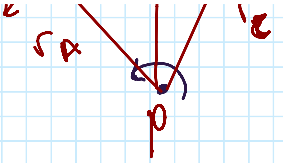
$I_C > I_{BC} > I_{AC}$

HARD EASY to rotate.

$r \perp$ to rotational axis

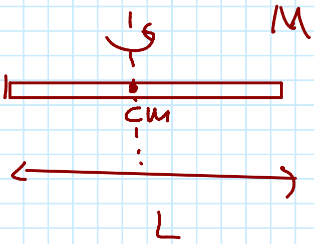


$I_p = \sum m_i r_i^2$ (point-like mass)

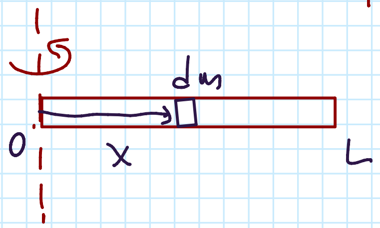


$I_p = \sum m_i r_i^2$ (point-like mass)

ROD (around) 1 dim object



$$I_{ROD,CM} = \frac{ML^2}{12}$$



$$\int x^2 dm = \frac{M}{L}\left[\frac{x^3}{3}\right]_0^L = \frac{ML^2}{3}$$

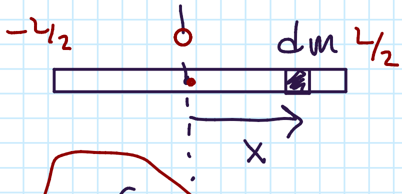
$$I = \int r^2 dm$$

$\rho = \frac{M}{L}$ = density (rho)

dm, dx

$$\rho = \frac{dm}{dx} = \frac{M}{L}$$

$$\left(dm = \frac{M}{L} dx\right)$$



$$\int x^2 dm$$

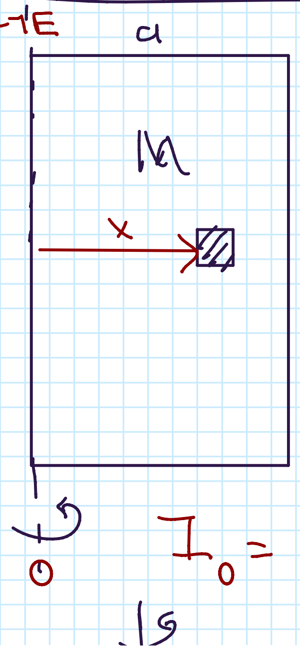
If I change location dm, X changes, X ⟷ dm related

$$\int x^2 dm = \int_{-L/2}^{L/2} x^2 \frac{M}{L} dx = \frac{M}{L}\left[\frac{x^3}{3}\right]_{-L/2}^{L/2}$$

$$= \frac{M}{3L}\left[\frac{L^3}{8} - \left(-\frac{L}{2}\right)^3\right]$$

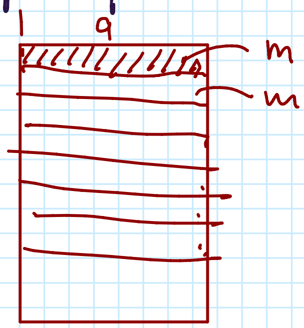
$$\frac{M}{3\cancel{L}} \frac{\cancel{2} L^{\cancel{3}\,2}}{\cancel{8}\,4} = \frac{ML^2}{12}$$

PLATE



$$I_0 = \frac{Ma^2}{3}$$

Rectangular plate

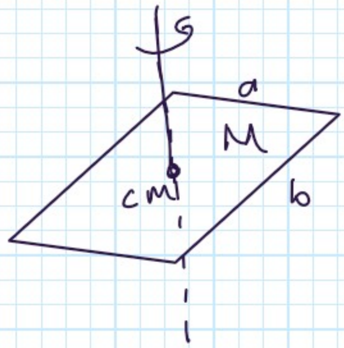


$$I = \frac{ma^2}{3}$$

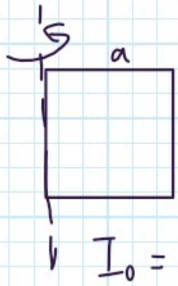
$$\sum m = M$$

$$I_{tot} = \sum m \frac{a^2}{3} = \frac{Ma^2}{3}$$

0 + 0 − 3   3 − 3



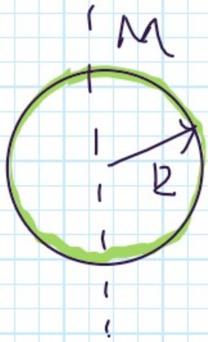
$$I_{cm} = \frac{M(a^2+b^2)}{12}$$



$$I_0 = \frac{Ma^2}{3}$$

SPHERE

HOLLOW SPHERE
(SPHERICAL SHELL)



$$I_{cm} = \frac{2}{3}MR^2$$

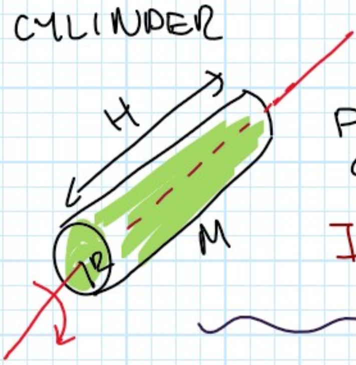
$$\frac{2MR^2}{3} > \frac{2}{5}MR^2$$

hollow > full

FULL SPHERE

$$I_{cm} = \frac{2}{5}MR^2$$

CYLINDER



FULL CYLINDER

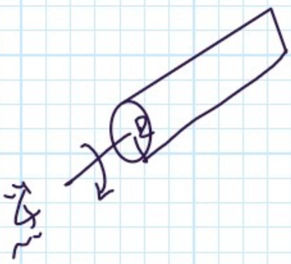
$$I_{cm} = \frac{MR^2}{2}$$

$\Leftrightarrow$

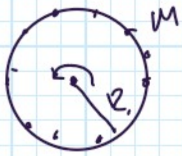
DISC

$$I_{cm} = \frac{MR^2}{2}$$

HOLLOW CYLINDER



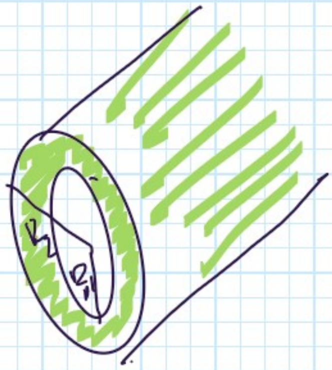
TOP VIEW



$$I_{cm} = MR^2$$

RING

$MR^2 = I_{cm}$



$$I_{cm} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$R_1 = R_2$ HOLLOW sphere

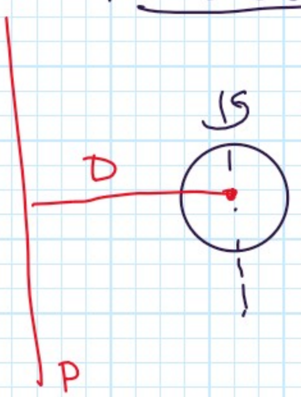
$\frac{1}{2}M(R^2+R^2) = MR^2$

$R1 = 0$ $R_2 = R$

FULL SPHERE

$\frac{MR^2}{2}$ ✓

PARALLEL AXIS THEOREM

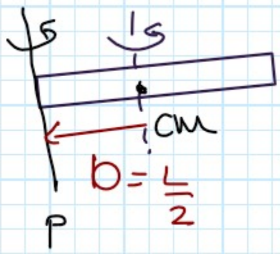


I_{cm} ✓

$$\boxed{I_p = I_{cm} + MD^2}$$

new axis // to cm axis

ex



$I_{cm, rod} = \frac{ML^2}{12}$

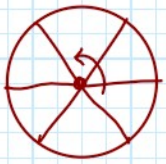
$I_p = I_{cm} + MD^2$

$I_p = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2$

$= \frac{ML^2}{3}$ ✓

The end of Ch 9

9.33)



; $I = ?$

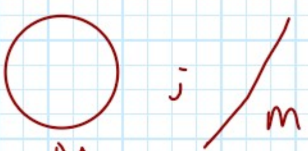
radius $= R$

wagon wheel

(at arabası tekeri)

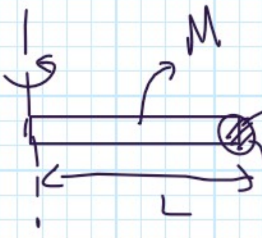
$\Rightarrow$ 

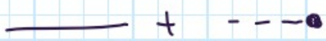
$MR^2 + 3\frac{m(2R)^2}{12}$



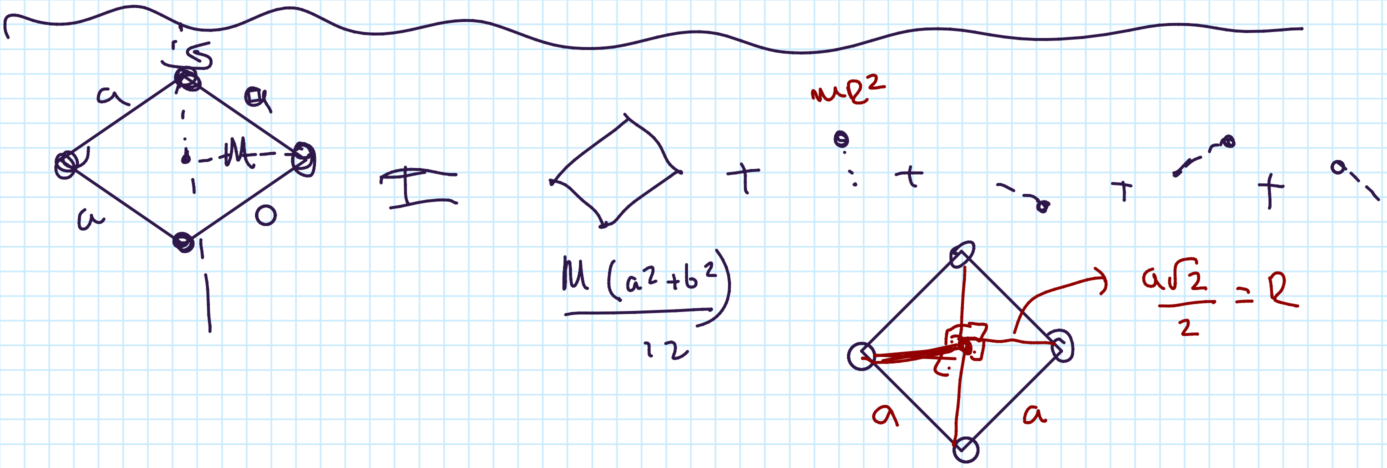
$MR^2 + mR^2 = (M+m)R^2$

$I_{cm} = MR^2$ $I_{cm} = \frac{mL^2}{12}$





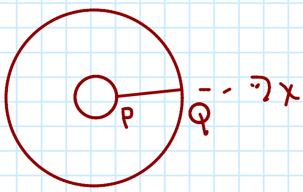
$\frac{ML^2}{3} + mL^2$ ✓



ex

CD disc is slowing down to stop.

at 0 s $\omega = 27.5$ rad/s $\alpha = -10$ rad/s²



PQ line lies along x axis at t = 0 s

a) ω_f at t = 0.3 s ?

$$\omega_f = \omega_i + \alpha t$$

$$= 27.5 + (-10)(0.3) = 24.5 \text{ rad/s}$$

b) angle of PQ line at t = 0.3 s ?

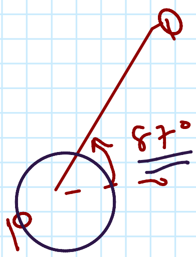
$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$= 0 + 27.5(0.3) + \frac{1}{2}(-10)(0.3)^2 = 7.8 \text{ rad}$$

$2\pi = 6.28$ rad $= 1$ rot $= 360°$

$$\frac{7.8 \text{ rad}}{6.28 \text{ rad}} \times 360° = 447° = 360° + 87°$$

final angle of PQ with x axis



CH 10: DYNAMICS of ROTATIONAL MOTION

$x_f = x_i + v_i t + \frac{1}{2}at^2$ $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

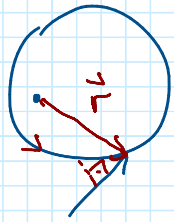
$v_f = v_i + at$ $m \leftrightarrow I$ $\omega_f = \omega_i + \alpha t$

$v_f = v_i + at$ $\qquad m \leftrightarrow I \qquad$ $\omega_f = \omega_i + \alpha t$

$v_f^2 = v_i^2 + 2a\Delta x$ $\qquad\qquad$ $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

a const $\qquad a = ?$ $\qquad\qquad$ $\alpha = ?$

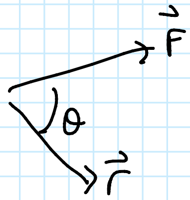
$\square \rightarrow \sum \vec{F} = m\vec{a}$ $\iff$ $\sum \vec{\tau} = I\alpha$



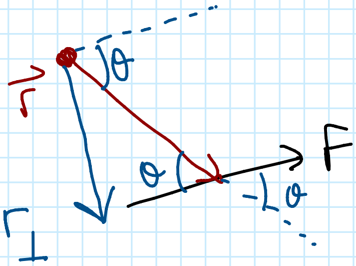
τ = torque

F = force $\Rightarrow \underline{a}$

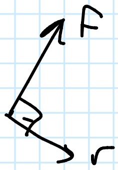
$\vec{\tau} = \vec{r} \times \vec{F}$



$|\vec{\tau}| = |F r \sin\theta|$

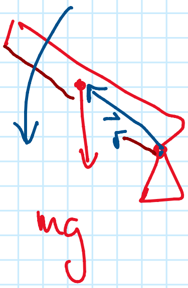


$\tau = F r_\perp = F r \sin\theta$



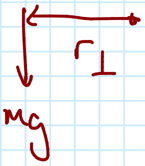
$\theta = 90^\circ$

$\tau = max$



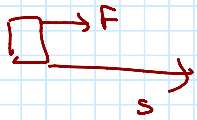
$\Rightarrow$

$\vec{r}$: rotation axis to the $\vec{F}$ point



$\vec{\tau} = \vec{r} \times \vec{F} = [N\,m] \neq$ joule

$\vec{F} \cdot \vec{s}$ = work

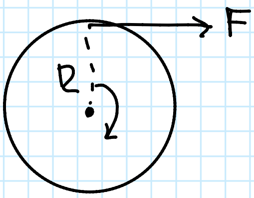


$\vec{\tau} = I\vec{\alpha}$

$\tau = I\alpha$

↳ ? $\quad$ I = is given most of the time

we will calculate it



$\tau = |\vec{r} \times \vec{F}| = FR \sin 90 = FR = I\alpha$

Disc $\rightarrow$ $I = \frac{MR^2}{2}$?

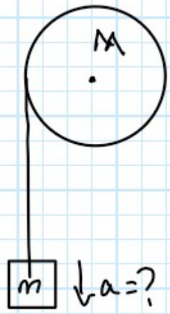
cylinder $\qquad \int r^2 dm = I_{cylinder}$

$FR = \frac{MR^2}{2}\alpha \qquad \alpha = \frac{2F}{MR} \qquad \alpha > 0$

FREE BODY DIAGRAM!! $\Rightarrow$ asked in FINAL

FREE BODY DIAGRAM!! ⟹ asked in <u>FINAL</u>

ex



$\downarrow a = ?$

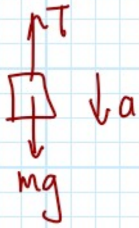
Disc

$\alpha = ?$

$a = ?$

$I = \frac{MR^2}{2}$

FBD



$\Sigma F = ma \quad \downarrow +$

$mg - T = ma$ (1) a? α? T?

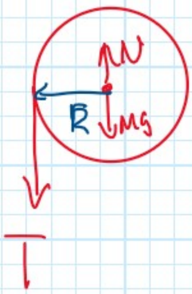
$\Sigma F = M\cancel{a}^{\,0}$

$T + mg - N = 0$ ✗

$\quad ? \qquad\quad ?$

Rotation ⟷ Linear

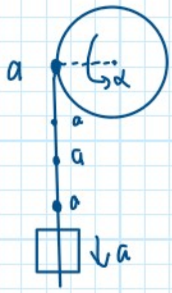
$\alpha R = a$ (3)



$$\Sigma \tau = \underbrace{\tau_T}_{TR} + \underbrace{\cancel{\tau_{mg}}}_{r=0} + \underbrace{\cancel{\tau_N}}_{r=0}{}^{0} = I\alpha$$

$TR = I\alpha$ (2) T? $\alpha = ?$

$$\begin{cases} \theta R = x \\ \omega R = v \\ \alpha R = a \end{cases}$$



$\alpha R = a$ (condition rolling without slipping)

$TR = I\alpha \qquad \alpha R = a \qquad mg - T = ma$

$I = \frac{MR^2}{2}$

$TR = I\frac{a}{R}$

$T = \frac{Ia}{R^2}$

$mg - \frac{Ia}{R^2} = ma$

$mg = \left(m + \frac{I}{R^2}\right) a$

$$a = \frac{mg}{\left(m + I/R^2\right)} = \frac{mg}{m + \frac{MR^2}{2R^2}} = \frac{m}{\left(m + \frac{M}{2}\right)} g < g$$

$$\alpha = \frac{a}{R} = \frac{mg}{R\left(m + \frac{M}{2}\right)}$$

How many rotations will disc experience in 2 second?

$$\theta_f = \theta_i + \underset{0}{\omega_i t} + \frac{1}{2}\alpha t^2$$

$$a = \quad \Box t \;-\; \Delta t \;-\; \Big|\; y = \frac{1}{2}at^2 \Rightarrow \frac{y}{2\pi R}$$

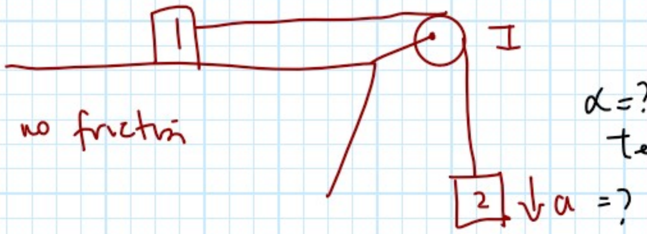
$$\Delta\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\frac{mg}{\left(m+\frac{M}{2}\right)R}(2^2) \Rightarrow \frac{\Delta\theta}{2\pi R} = \text{\# of rotations}$$

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

$$(\omega_i R = v_i)$$

* FROM now on; DISCS = PULLEYS are rotating!!

(ex)



no friction

$\alpha = ?$

tension?

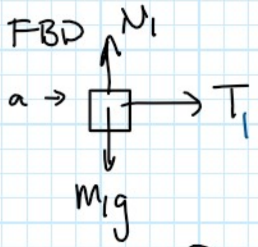
$\downarrow a = ?$

Before

$$I = 0 = \frac{MR^2}{2}$$

$M \simeq 0$ not rotating

FBD



rotates $\Sigma\tau > 0 \quad \tau = I\alpha$

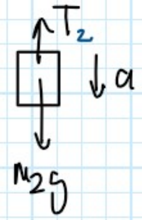
$$\tau_{T_1} + \tau_{T_2} + \underset{0}{\tau_{mg}} + \underset{0}{\tau_N}$$

$$-T_1R + T_2R$$

$$\Sigma\tau = \underbrace{T_2R - T_1R = I\alpha}, \ast ③$$

$$T_1 = m_1 a \quad ①$$

$$m_2 g - T_2 = m_2 a \quad ②$$



④ rolling w/o slipping

$$\alpha R = a \quad ④$$

① + ②

$$m_2 g - T_2 + T_1 = (m_1 + m_2)a$$

$$m_2 g - \frac{Ia}{R^2} = (m_1 + m_2)a$$

$$m_2 g = \left(m_1 + m_2 + \frac{I}{R^2}\right)a$$

③ & ④ $\quad (T_2 - T_1)R = I\frac{a}{R}$

$$T_2 - T_1 = I\frac{a}{R^2}$$

$$\boxed{a = \frac{m_2}{\left(m_1 + m_2 + \frac{I}{R^2}\right)}g}$$

$$I = \frac{M_3 R^2}{2}$$

$$\alpha = \frac{a}{R}$$

$\alpha = \frac{\cdot}{R}$

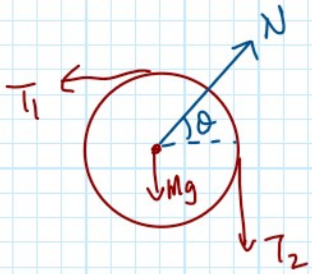
$T_1 = m_1 a = m_1 (\dots)$ $\qquad m_2 g - m_2 a = T_2$ ✓

If $I = 0$ — not rotating disc

$$\begin{cases} a = \dfrac{m_2}{m_1 + m_2} g \\ T_1 = T_2 \end{cases}$$

$\alpha = 0$

$\tau = 0 = I\alpha$

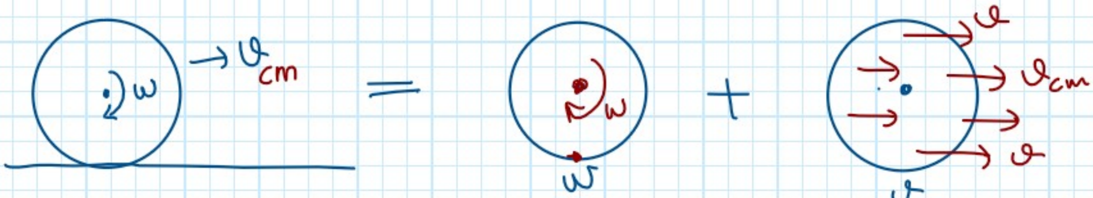


$\theta = ?$

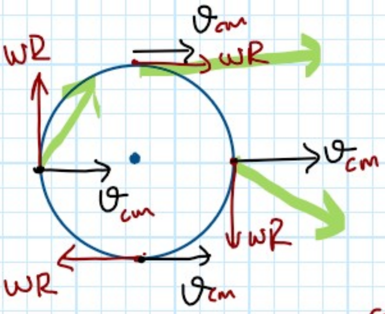
$N = ?$

$$\sum \vec{F} = 0 \quad \begin{cases} N\cos\theta = T_1 \Rightarrow \sum F_x = 0 \\ N\sin\theta = Mg + T_2 \end{cases}$$

ROTATING WITHOUT SLIPPING $\quad v = \omega R$



rotating about its central axis $\qquad$ translating



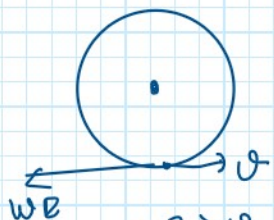
contact point should have NET NO VELOCITY

$\omega R = v_{cm} \equiv$ rotates w/o slipping $\qquad \overleftarrow{\omega R} \; \overrightarrow{v_{cm}} = 0$

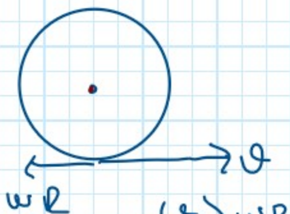
$\frac{d}{dt}$

$$\frac{d\omega}{dt} R = \frac{dv}{dt}$$

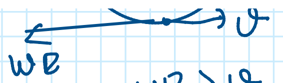
$$\boxed{\alpha R = a_{cm}} \equiv \boxed{\omega R = v_{cm}} \equiv \boxed{\theta R = X_{cm}}$$



$\omega R > v$ (patinaj)



$v > \omega R$ (lastik izi)



wR > v (patinaj)

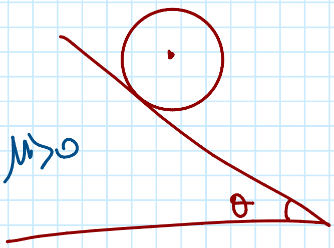
wR

v > wR (lastik izi)
lastik kayıyor.

!! SLIPPING !!

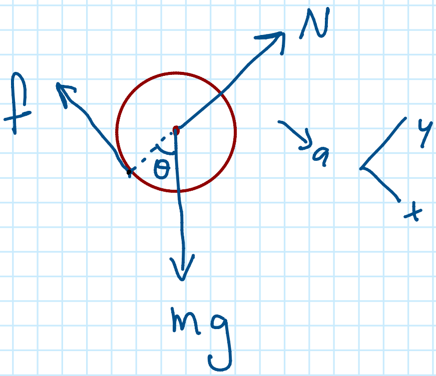
(ex) rolling on an inclined plane

Rolling w/o slipping



a=? friction force f=?

friction constant μ=?

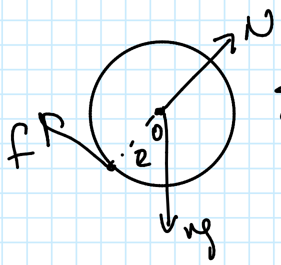


$\Sigma F_y = N - mg\cos\theta = m\cancel{a_y}^{\,0}$

$N = mg\cos\theta$ $\mu N = f$

$\Sigma F_x = mg\sin\theta - f = ma$

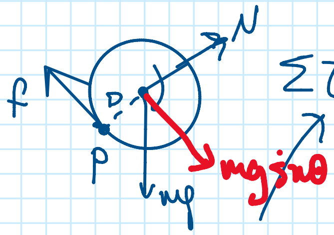
$\Sigma \tau = I\alpha$ $\alpha R = a$



$\Sigma \tau_o = fR = I_o \alpha_o$

$I_o = \frac{2}{5} mR^2$

$\Sigma \tau = fR = \left(\frac{2}{5} mR^2\right)\alpha_o$

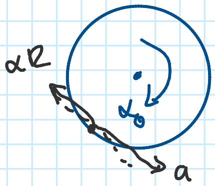


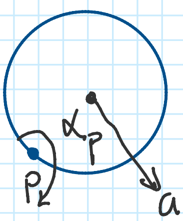
$\Sigma \tau_p = I_p \alpha_p$

$I_p = I_o + MD^2$ parallel axis theorem

$I_p = \frac{2}{5} mR^2 + mR^2$

$I_p = \frac{7}{5} mR^2$





$a = R\alpha_p$

$\Sigma \tau_p = \tau_{mg} + \cancel{\tau_N}^{\,0} + \cancel{\tau_f}^{\,0}$

$\tau_p = mgR\sin\theta = \frac{7}{5} mR^2 \alpha_p$

$\alpha, a, \underline{\mu}$

(1) $mg\sin\theta - f = ma$

$mg\sin\theta - \mu mg\cos\theta = ma$

$g(\sin\theta - \mu\cos\theta) = a$

$g\sin\theta - \mu g\cos\theta = \frac{5}{2}\mu g\cos\theta$

(2) $fR = I\alpha$

$\mu \cancel{m} g\cos\theta \cancel{R} = \frac{2}{5}\cancel{m}R^{\cancel{2}}\underbrace{\alpha}_{a}$

$\mu g\cos\theta = \frac{2}{5} a$

(3) Rolling w/o slipping

$\alpha R = a$

$$g\sin\theta - \mu g\cos\theta = \frac{5}{2}\mu g\cos\theta \qquad \mu g\cos\theta = \frac{2}{5}\overset{\alpha}{a}$$

slipping
$\alpha R = a$

$$g\sin\theta = \left(\frac{5}{2}+1\right)\mu g\cos\theta$$

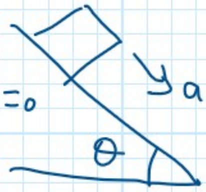
$$\sin\theta = \frac{7}{2}\mu\cos\theta \qquad \boxed{\mu = \frac{2}{7}\tan\theta}$$

$\theta > \tan^{-1}\left(\frac{7\mu}{2}\right)$

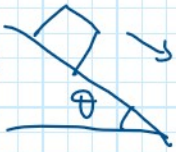
SLIPS !!

$$a = \frac{5}{2}\mu g\cos\theta = \frac{5}{2}\,\frac{2}{7}\tan\theta\, g\cos\theta = \frac{5}{7}g\sin\theta = a \qquad \alpha = \frac{a}{R}$$

$$f = \mu mg\cos\theta = \frac{2}{7}\tan\theta\, mg\cos\theta = \frac{2}{7}mg\sin\theta$$



$\mu = 0$ $\quad a = g\sin\theta$ $\quad a_1$



μ $\quad a = g(\sin\theta - \mu\cos\theta)$; if $\mu = \frac{2}{7}\tan\theta$ $\quad a_2$ $\rightarrow \frac{5}{7}g\sin\theta$

$$a_1 > \frac{5}{7}g\sin\theta = a_2$$

x	θ
$v \longleftrightarrow$	ω
a	α
F	$\tau = \vec{r}\times\vec{F}$
$m \longleftrightarrow$	I

$I = mr^2$

Work

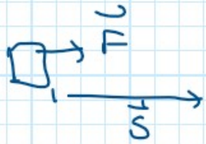
$K = \frac{1}{2}mv^2 \rightarrow \frac{1}{2}I\omega^2 = K$

$P = mv \rightarrow I\omega = L$

$\Sigma P_i = \Sigma P_f \Rightarrow \Sigma L_i = \Sigma L_f$

WORK in Rotational motion

$W = \tau\theta \equiv$ joule
$= RF\theta$
$= F\underbrace{R\theta}_{x} = W$



$W = \vec{F}\cdot\vec{s} = \vec{F}\cdot\vec{x}$

$$K = \frac{1}{2}I\omega^2$$

$$= \left[kg\, m^2 \frac{rad^2}{s^2}\right]$$

$$= \left\{kg\, m^2/s^2 = \text{Joule}\right\}$$

KE Theorem $\quad \Sigma W = \Delta K$

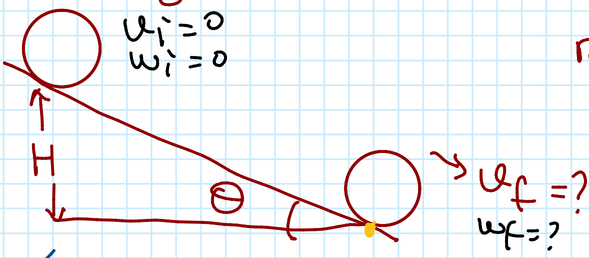
$$\tau\theta = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$

$$P = \text{Power} = \frac{W}{t} = F\frac{x}{t} = \vec{F}\cdot\vec{v}$$

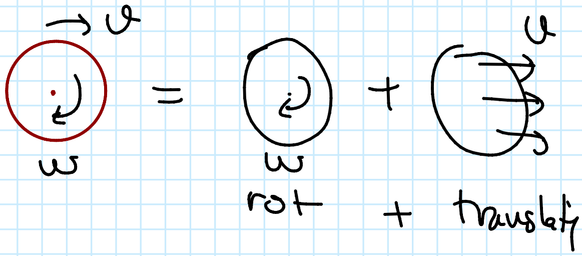
$$\boxed{P = \tau\omega} \; *$$

horsepower = watt vs torque

Energy Conservation in Rotational Motion



Rolling w/o slipping



$$E_i = E_f$$

$$U_i + K_i = U_f + K_f$$

$$mgy_i + \frac{1}{2}m\cancel{v_i^2} + \frac{1}{2}I\cancel{\omega_i^2} = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega^2 + mg\cancel{y_f}$$

$y_i = H$, 0, 0, 0

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$+ \quad \omega R = v$ rolling w/o slipping

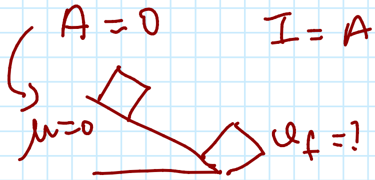
$$mgH = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 \quad ; \quad I = \frac{1}{2}mR^2 ; \ \frac{2}{5}mR^2$$

$$= \frac{1}{2}mv_f^2 + \frac{A}{2}mR^2\omega_f^2 \qquad I = A\,mR^2$$

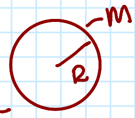
$R^2\omega_f^2 = v_f^2$

$$mgH = \frac{1}{2}m(1+A)v_f^2 \Rightarrow \underbrace{v_f = \sqrt{\frac{2gH}{(1+A)}}} \qquad \omega_f = \frac{1}{R}\sqrt{\frac{2gH}{1+A}}$$

$A = 0 \quad I = A\,mR^2 = 0 \equiv$ no rotation $\quad v_f = \sqrt{2gH}$



$A = 1 \quad I = mR^2$ RING



$$v_f = \sqrt{\frac{2gH}{2}} = \sqrt{gH} = v_1$$

$A = 1/2$ DISC

$$v_f = \sqrt{\frac{2gH}{1+1/2}} = \sqrt{\frac{4}{3}gH} = v_2$$

$A = 2/5$ full sphere

$$v_f = \sqrt{\frac{2gH}{1+2/5}} = \sqrt{\frac{10gH}{7}} = v_3$$

$$v_3 > v_2 > v_1 \qquad \omega_3 > \omega_2 > \omega_1 \qquad R = \text{same}$$

$$A_3 < A_2 < A_1$$

$$\frac{2}{5} < \frac{1}{2} < 1$$

$$I = \frac{mR^2}{2}$$