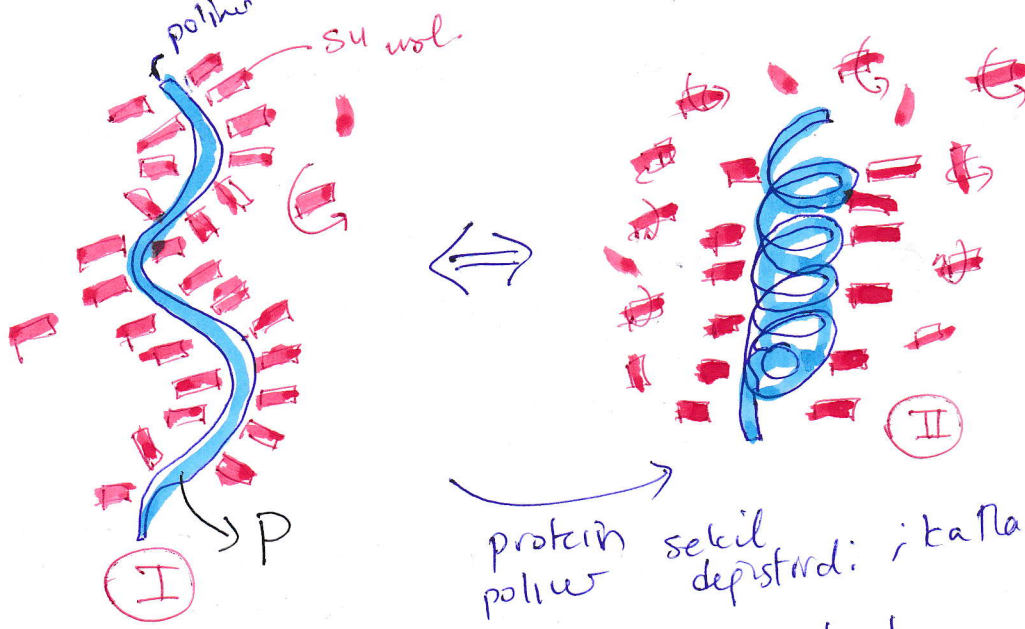




proteinin şekil değişti; katlandı $\Rightarrow$ düzenli

entropileri karşılaştırdık.

$$\begin{aligned}
 & S_{IP} > S_{IIP} \\
 & S_{ISu} < S_{IISu} \\
 & V_{ISu} < V_{IISu}
 \end{aligned}$$

sistemi toplam

$$S = S_p + S_{su}$$

$$S_I ; S_{II}$$

$$F = E - TS$$

$$\Rightarrow F_I \quad F_{II}$$

$$V \uparrow ; S \uparrow$$

Basınç P ile $\uparrow$; moleküllerin polimerin düzeni $\downarrow$

$$P \uparrow ; V_{su} \downarrow$$

Suyun entropisinin artmasında dolayı; $\Rightarrow$ oluşan kümelerde moleküller birbirlerini çekiyor.

Sudaki iyonlar

ve

Debye-Hückel yaklaşımı

$$\epsilon_{su} = 80$$



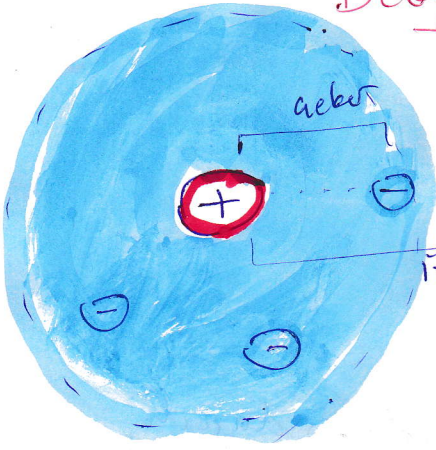
100mM NaCl
azaltısında
 $\rightarrow$ yoğunluk;

iyonlar arası mesafe $\sim 2nm = 2 \times 10^{-9}m$

iyon sayısı $\Rightarrow$ etkileşimi belirler etkiler.

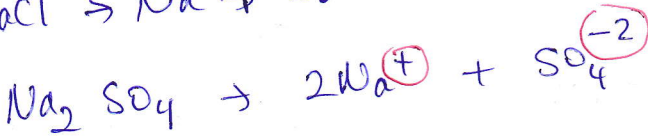
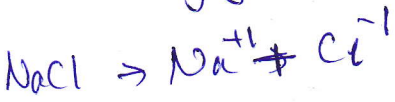
Debye - Hückel Teorisi

⇒ iyon aerbasi
plasma durumu



- $\oplus \Rightarrow$ iyon
- $\ominus \Rightarrow$ karst iyon (counter-ion)
- $\oplus \Rightarrow$ aym iyon (co-ion)

* mörkerde bulunan iyon etrafında küresel snetriye uygun iyon aerbasi / plazma bulutu bulunur



yük sayısı z_i

$$\left\{ \begin{array}{l} z_{\text{Na}^{+}} = +1 \\ z_{\text{Cl}} = -1 \\ z_{\text{SO}_4} = -2 \end{array} \right\}$$

* uzayda bir noktada bulunan ve ϕ potansiyel sahip iyonun potansiyel enerjisi

$$\frac{qV}{z_i e V}$$

* Boltzman denklemi enerji dağılımlarını öngörme yapar.

- 1) Düşük enerji li durumlar yüksek enerjili durumlara göre daha çok gözlenir.
- 2) $k_B T$ termal enerjiye sahip bir ortamda E_i enerjili sahip durum sayısı

$$n_i = C e^{-E_i/k_B T}$$

$$\left\{ \frac{n_i}{n_{\text{toplam}}} = \frac{e^{-E_i/k_B T}}{\sum_{i=0}^{\infty} e^{-E_i/k_B T}} \right\} \quad E_i = \text{enerji} = z_i e \phi$$

yoğunluk = konsantrasyon

$$C_i(\phi) = C_{i0} e^{-E_i/kT}$$

$$C_i(\phi) = C_{i0} e^{-z_i e \phi / kT}$$

$$E_i \leftrightarrow kT$$

C_{i0} = iyon merkezından çok-cok uzaktaki yoğunluk

$\phi \Rightarrow ?$ bilmiyoruz!!

$\phi \Rightarrow$ iyon yoğunluğuna bağlı

iyon yoğunluğu $\Rightarrow \phi$ 'ye bağlı

$C_i(\phi)$ bilmeden; $C_i(r)$; $\phi(r)$

bulmak önemli

Poisson Denklemi

$$\vec{\nabla} \cdot \vec{D} = \frac{\rho}{\epsilon_0}$$

$$\nabla(-\nabla\phi) = \frac{\rho}{\epsilon_0 \epsilon}$$

$$\nabla^2 \phi = -\frac{\rho}{\epsilon \epsilon_0}$$

$$\rho = \text{iyon yoğunluğu} \left[\frac{\text{coulomb}}{\text{m}^3} \right]$$

$$\rho = \sum_{i=1}^N C_i z_i \frac{N e}{F}$$

$$\text{Faraday sbt} = e N_A = \{1.6 \times 10^{-19} \times 6 \times 10^{23}\}$$

$$\rho = F \sum_{i=1}^N C_i z_i$$

$$\nabla^2 \phi = -\frac{F}{\epsilon \epsilon_0} \sum_{i=1}^N z_i C_{i0} e^{-\frac{z_i e \phi}{kT}}$$

Poisson - Boltzman denklemi

Yaklaşımları

- x) tek tip iyonlar
- x) $z_i e \phi \ll kT$; $e^{-x} = 1 + x + \frac{x^2}{2} + \dots$
- x) küresel simetri

kabul ederssek

PB - ağırlıklı yaklaşımlarla (sınırlı şekilde) elde edilir.

$$\phi(r) = \frac{z_i e}{4\pi\epsilon_0\epsilon} \frac{e^{-\kappa r}}{r}$$

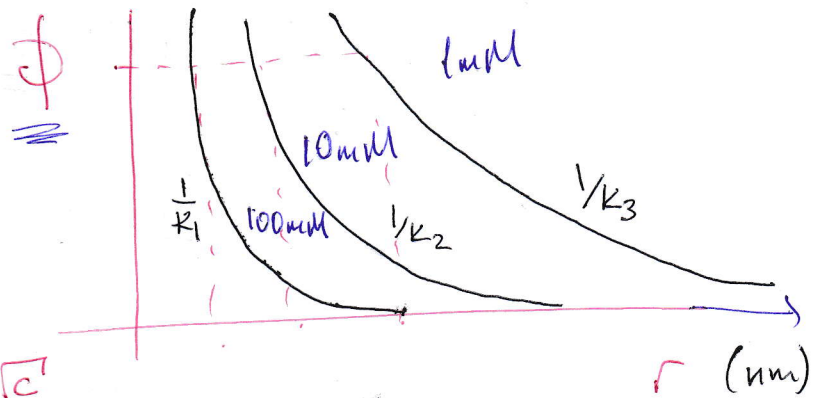
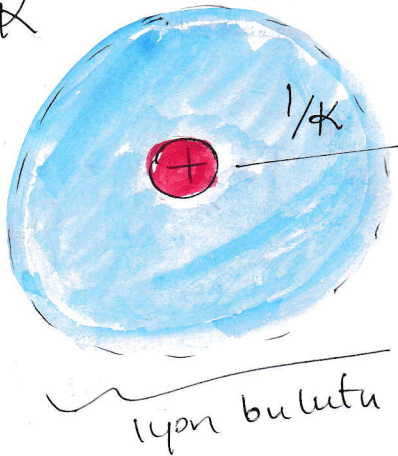
; $\kappa \equiv \text{Kappa}$

$$\kappa = \sqrt{\frac{e^2 N}{\epsilon_0 \epsilon k T} \sum_{i=1}^n c_{i0} z_i^2} \quad \left. \vphantom{\kappa = \sqrt{\frac{e^2 N}{\epsilon_0 \epsilon k T} \sum_{i=1}^n c_{i0} z_i^2}} \right\} \kappa = \left[\frac{l}{m} \right] = m^{-1}$$

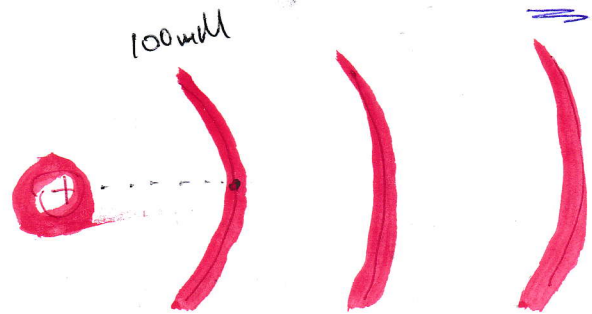
$$\Rightarrow \left\{ \begin{array}{l} N_e = F; \quad k_B N = \underline{R} \\ I \equiv \text{iyonik kuvvet} = \frac{1}{2} \sum_{i=1}^n c_{i0} z_i^2 \end{array} \right\}$$

$$\kappa = \sqrt{\frac{2 F^2}{\epsilon_0 \epsilon R T}} \sqrt{I}$$

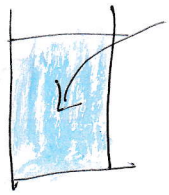
$\frac{1}{\kappa}$ = uzunluk biriminden $\equiv$ Debye yarıçapı uzunluğu
elektrostatik etkileşimin etkin yarıçapını temsil eder.



$$\phi \sim \frac{e^{-\kappa r}}{r}; \quad \kappa \sim \sqrt{I} \propto \sqrt{c}$$



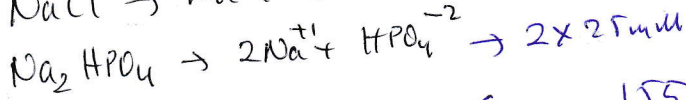
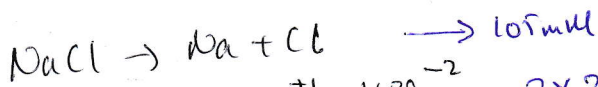
(ör)



105 mM NaCl
 5 mM KCl
 25 mM Na₂HPO₄
 2 mM CaCl₂

I = ?

$$I = \frac{1}{2} \sum z_i^2 C_{io} = \frac{1}{2} \left[C_{Na} z_{Na}^2 + C_K z_K^2 + C_{Cl} z_{Cl}^2 + C_{HPO_4} z_{HPO_4}^2 + C_{Ca} z_{Ca}^2 \right]$$



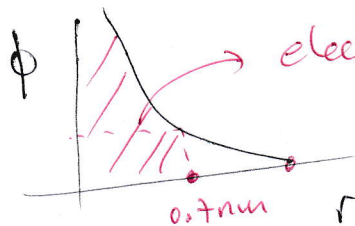
$C_{Na} = 155 \text{ mM}$

$$I = \frac{1}{2} \left[0.155(1^2) + 0.005(1)^2 + 0.114(1)^2 + 0.025(2)^2 + 0.002(2)^2 \right]$$

$= 0.191 \text{ mol/l} = 0.191 \text{ M}$

$\frac{1}{K} = \text{sb}t \frac{1}{\sqrt{I}} \text{ (nm)} = \frac{0.304}{\sqrt{I}} \text{ nm}$

$\frac{1}{K} = \frac{0.304}{\sqrt{0.191}} = 0.696 \approx \underline{\underline{0.7 \text{ nm}}}$ etkin
elektrostatik
etkileşim
uzunluğu



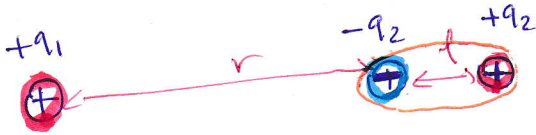
↳ merkez için
etkili $r < 0.7 \text{ nm}$
kissediliyor.

Moleküller arası etkileşim

Dipol

yon-dipol

yük-dipol etkileşimi (1 boyut)



Enerjisi

$$E_{id} = \left[-\frac{q_1(-q_2)}{4\pi\epsilon_0 r} \right] + \left[-\frac{q_1 q_2}{4\pi\epsilon_0 (r+l)} \right]$$

$$E = -W = \int_{\infty}^r \frac{q_1 q_2}{\epsilon_0 4\pi r^2} dr$$

$$W = \frac{-q_1 q_2}{4\pi\epsilon_0 r}$$

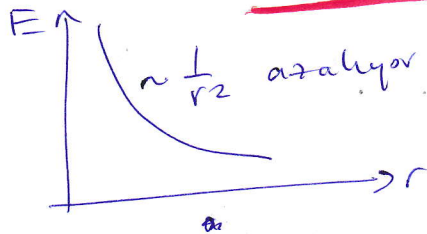
$\oplus q_2$ $\bar{r}_1$ $\oplus q_1$

$\mu = q_2 l$
dipol
moment

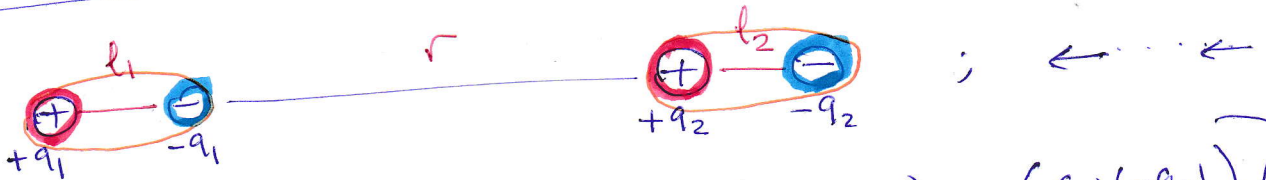
$$E_{id} = \frac{q_1 q_2}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{r+l} \right] = \frac{q_1 q_2 l}{4\pi\epsilon_0 r(r+l)}$$

$r \gg l$

$E_{id} \propto \frac{1}{r^2}$;



Dipol-Dipol

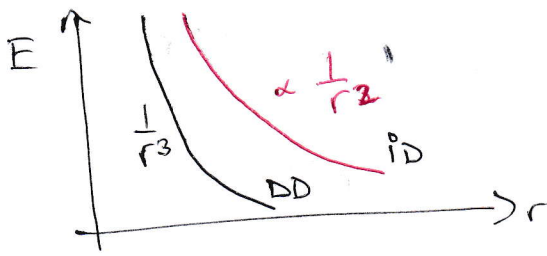


$$E_{dd} = \left[-\frac{1}{4\pi\epsilon_0} \left(\frac{q_1 q_2}{(r+l_1)} + \frac{q_1(-q_2)}{(r+l_1+l_2)} + \frac{(-q_1)q_2}{r} + \frac{(-q_1)(-q_2)}{r+l_2} \right) \right]$$

$$E_{dd} = \left\{ \frac{q_1 q_2 l_1 l_2}{4\pi\epsilon_0} \frac{l_1 + l_2 + 2r}{(r+l_1)(r+l_1+l_2)r(r+l_2)} \right\}$$

$r \gg l_1, l_2$

$$E_{dd} \propto \frac{2r}{r^4} \approx \frac{\mu_1 \mu_2}{4\pi\epsilon_0 \epsilon} \frac{2}{r^3}$$



$$E_{iD} \propto \frac{1}{r^2}$$

$$E_{DD} \propto \frac{1}{r^3}$$

⇒ indüklenmiş iki dipol
(dış etki ile) arası etkileşim

$$E \propto \frac{1}{r^4}$$

⇒ birbirini indükleyen iki dipol
arası etkileşim

$$E \propto \frac{1}{r^6}$$

$E \propto r^{-\alpha}$; α büyükse etkileşim o kadar KISA mesafelidir.

⇒ van der Waals, London dispersiyon kuvvetleri - - - - -

⇒ yük salınımları önemli

- termal olabilir

- e^- salınımları olabilir.



$$0 \Rightarrow 0 ; \underline{QM}$$

$$\frac{\text{Hamaker sbt}}{f(r^{-n})}$$

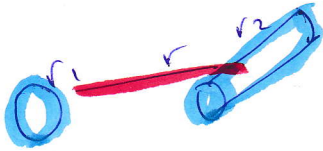
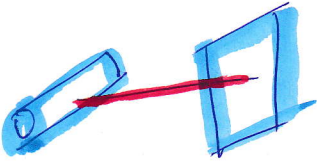
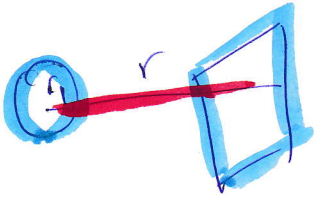
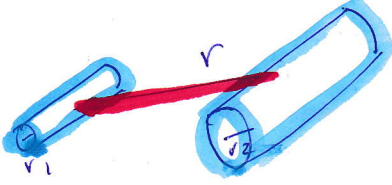
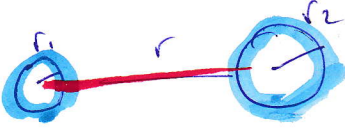
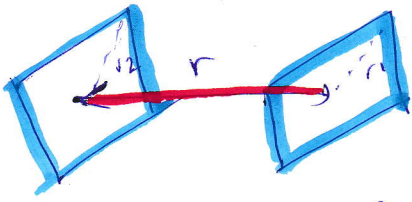
⇒ DLVO teorisi

Derjaguin
Landau
Verwey
Overbeek

Debye-Hückel teorisi ile elektrostatik geometrik parametreler hesabı katılarak etkileşimin çözümü.

vdW kuvvetleri molekülleri birbirine (günde) çeken kuvvetlerdir.
↳ elektrostatik + ~~termal~~ e^- salınımları

Geometri



$r_1, r_2 \gg r$	$r_1, r_2 \ll r$
r^{-2}	r^{-2}
r^{-1}	r^{-6}
$r^{-3/5}$	r^{-5}
r^{-1}	r^{-3}
$r^{-3/2}$	r^{-3}
r^{-1}	r^{-5}

DLVO

* Etkileşim kuvvetlerinin toplamından neye göre gelir.
 $r \uparrow$ Etkileşim $\downarrow$

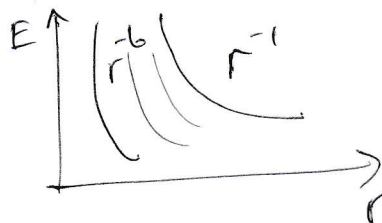
x) vdW çekimi ; Elektrostatik etkileşim itizi

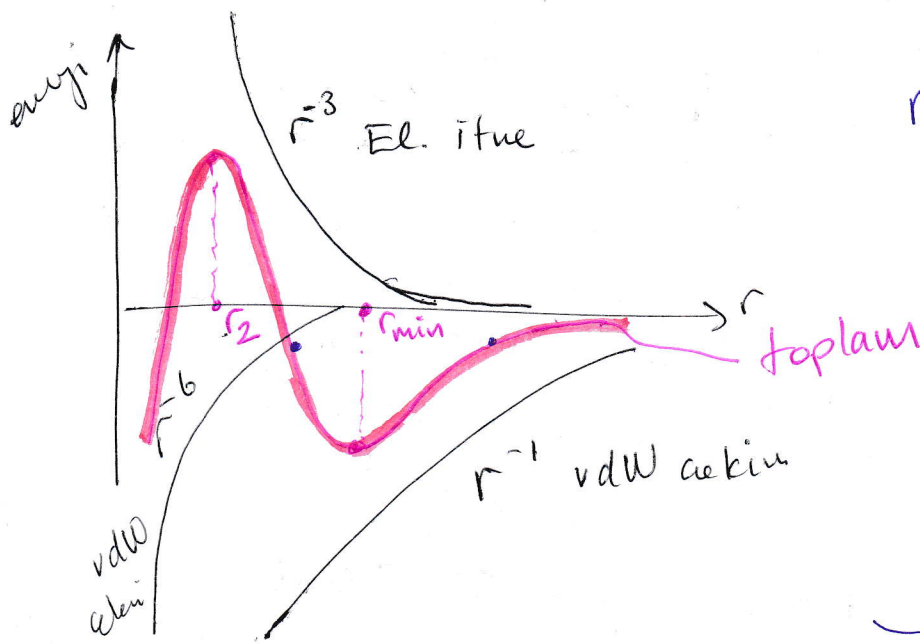
Kısa mesafede etkin etkileşimler.

$$r^{-5}; r^{-6}$$

Uzun mesafede etkin etkileşim

$$r^{-1}$$





5
 $r_{min} \equiv$ sistemin min - enerjiye iki mesafesi

$$r > r_{min}$$

1 — r — 2
 iki molekül birbirini çekiyor

$$r < r_{min}$$

1 — r — 2
 iki molekül birbirini iter.

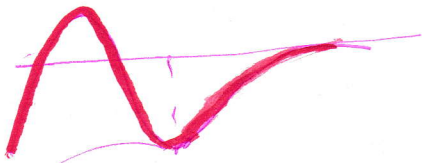
$$r_{min} \text{ da } \frac{\partial E}{\partial r} = 0$$

r_2 toplam enerjinin max olduğu

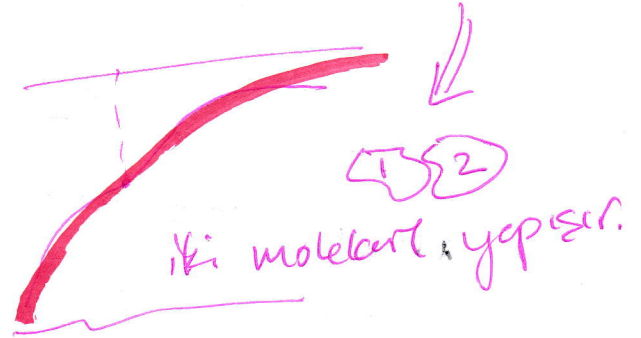
$r < r_2$ olursa

moleküller birbirini çeker - yapışır $\Sigma 1 2$ enerji

Tuz yoğunluğu arttırılırsa r_2 'deki 1 biriyor yok olur.



$\Rightarrow$



Biyolojik moleküllerin etkileşimini ıyon - pH değişimleri ve dielektrik sgt değiştirilerek kontrol edilir.



nature
katlanmış

Protein (Katlanması - Folding)
(Folding) (nature)
(denature)



acılmış
denature olmuş

} - iyon yoğunluğu
- sıcaklık
- pH
} dışarı

$$\Delta G \equiv \text{Gibbs enerji farkı} \sim 50 \frac{\text{kJ}}{\text{mol}}$$

protein acılamak/bözmek için a-a arası - 100 bağın bozulması lazım.

$$T = 300 \text{ K} ; k_B T ; RT = 2500 \text{ J/mol}$$

⇒ proteinlerde termal salınım var; fonksiyonu için önemli
(enzim aktivasyonu inaktivasyonu için vs)

⇒ hacim değişimi protein de nedir?

$$\langle \Delta V^2 \rangle ; \langle \Delta U^2 \rangle$$

hacim sapma

m = kütle

V = moleküler hacim

C_V = ısı sığası

β_T = izotermik hacim sıkıştırma sayısı

$$\langle \Delta U^2 \rangle = k_B T^2 m C_V$$

$$\langle \Delta V^2 \rangle = k_B T V \beta_T$$

$$M = 25000 \text{ olsun}$$

$$m = \frac{M}{N} = 4.15 \times 10^{-23} \text{ kg}$$

$$V = 3.2 \times 10^{-26} \text{ m}^3$$

$$C_V = 1.34 \text{ kJ/kg}$$

$$T = 300 \text{ K}$$

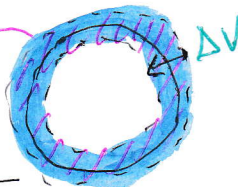
$$\beta_T = 2 \times 10^{-11} \text{ Pa}^{-1}$$

$$\Delta U = 2.6 \times 10^{-19} \text{ J}$$

$$\Delta U \times N = 157 \text{ kJ/mol}$$

$$\Delta V = 5 \times 10^{-29} \text{ m}^3$$

$$\frac{\Delta V}{V} = 0.16 \%$$



$$RT = 2500 \text{ J/mol}$$

157 kJ/mol enerjide bütün moleküller sentron olarak titreşir.
(enerjileri yüksek) = proteini denature etmek/acılamak için yeterli

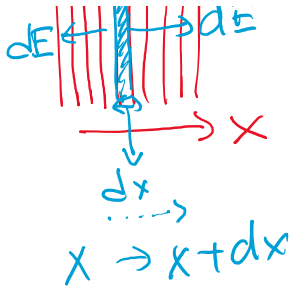
$$\frac{\Delta V}{V} = \% 0.16 \text{ küçülmüş olmasına rağmen}$$

proteini bozulmasına yeterli



çok büyük hacim bozulması sebep olur

iyonun gelip ile etkileşime yeterli



$$dE = \frac{\rho(x) dx}{2\epsilon}$$

$$E(x+dx) - E(x) = 2dE' = \frac{\rho(x) dx}{\epsilon}$$



$$\frac{dE}{dx} = \frac{\rho(x)}{\epsilon}$$

$$\frac{d}{dx} \left(-\frac{d\phi}{dx} \right) = -\frac{d^2\phi}{dx^2} = \frac{\rho(x)}{\epsilon}$$

$$\frac{d^2\phi(x)}{dx^2} = -\frac{1}{\epsilon} \sum_{\alpha} q_{\alpha} n_{\alpha}^0 e^{-\frac{q_{\alpha} \phi(x)}{kT}}$$

$$\nabla^2 \phi(r) = -\frac{1}{\epsilon} \sum_{\alpha} q_{\alpha} n_{\alpha}^0 e^{-\frac{q_{\alpha} \phi(r)}{kT}} ; \text{Poisson Boltzmann denk.}$$

$$n(r) = n_{\alpha}^0 e^{-\frac{q_{\alpha} \phi(r)}{kT}}$$

Basitlestirne: $\frac{q_{\alpha} \phi}{kT} \ll 1$

$$e^{-x} = (1 - x + \dots)$$

$$\nabla^2 \phi(r) = -\frac{1}{\epsilon} \sum_{\alpha} q_{\alpha} n_{\alpha}^0 \left[1 - \frac{q_{\alpha} \phi(r)}{kT} + \dots \right]$$

$$= -\frac{1}{\epsilon} \underbrace{\sum_{\alpha} q_{\alpha} n_{\alpha}^0}_{\text{toplam yük} \ll 0} + \frac{1}{\epsilon} \sum_{\alpha} (q_{\alpha}^2 n_{\alpha}^0) \frac{\phi(r)}{kT}$$

$N_{\text{aCl}} \rightarrow N_{\text{a}^+} + \text{Cl}^-$
nötr

$$\nabla^2 \phi(r) = \frac{1}{\epsilon kT} \left(\sum_{\alpha} q_{\alpha}^2 n_{\alpha}^0 \right) \phi(r) = \frac{1}{\lambda_D^2} \phi(r)$$

$\lambda_D = \text{Debye uzunluğu} \approx \kappa^{-1} (\text{kappa})$

$$\left\{ \frac{1}{[L]^2} \phi \right\} = \left\{ \frac{1}{[L]^2} \phi \right\} \quad \lambda_D = \text{Debye uzunluğu} -$$

$q \rightarrow e$
 $A \rightarrow C$

1 Boyutlu PB

$$\frac{d^2\phi}{dx^2} = -\frac{1}{\epsilon} \sum n_d^0 e^{-\frac{q\phi}{kT}}$$

* σ = pozitif {yüzey - σ yükü}

$$\int_0^{\infty} \rho(x) dx = \sigma$$

* $\phi \rightarrow$ rostgele $\phi(0) = 0$!

* $E = -\frac{d\phi}{dx}$

* $E(x < 0) = 0$

* $E(\infty) = 0$

* $E(0^+) = -\frac{\sigma}{\epsilon}$ (metal yüzey gibi)

$$\frac{d^2\Phi(x)}{dx^2} = -\frac{q}{\epsilon} n_0 e^{-\frac{q\Phi(x)}{kT}}$$

$\sigma = \int_0^{\infty} \rho(x) dx = n_0 \int_0^{\infty} dx e^{-\frac{q\Phi(x)}{kT}}$

Birimsiz: $\frac{q\Phi(x)}{kT} = \phi(x)$ birimsiz

$$\Rightarrow \Phi(x) = \frac{\phi(x) kT}{q}$$

$$\frac{d^2\phi(x)}{dx^2} = -\frac{4\pi q^2}{\epsilon kT} n_0 e^{-\phi(x)} = -4\pi \ell_B n_0 e^{-\phi(x)}$$

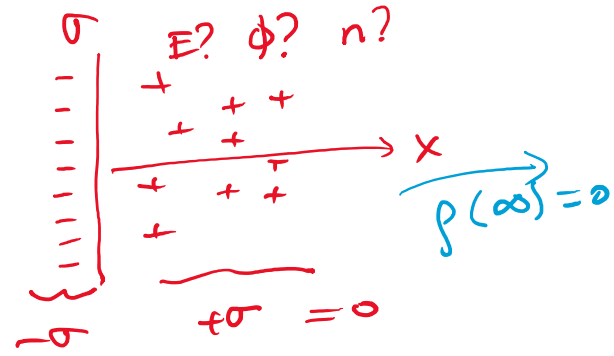
* ℓ_B = Bjerrum uzunluğu

$$\ell_B = \frac{q^2}{4\pi\epsilon k_B T}$$



$$\left\{ \frac{q^2}{4\pi\epsilon \ell_B} = k_B T \right.$$

termal enerjiye



... } $4\pi\epsilon l_B$
 elektrostatik enerji = termal enerjiye
 eşit olduğu
 uzunluk.

$$T = 300K$$

$$\epsilon = 80$$

su

$$q = e$$

$$l_B = 0.7nm$$

$$z \equiv \frac{x}{\lambda}$$

$$dz = \frac{dx}{\lambda}$$

$$\frac{1}{\lambda^2} = 8\pi l_B n_0 = \cancel{8\pi} \frac{q^2}{4\pi\epsilon kT} n_0$$

$$\left[\frac{1}{m^2} \right] = \left[m \frac{1}{m^3} \right]$$

$-\phi(x)$

$$\frac{d^2\phi}{dx^2} = -4\pi l_B n_0 e$$

$$\frac{1}{\lambda^2} \frac{d^2\phi}{dz^2} = -\frac{1}{2\lambda^2} e^{-\phi(z)}$$

$$\boxed{\frac{d^2\phi(z)}{dz^2} = -\frac{1}{2} e^{-\phi(z)}}$$

$$F = m \frac{d^2x}{dt^2} = -\frac{dV}{dx} \cdot \frac{dx}{dt}$$

$$F \frac{dx}{dt} = m \frac{d^2x}{dt^2} \frac{dx}{dt} = -\frac{dV}{dx} \frac{dx}{dt} = \frac{d}{dt} (-V(x))$$

$$m \frac{d}{dt} \frac{dx}{dt} \frac{dx}{dt} + \frac{d}{dt} \left(m \frac{v^2}{2} \right) = -\frac{d}{dt} (-V(x))$$

$$m v^2 + V(x) = E = \text{const}$$

$$\frac{d}{dt} \left(\frac{mv^2}{2} \right) \quad \left\{ \frac{mv^2}{2} + V(x) = E = \text{const} \right\}$$

$$\frac{d^2\phi}{dz^2} \frac{d\phi}{dz} = -\frac{1}{2} e^{-\phi(z)} \frac{d\phi}{dz}$$

$$\frac{d}{dz} \left[\frac{1}{2} \left(\frac{d\phi}{dz} \right)^2 \right] = \frac{d}{dz} \left[\frac{1}{2} e^{-\phi(z)} \right]$$

$$\left(\frac{d\phi}{dz} \right)^2 = e^{-\phi(z)} + sbt$$

$$\frac{d\phi}{dz} = e^{-\frac{\phi(z)}{2}}$$

$$\int e^{+\frac{\phi}{2}} d\phi = \int dz$$

$$2 e^{\frac{\phi}{2}} = z + sbt$$

$$\ln(e^{\phi/2}) = \ln\left(\frac{z}{2} + sbt\right)$$

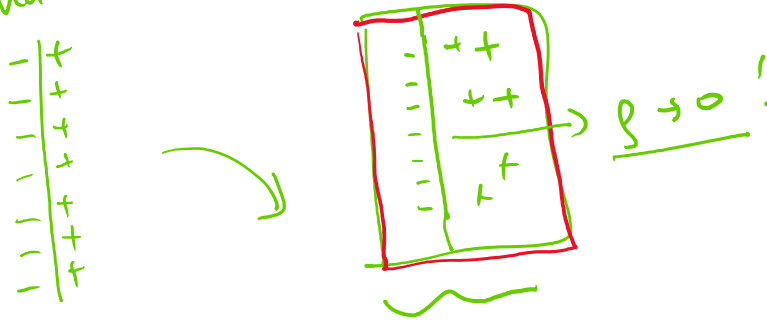
$$\frac{\phi}{2} = \ln\left(\frac{z}{2} + sbt\right) \rightarrow 1$$

$$\phi = 2 \ln\left(1 + \frac{z}{2}\right)$$

$$\phi = 2 \ln \left(1 + \frac{z}{2} \right)$$

sınır koşulları $\phi(0) = 0$ aldık

levha $\phi(z) \xrightarrow{z \rightarrow \infty} \infty$; $p(\infty) = 0$



$$\phi(z) = 2 \ln\left(1 + \frac{z}{2}\right) ; z = \frac{x}{\lambda} ; \lambda = \dots$$

$$\Phi(x) = \frac{2k_B T}{q} \ln\left(1 + \frac{x}{2\lambda}\right)$$

pot.

$$E(x) = -\frac{d\Phi}{dx} = -\frac{k_B T}{q\lambda} \left(\frac{1}{1 + \frac{x}{2\lambda}}\right)$$

El. alan

$$g(x) = -\varepsilon \frac{d^2\Phi}{dx^2} = \frac{\varepsilon k_B T}{2q\lambda^2} \frac{1}{\left(1 + \frac{x}{2\lambda}\right)^2}$$

yük

$$\Phi(x) = \frac{2k_B T}{q} \ln\left(1 + \frac{q\sigma}{2\varepsilon k_B T} x\right)$$

$$E = -\frac{\sigma}{\varepsilon} \frac{1}{\left(1 + \frac{q\sigma}{2\varepsilon k_B T} x\right)}$$

$$g(x) = \frac{q\sigma^2}{2\varepsilon k_B T} \frac{1}{\left(1 + \frac{q\sigma}{2\varepsilon k_B T} x\right)^2}$$

$$n(x) = \frac{g(x)}{q}$$

$$n(x) = \frac{\sigma^2}{2\varepsilon k_B T} \frac{1}{\left(1 + \frac{q\sigma}{2\varepsilon k_B T} x\right)^2}$$

$$\Phi(0) = 0 !!$$

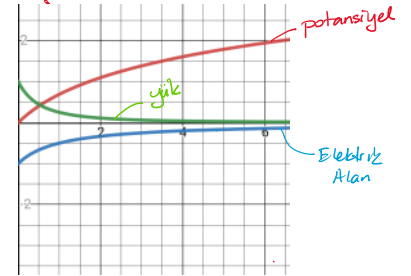
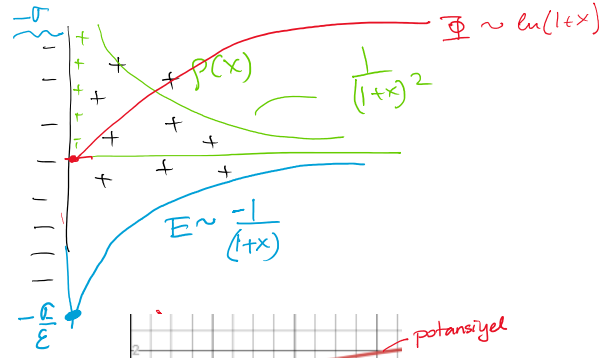
$$E(0^+) = -\frac{q}{2\varepsilon}$$

$$-\frac{k_B T}{q\lambda} = -\frac{q}{2\varepsilon} \Rightarrow \lambda = \frac{\varepsilon k_B T}{q^2}$$

$$\sim \ln(1+x) = \Phi$$

$$\sim -\frac{1}{(1+x)} = E$$

$$\sim \frac{1}{(1+x)^2} = g$$



* yük bulutu $g(x) \sim \frac{1}{x^2}$ ile azalır.

* karakteristik uzunluk

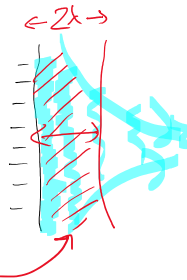
$$2\lambda = \frac{2\varepsilon k_B T}{\frac{\sigma}{e} e} \frac{2\pi}{2\pi}$$

$$\frac{q}{e} = \frac{\sigma}{e} = \text{yük sayısı yoğunluğu}$$

$$2\lambda = \frac{1}{2\pi \varepsilon_0 n_0} \equiv \frac{\text{Gouy-Chapman}}{\text{uzunluğu}} \Rightarrow n_0 \uparrow \lambda \downarrow$$

$$[m] = \frac{1}{\left[m \frac{\#}{m^2}\right]} = [m]$$

* Gouy-Chapman uzunluğu (bulut) + iyonların oluşturduğu katman uzunluğu

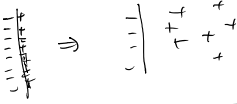


elektros. enerji $\oplus$ yükleri yüzeyde sıkılaştırdıkça entropi dağıtmaya çalışacak !!

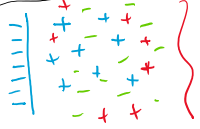
dinamik $\Rightarrow$ düzeni yaparak $\rightarrow$ enerji $\leftrightarrow$ entropi

yüzey + sy $\rightarrow \sigma$: yüzeydeki toplam yük

yüzey + su
(cam) $\rightarrow \sigma$: yüzeydeki toplam yük
{ hücre zarı }



yüzey + su + iyon/tuz $\rightarrow$ Na⁺ Cl⁻

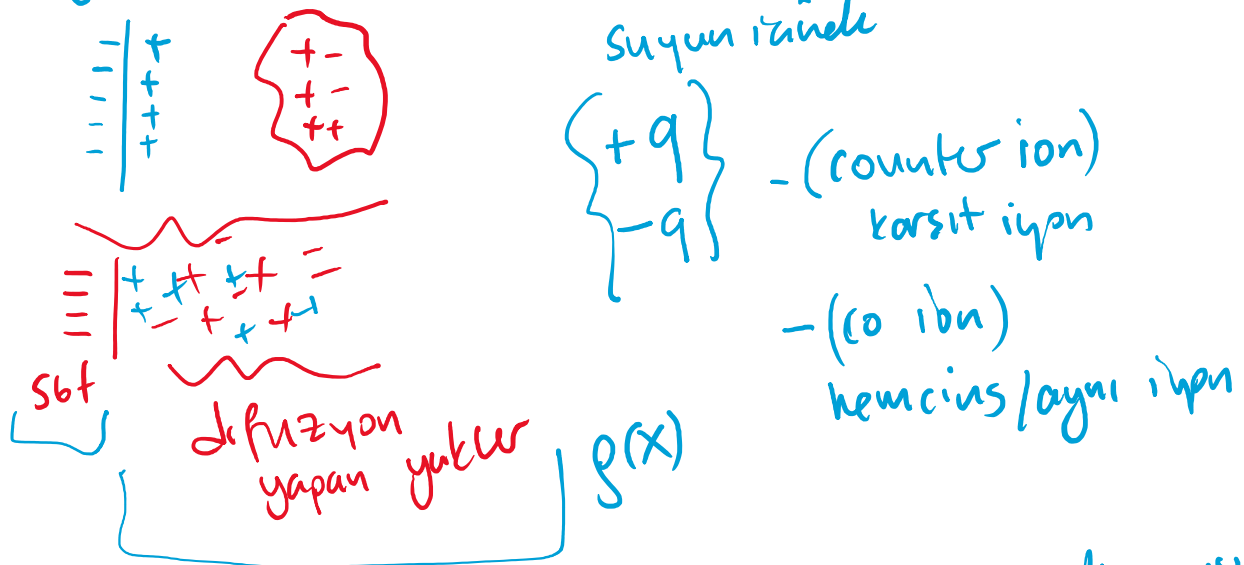


tuz yokken
nasıl değişir?

tuz yokken

Φ , E, ρ ... nasıl etkiler?!

Eğer tuz ilave edilirse;



$$\rho(x) = qn_+(x) + [-qn_-(x)] ; \quad n(x) \rightarrow \text{yük sayısı dağılım}$$

$$n_+ = n_0 e^{-\frac{q\phi}{kT}}$$

$$n_- = n_0 e^{-\frac{(-q)\phi}{kT}}$$

$$n_- = n_0 e^{\frac{q\phi}{kT}}$$

$$\rho(x) = qn_0 \left[e^{-\frac{q\phi(x)}{kT}} - e^{\frac{q\phi(x)}{kT}} \right]$$

$$= \left\{ qn_0 e^{-q\phi/kT} - qn_0 e^{q\phi/kT} \right\}$$

$$\frac{e^x - e^{-x}}{2} = \sinh x$$

$$\left\{ \frac{d^2 \phi(x)}{dx^2} = -\frac{\rho(x)}{\epsilon} = -\frac{q}{\epsilon} n_0 \left[\exp\left(-\frac{q\phi(x)}{kT}\right) - \exp\left(\frac{q\phi(x)}{kT}\right) \right] \right.$$

$$= \frac{2qn_0}{\epsilon} \sinh\left(\frac{q\phi}{kT}\right)$$

birimsizleştirme
 $x \rightarrow z$

$$z = \frac{x}{\lambda_D} ; \quad \lambda_D = \sqrt{\frac{\epsilon kT}{2q^2 n_0}}$$

$$= \frac{2q n_0}{\epsilon} \sinh\left(\frac{q\phi}{2T}\right)$$

birimsizleştirme

$x \rightarrow z$

$$z = \frac{x}{\lambda_D}$$

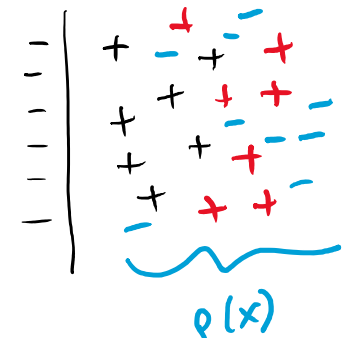
$$\lambda_D = \sqrt{\frac{\epsilon kT}{2q^2 n_0}}$$

$$\frac{d^2\phi(z)}{dz^2} = \sinh\phi$$

$$* \left(\frac{d\phi}{dz}\right)^2 = 2 \cosh\phi + sbt$$

* sınır koşulu

$$\begin{aligned} \phi(z \rightarrow \infty) &= 0 \\ E(z \rightarrow \infty) &= 0 \\ sbt &= -2 \end{aligned}$$

$$\sum q_- = \sum q_+$$


$g(x)$

* sınır koşulu

$$E(z=0) = -\frac{\sigma}{\epsilon}$$

$$\sigma = q \int_0^\infty dx [n_+(x) - n_-(x)]$$

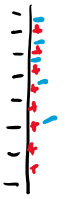
$$\left(\frac{d\phi}{dz}\right)^2 = 2 \cosh\phi - 2$$

$$\frac{d\phi}{dz} = \sqrt{2(\cosh\phi - 1)}$$

$$\frac{d\phi}{\sqrt{2(\cosh\phi - 1)}} = dz$$

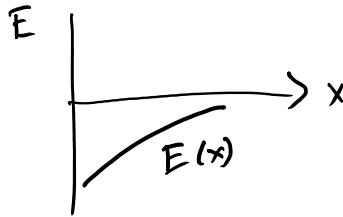
$$\phi(z) = 2 \ln \left(\frac{1 - C e^{-z}}{1 + C e^{-z}} \right) ; z = \frac{x}{\lambda_D}$$

$$1-c \approx 0$$



yüzeyde - iyon sayısı çok az
sıfır olabilir
{ olmayabilir de!! }

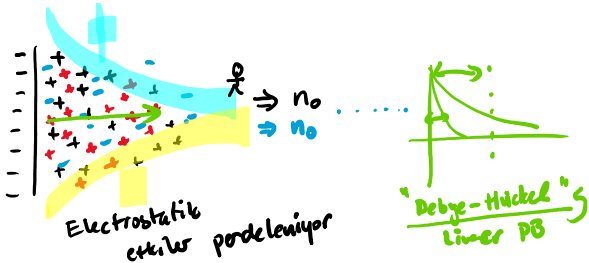
$$\Phi(x=0) = \frac{2k_B T}{q} \ln\left(\frac{1-c}{1+c}\right) \quad ; 0 < c < 1$$



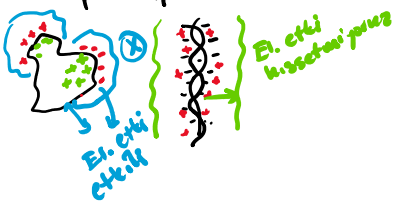
PB → lineerleştirip
azutulebilir

$$e^{-\frac{q\Phi}{k_B T}} = e^{-z} = \left\{ \underline{\underline{1 - z + \frac{z^2}{2} \dots}} \right\} \quad z \ll 1$$

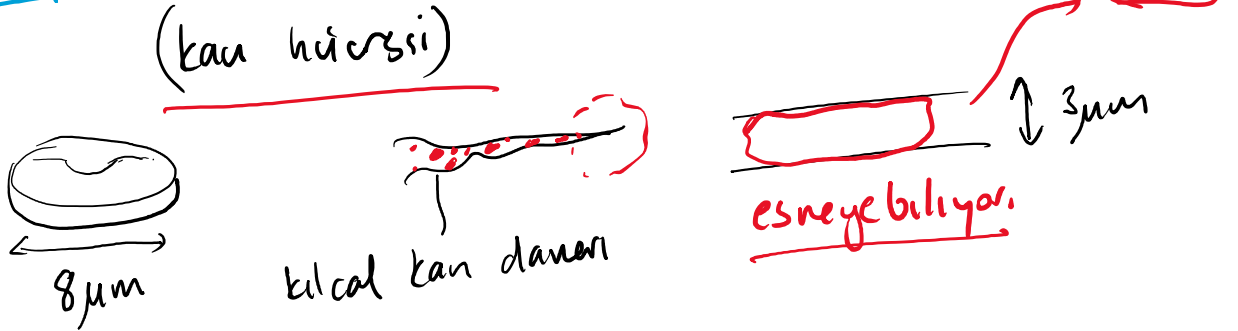
$$z \ll 1 ; \quad q\Phi \ll k_B T$$



H₂O iyonlaşma var!!
protein / polimer / hücre zarı



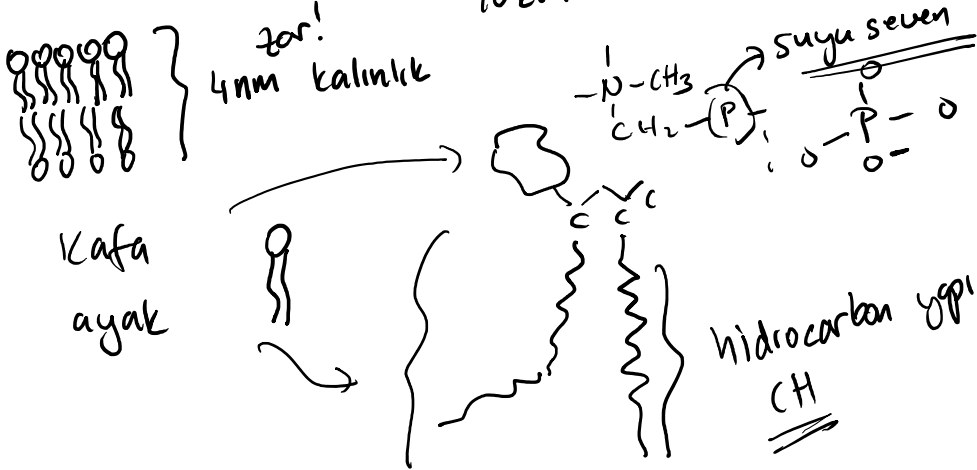
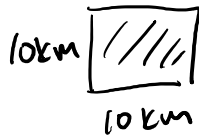
Biyo Zarlar (Biomembrane)



ESNEK!

Kan hücreleri 120 gün damarlarda, 10^5 kez kalbe/görüp çıkıyor

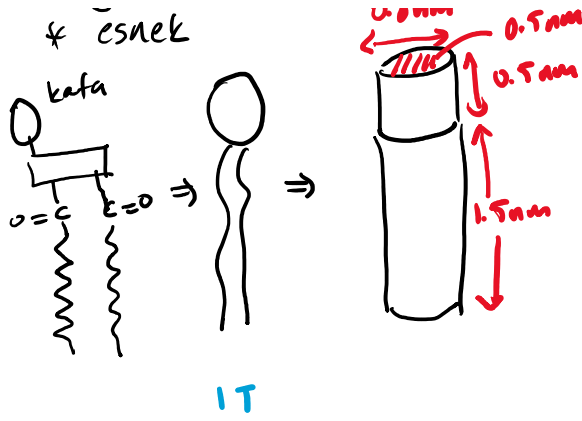
insanda
hücre zarı alanı $\sim 10-100 \text{ km}^2$!!



zar : (membrane)

- * Su geçişi mümkündür
- * diğer " mümkün değil? !
- * büyük molekül geçişleri " değıl.
- * "yumuşak"
- * esnek
- ~ kafa





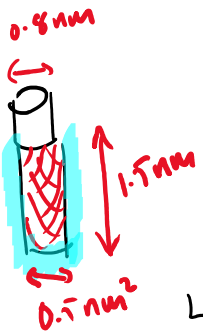
$C=C$ 240 kT
 $C-C$ 140 kT
 $H-bond$ 5-10 kT
 $C \dots C$ vdW 3 kT

} "yumuşak" !!

hidrofobik yüzeyin suyla temas etmesi için gerekli serbest enerji $\rightarrow \Delta F = \Delta E - T\Delta S$

$$\sigma = 10 \text{ kT/nm}^2$$

$$\sigma = 0.04 \text{ J/m}^2$$



bir lipid mol. suyla temas etme "cezası" !!

$L \Rightarrow [2\pi r L + \pi r^2] \sigma = \Sigma \text{ enerji}$

Alan

$r = 0.4 \text{ nm}$
 $L = 1.5 \text{ nm}$

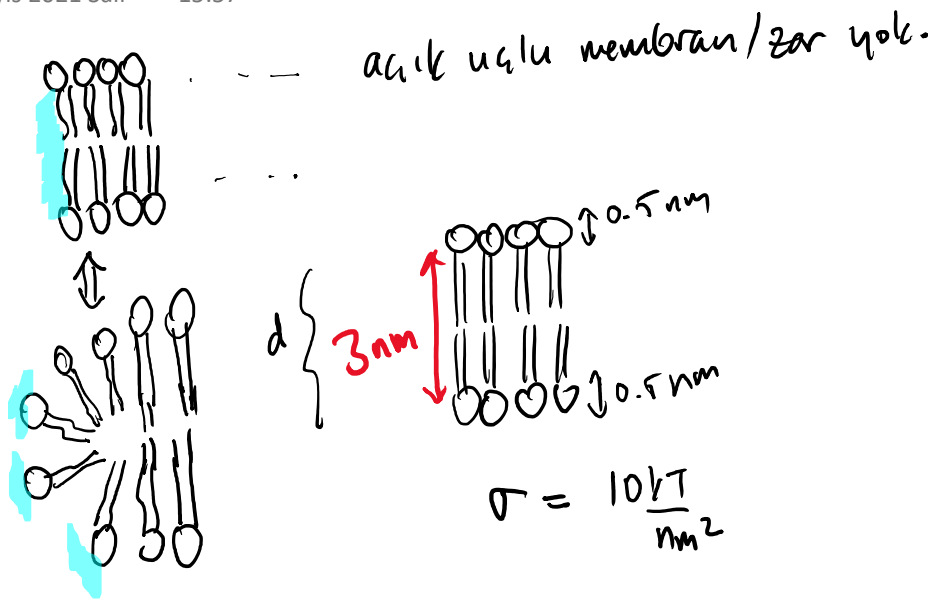
$$\left[2 \times 3 \times 0.4 \times 1.5 + 3 \times 0.4 \times 0.4 \right] 10 \frac{\text{kT}}{\text{nm}^2}$$

4 nm² $\times 10 \text{ kT/nm}^2$

$$\boxed{40 kT}$$

$$>> \underline{\underline{kT}}$$

biyolojik ortamda bu durumu game olasılığı $c = e^{-\frac{\Delta F}{kT}} = e^{-40} \approx 10^{-12}$



$$\sigma d = \lambda = 30 \frac{kT}{\text{nm}}$$

qizgi gerilimi $\equiv \lambda$

$$\lambda = \frac{30 (1.38 \times 10^{-23}) (300)}{10^{-9} \text{ m}} = 1.2 \times 10^{-10} \text{ N} = 120 \text{ pN}$$

Su moleküllerinin zardan geçişi



küre.

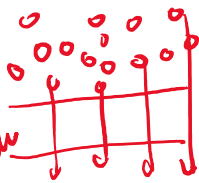
$r = 0.15 \text{ nm}$

$$\begin{aligned} \Delta F &= \sigma \text{ Alan} \\ &= (10 kT) (4\pi (0.15)^2) \\ &\approx \underline{\underline{3 k_B T \approx kT}} \end{aligned}$$

$P \sim e^{-3}$

N : su moleküllerinin

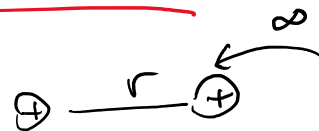
PN



iyonun zardan geçişi

Su $\epsilon \sim 80$

$$\frac{q^2}{4\pi\epsilon r} = W$$



$\sim 550 \sim P$

$$\frac{q^2}{4\pi\epsilon r} = W$$

$$\frac{e^2}{4\pi\epsilon r} = W$$

$$\frac{e^2}{4\pi\epsilon r} \approx 2.3 \times 10^{-18} \text{ J} \approx 550 kT$$

$$\epsilon \approx 1$$

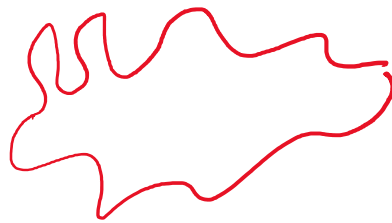
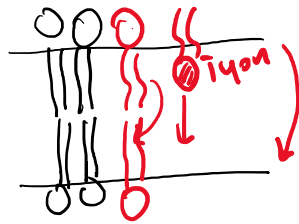
$$\epsilon \approx 80$$

$$\epsilon \approx 1$$

$$e^{-550} \approx 8$$

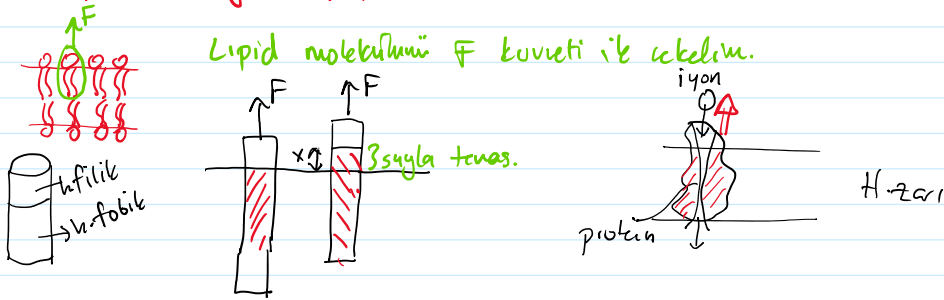
İyon geçişi çok düşük kadar az!!

lipid
nb. içinden geçen
yer deprotonasyonu



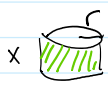
Biyoloji = biyomembran

Hidroforobik enerji ölçeri / zar davranışı



X uzunluğunda suyla temas ediyor.

$$\Delta E = \sigma A \quad ; \quad \sigma = 10 \text{ kBT} / \text{nm}^2 = 0.04 \text{ J/m}^2$$



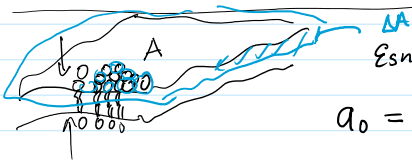
$$= (0.04) \frac{\text{J}}{\text{m}^2} (2\pi r x) \quad ; \quad r = 0.4 \text{ nm}$$

$$\Delta E = \text{enerji} = \sigma A = F x$$

$$\sigma 2\pi r x = F x \Rightarrow F = \sigma 2\pi r = \text{Newton}$$

$$F = \sigma 2\pi r = (0.04) (2 \times 3.14 \times 0.4) \frac{\text{J}}{\text{m}^2} \times 10^{-9} \text{ m}$$

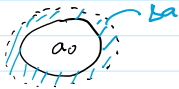
$$\approx 10^{-10} \text{ N} = 0.1 \text{ nN} = \underline{\underline{100 \text{ pN}}}$$



Esneeme modülü (katsayısı)

$$a_0 = \text{lipid mol. başına düşen alan} = 0.5 \text{ nm}^2$$

$$a = a_0 + \Delta a$$



$$\frac{\Delta a}{a_0} = \text{esneeme oranı} = \text{gerileme oranı} = \frac{a - a_0}{a_0} = \frac{\Delta a}{a_0}$$

$$\left\{ \frac{\text{Esneeme enerjisi}}{\text{Alan}} \right\} \Rightarrow \epsilon_s = \frac{1}{2} K_s \left(\frac{a - a_0}{a_0} \right)^2 \Rightarrow \left\{ \frac{1}{2} k \Delta x^2 \right\}$$

$\downarrow$ $\frac{\text{J}}{\text{m}^2}$ $\downarrow$ $\frac{\text{J}}{\text{m}^2}$ $K_s = \text{modülü}$

$$\epsilon_s = K_s = \left[\frac{E}{A} = \frac{F}{L} \right]$$

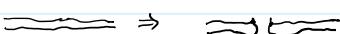
$$\epsilon = \frac{1}{2} K_s \frac{\Delta a^2}{a_0^2}$$

$$\ast \text{ membran gerilimi } \approx \text{kuvvet} = \frac{\text{enerji}}{\Delta \text{yol}} = \tau = \frac{d\epsilon}{d\left(\frac{\Delta a}{a_0}\right)}$$

$$= \frac{1}{2} K_s \frac{\Delta a}{a_0}$$

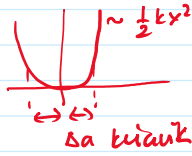
$$\tau = K_s \frac{\Delta a}{a_0} = K_s \frac{\Delta A}{A_0}$$

$\ast$ Biyoloji Lysis = zar yırtılması / açılması



Deweyse $\frac{\Delta A}{A_0} \gg 0.2$ ise $\Rightarrow$ 2arda yığılma meydana gelir.

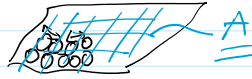
* Kuant harmonik



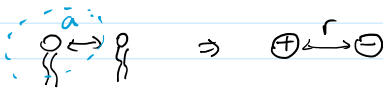
$$\zeta = K_S \frac{\Delta A}{A_0} = 0.02 K_S$$

yığılmadan önceki ortalama gerilim miktarı

Denge durumunda zar yapısı



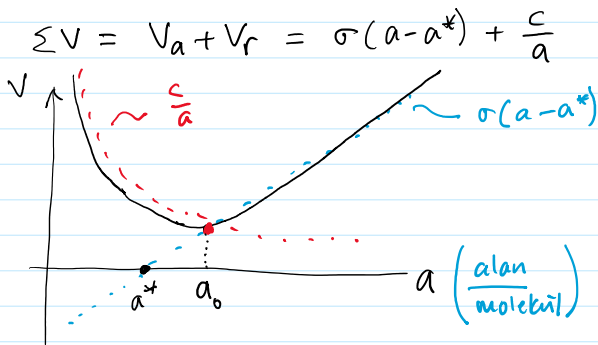
çekici İtici



Lipid başına çekici potansiyel enerji
Lipid başına itici pot. enerji

$$V_{att}(a) = \sigma(a - a^*)$$

$$V_{repel}(a) = \frac{c}{a}$$



Denge durumu:

$$\frac{dV}{da} = 0 = \sigma - \frac{c}{a_0^2} = 0$$

$$a = a_0$$

$$a_0^2 = \frac{c}{\sigma}$$

$$c = \sigma a_0^2 \quad ; \quad a_0 = 0.5 \text{ nm}^2$$

$$\frac{c}{a_0} = \sigma a_0 = 10 \frac{k_B T}{\text{nm}^2} \cdot 0.5 \text{ nm}^2 = 5 k_B T$$

$$\begin{aligned} \epsilon_s &= \frac{1}{2} K_S \frac{\Delta a^2}{a_0^2} \quad ; \quad V(a) = V_{att} + V_{rep} = \sigma(a - a^*) + \frac{c}{a} \\ &= \frac{1}{2} K_S \frac{(a - a_0)^2}{a_0^2} \quad \quad \quad = \sigma(a - a^*) + \frac{\sigma a_0^2}{a} \end{aligned}$$

$$\epsilon_s = 5kT + \frac{1}{2} K_S \frac{(a - a_0)^2}{a_0^2} + \dots (a - a_0)^3 + \dots$$

$$(?) \quad \epsilon_s = \frac{2}{a_0} V(a) \quad \Rightarrow \quad \frac{2}{a_0} \frac{d^2 V(a)}{da^2} = \frac{K_S}{a_0^2} \Rightarrow$$

$$K_S = 2 a_0 \left(\frac{2c}{a_0^3} \right) = 4 \frac{c}{a_0^2} = 4 \sigma = 4(0.04) \frac{\text{J}}{\text{m}^2}$$

$K_s = 0.16 \text{ J/m}^2 \Rightarrow$ zarı çok esnek / narin
metal $K \sim 10^2 - 10^3 \text{ J/m}^2$

$\gamma = 0.02 K_s = 3.2 \times 10^{-3} \text{ N/m}$ { yırtılmadan önceki kuvvet / gerilim }



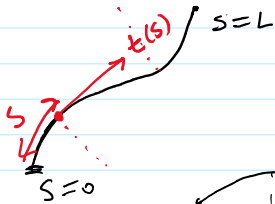
$3.2 \times 10^{-3} \frac{10^9 \text{ N}}{10^9 \text{ nm}}$

$K_s = \text{"yırtılma" modülü}$

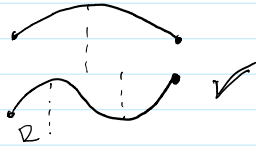
$K_b = \text{eğilme / bükülme modülü}$



Sert polimerin bükülmesi

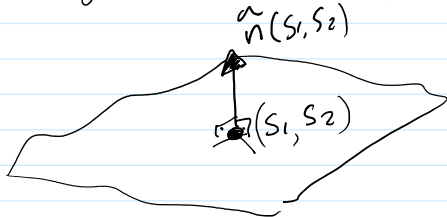


$E_{\text{bükme}} = \frac{K}{2} \int_0^L ds \frac{1}{R^2(s)}$



$R \downarrow; E \uparrow$

$\frac{K}{2} \int_0^L ds \frac{1}{R^2(s)} = \frac{K}{2} \int_0^L ds \frac{d\hat{t}}{ds} \cdot \frac{d\hat{t}}{ds} \dots$

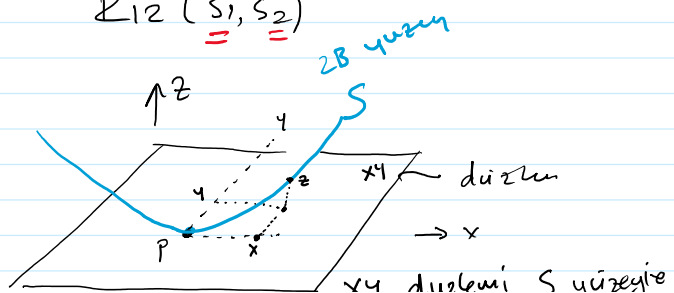
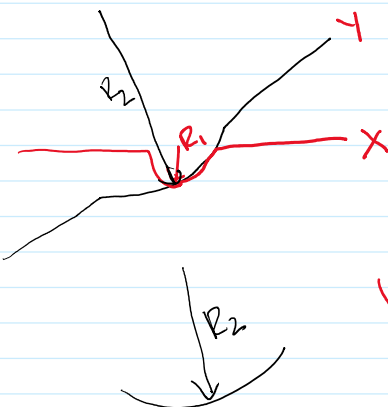


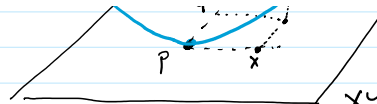
* Yüzeyde her nokta s_1, s_2 ile tanımlı

* Her noktada $\hat{n}$ normal vektörü var.

* Her noktada iki tane eğrilik parametresi / yarıçapı var.

$R_{12}(s_1, s_2)$

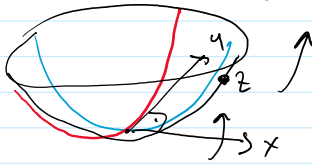




xy düzlemi S yüzeyine
P noktasında teğet

S yüzeyinde $z(x, y) = a_{xx} x^2 + a_{xy} xy + a_{yy} y^2 + \dots (x^3, y^3, x^2y, \dots)$

teğet eksenleri seçip!!

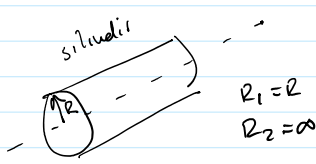


$$z(x, y) = \frac{1}{2R_1} x^2 + \frac{1}{2R_2} y^2 + (x^3, y^3, \dots) \Rightarrow a_{xy} = 0$$

eksen seçimi.

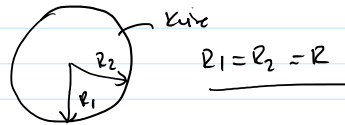
R_1, R_2 teğet eksenindeki
eğrilik yarıçapları

eğrilik = curvature

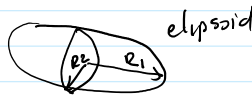


$$R_1 = R$$

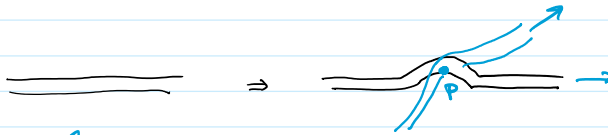
$$R_2 = \infty$$



$$R_1 = R_2 = R$$



elipsoid

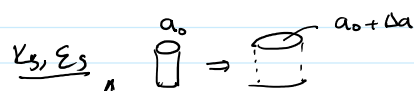
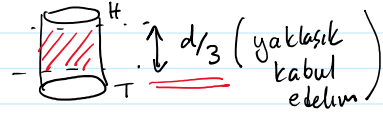
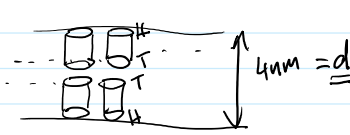


Bükülme enerji yoğunluğu
(bending)

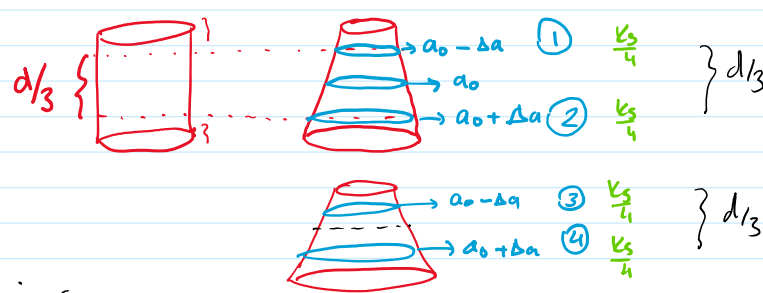
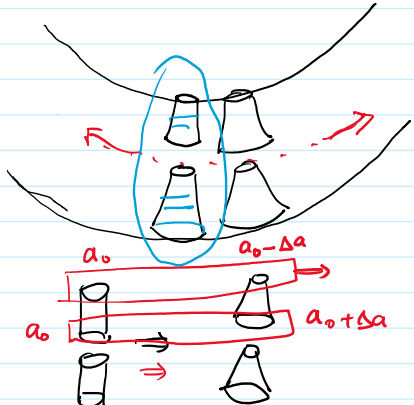
$$E_b(P) = \frac{K_b}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^2 = \left[\frac{\text{enerji}}{\text{alan}} \right]$$

$$\left\{ E_s = \frac{\text{enerji}}{\text{alan}} \right\}^v ; K_b = [\text{enerji}]$$

$$K_b \Rightarrow K_s ?$$



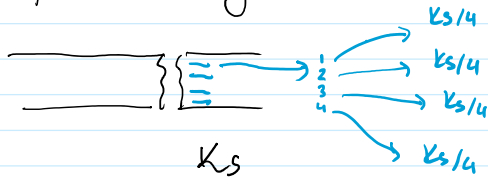
$$\left\{ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \right\}$$



2ar bükülme

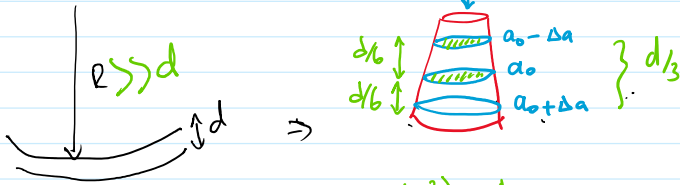
4 tane alan değişiyor
① ② ③ ④

4 tane alan değişiyor
 ① ② ③ ④
 değişimin aynı olsun.



$$\epsilon_b \approx 4 \left[\frac{1}{2} \left(\frac{K_s}{4} \right) \left(\frac{\Delta A}{A_0} \right)^2 \right] ; \frac{\Delta A}{A_0} = \frac{\Delta a}{a_0} = ?$$

$$\epsilon_b = \frac{K_b}{2} \left(\frac{1}{r_1} + \frac{1}{r_2} \right)^2$$



$$\frac{\Delta A}{A_0} \sim \frac{\pi R^2 - \pi (R \pm \frac{d}{6})^2}{\pi R^2} \quad \pi (R^2) \sim A_0$$

$$\pi (R \pm \frac{d}{6})^2 \sim A_0 \pm \Delta A$$

$$\frac{d/6}{R}$$

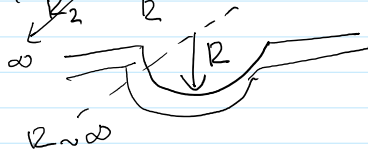
$$\epsilon_b = \cancel{4} \frac{1}{2} \frac{K_s}{\cancel{4}} \left(\frac{\Delta A}{A_0} \right)^2$$

$$= \frac{K_s}{2} \left(\frac{d/6}{R} \right)^2$$

$$= \frac{1}{2} \left(\frac{K_s d^2}{36} \right) \frac{1}{R^2} = \frac{1}{2} K_b \frac{1}{R^2}$$

silindirik

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{R}$$



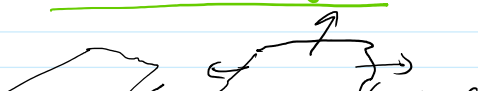
$$d = 4 \text{ nm}$$

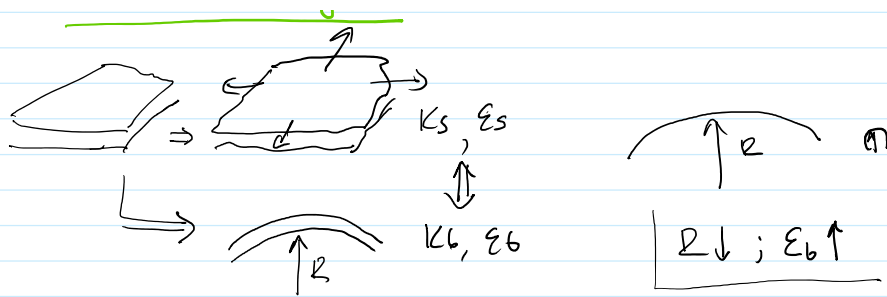
$$K_b = \frac{K_s d^2}{36} = \frac{(0.16) (4 \times 10^{-9})^2}{36} = 7.1 \times 10^{20} \text{ J}$$

$$K_b = 17 k_B T \quad (\text{modelin sonucu})$$

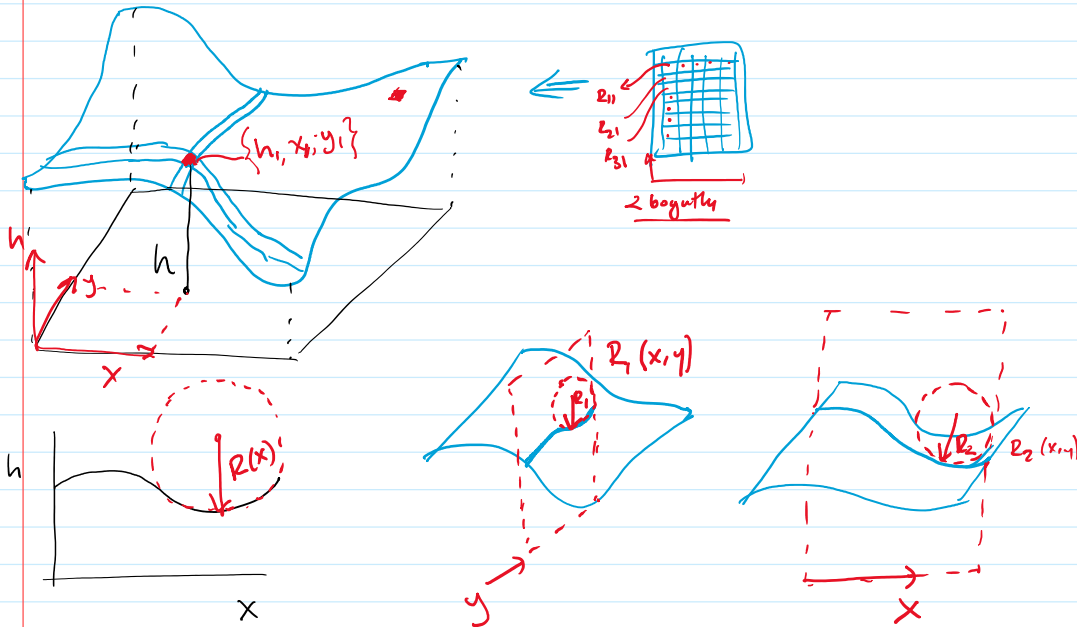
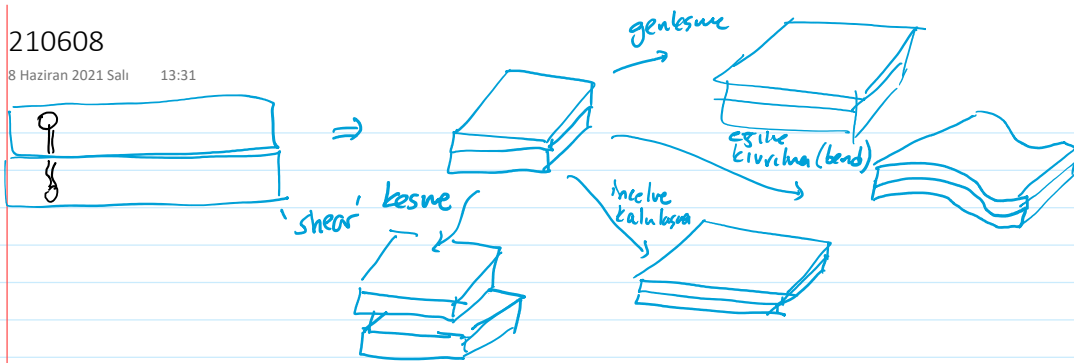
$$* \text{ deneylerde insan kan hücreleri } \epsilon_b = 25 k_B T$$

17-25 birbirine yakın !!





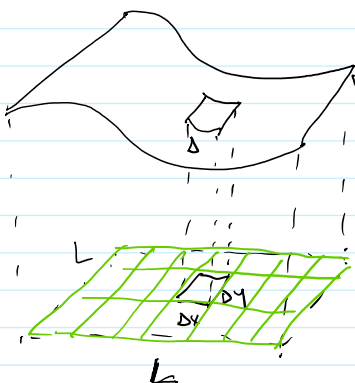
$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2}$$



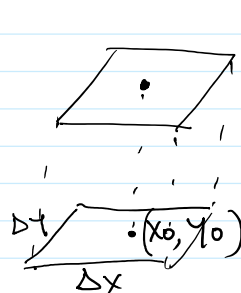
$$\begin{aligned} x_1 &= x \\ x_2 &= y \end{aligned} \quad h(x_1, x_2) = \sum_{i,j=1}^2 K_{ij} x_i x_j$$

$$K_{ij} = \frac{\partial^2 h}{\partial x_i \partial x_j}$$

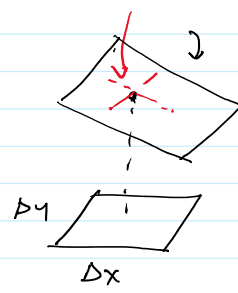
$$h(x, y) = \underbrace{h(x_0, y_0)}_{0^{\circ}} + \underbrace{\frac{\partial h}{\partial x} \Delta x + \frac{\partial h}{\partial y} \Delta y}_{1 \text{ derece}} + \underbrace{\frac{1}{2} \left(\frac{\partial^2 h}{\partial x^2} \Delta x^2 + \frac{\partial^2 h}{\partial y^2} \Delta y^2 + 2 \frac{\partial^2 h}{\partial x \partial y} \Delta x \Delta y \right)}_{2 \text{ derece}} + \dots$$



1. m

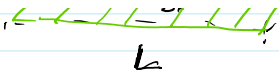


$h(x_0, y_0)$



DÖNME

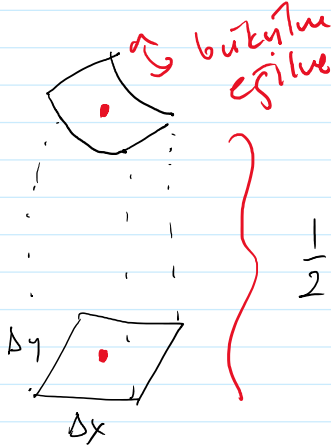
2. m . 3. m .



$h(x_0, y_0)$

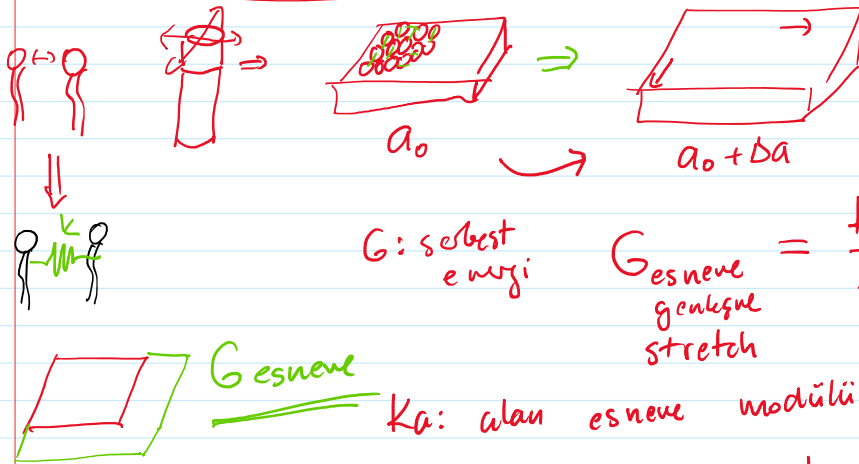
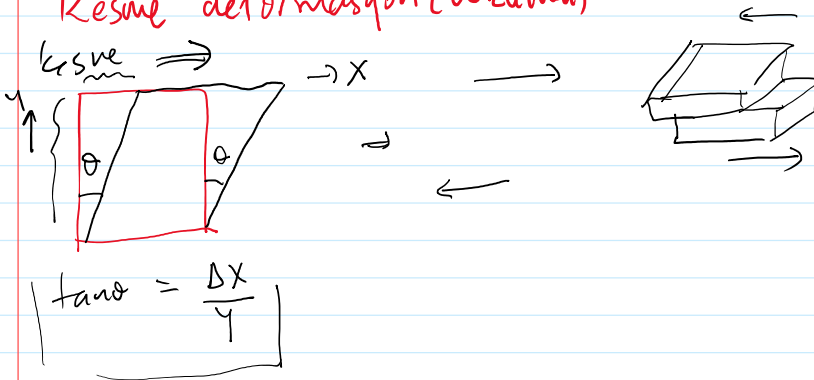
DÖNME

$$\frac{\partial h}{\partial x} \Delta x + \frac{\partial h}{\partial y} \Delta y$$



$$\frac{1}{2} \frac{\partial^2 h}{\partial x^2} \Delta x^2 + \frac{1}{2} \frac{\partial^2 h}{\partial y^2} \Delta y^2 + \frac{\partial^2 h}{\partial x \partial y} \Delta x \Delta y$$

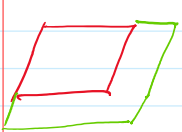
Kesme deformasyon (bozulma)



G : serbest enerji

G_{esneve}
genleşme stretch

$$G_{esneve} = \frac{k_a}{2} \int \left(\frac{\Delta a}{a} \right)^2 da$$



G_{esneve}

k_a : alan esneve modülü = $\frac{J}{m^2} = \left[\frac{\text{enerji}}{\text{alan}} \right]$

$$k_a \sim 55-70 \frac{k_B T}{nm^2}$$

Eğilme, bükülme (Bending)

serbest enerjisi

$$G_{eğilme} = \frac{k_b}{2} \int da \left(\underbrace{k_1(x,y)}_{\text{tanel eksenlerdeki sb+ indirgenmiş}} + \underbrace{k_2(x,y)}_{\text{tanel eksenlerdeki sb+ indirgenmiş}} \right)^2$$

egilme 2 J

temel eksenlerdeki sb+
indigenmis

$$K_{ij} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix}$$

köşegenleştirilirse

$$\begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix}$$

K_b = eğilme modülü

$$\bar{K} = \frac{K_1 + K_2}{2}$$

curvature
(eğrilik değeri) $\sim \frac{1}{R}$

$$\Rightarrow \frac{2}{R}$$

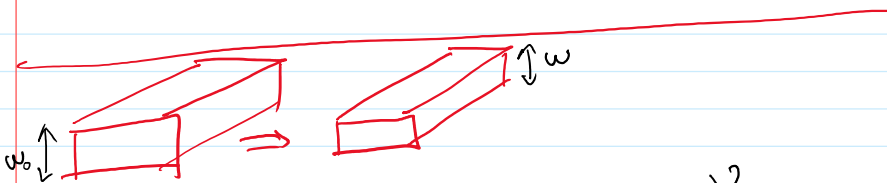
$$G_{\text{eğilme}} = \frac{K_b}{2} \int da \left(\frac{K_1 + K_2}{2} \right)^2$$

$\Downarrow$
enerji

$$\int m^2 \left(\frac{1}{m} \right)^2$$

$$K_b = \text{enerji}$$

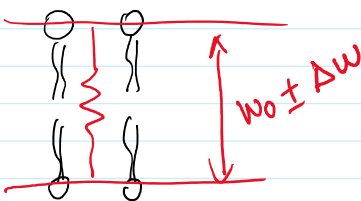
$$\text{eğrilik} \sim \left\{ \frac{\partial^2 h}{\partial x \partial y}; \frac{\partial^2 h}{\partial x^2}; \frac{\partial^2 h}{\partial y^2} \right\} \sim \left[\frac{1}{m} \right]$$



$$G_{\text{kalmık}} = \frac{K_t}{2} \int da \left(\frac{w(x,y) - w_0}{w_0} \right)^2$$

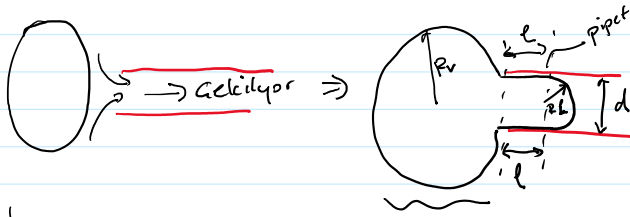
K_t : kalmık değışimini modülü

$$K_t = \left[\frac{\text{enerji}}{\text{alan}} \right]$$



Hücre zarı esnekliğinin ölçülmesi

Hücre zarı esnekliğinin ölçülmesi pipetleme tekniği



$$d \sim 10 \mu m$$

P : basınç

ΔP_{out} : mikropipet ile dış ortam arası basınç farkı

R_v : vesicle yarıçapı

R_e : pipet içindeki yarıçap

k_a, k_b ölçülebilir.

l : pipet içindeki uzayan kısım uzunluğu

τ = zarın gerilmesi

$$\Delta P_{out} = \frac{2\tau}{R_v}$$

$$\Delta P_{in} = \frac{2\tau}{R_e}$$

ΔP_{in} : vesicle içindeki basınca ile pipetin içindeki basınç farkı

$$\Delta P = \Delta P_{out} - \Delta P_{in} \Rightarrow$$

$$\tau = \frac{\Delta P}{2} \frac{R_e}{\left(1 - \frac{R_e}{R_v}\right)}$$

$$\tau = k_a \frac{\Delta a}{a_0}$$

$$\Delta a \Rightarrow \begin{array}{c} a_0 \\ \text{circle with } R_v \end{array} \Rightarrow \begin{array}{c} a_0 + \Delta a \\ \text{irregular shape} \end{array}$$

$$\Delta a = 2\pi R_e l + 2\pi R_e^2$$

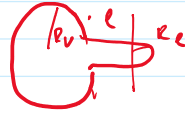
$$a_0 = 4\pi R_v^2$$

$$; \quad \tau = k_a \left(\frac{\Delta a}{a_0} \right)$$

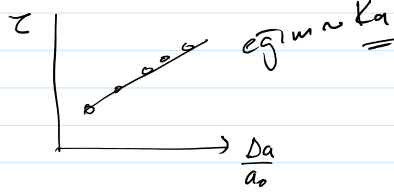
bu - 21111 - 21111

$$a_0 = 4\pi R_v^2 \quad ; \quad \tau = k_a \left(\frac{\Delta a}{a_0} \right)$$

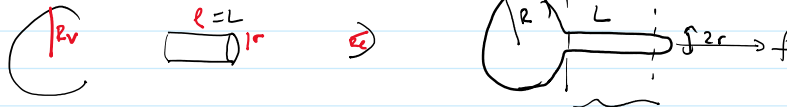
$$\frac{\Delta a}{a_0} = \frac{R_0^2 (1 + l/R_0)}{2R_v^2}$$



$$\tau = \frac{\Delta p}{2} \frac{R_e}{1 - \frac{R_1}{R_v}}$$



$G_{\text{elastik}} = \text{küresel} + \text{silindirik} + \text{yaprıkine sapka}$



$$\begin{aligned} &= \underline{8\pi k_b} + \pi k_b \frac{L}{r} + \underline{4\pi k_b} \\ &= 12\pi k_b + \pi k_b \frac{L}{r} \\ &= \pi k_b \left(12 + \frac{L}{r} \right) \end{aligned}$$

$$G_{PV} = -\Delta p V = -\Delta p \left(\frac{4}{3}\pi R^3 + \pi r^2 L \right)$$

Δp basınca karşı yapılan iş

$G_{\text{esneke (stretch)}}$

$$G_{\text{esneke}} = \frac{k_a}{2} \frac{(a-a_0)^2}{a_0} = \frac{\partial G_{\text{esneke}}}{\partial R} = k_a \frac{\Delta a}{a_0} \frac{\partial a}{\partial R} = \tau 8\pi R$$

dış f kuvvet ile çekilme $\Rightarrow G_{\text{yüklenme (load)}}$ $G_{\text{load}} = -fL$

$$G_{\text{toplam}} = \underline{12\pi k_b} + \underline{\pi k_b \frac{L}{r}} + G_{\text{esneke}} - \Delta p \left(\frac{4}{3}\pi R^3 + \pi r^2 L \right) - fL$$

sistemin toplam serbest enerjisi $\tau = k_a \frac{a-a_0}{a_0} = k_a \frac{\Delta a}{a_0}$

$$\frac{\partial G_{\text{top}}}{\partial R} = 0 \Rightarrow 8\pi \tau R - 4\pi \Delta p R^2 = 0$$

$$\frac{\partial G_{\text{top}}}{\partial r} = 0 \Rightarrow -\pi k_b \frac{L}{r^2} + 2\pi \tau L - 2\pi \Delta p r L = 0$$

$$\frac{\partial G_{\text{top}}}{\partial L} = 0 \Rightarrow \pi k_b \frac{1}{r} + 2\pi \tau r - \pi \Delta p r^2 - f = 0$$

$$\left\{ \frac{\partial G_{\text{esneke}}}{\partial L} = \tau 2\pi r \quad ; \quad \frac{\partial G_{\text{esneke}}}{\partial r} = \tau 2\pi r \right\}$$

$$\left\{ \frac{\partial G_{\text{esnek}}}{\partial r} = \tau 2\pi L ; \quad \frac{\partial G_{\text{esnek}}}{\partial L} = \tau 2\pi r \right\}$$

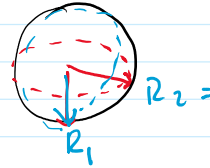
$$\Delta p = \frac{2\tau}{R}$$

$$r = \sqrt{\frac{K_b}{2\tau}} ; \quad f = 2\pi \sqrt{K_b 2\tau}$$

$f \sim 10 \text{ pN}$ (golgi aparatlarının çekerken)

$$G_{\text{esnek küresel}} = \frac{K_b}{2} \int da (k_1 + k_2)^2$$

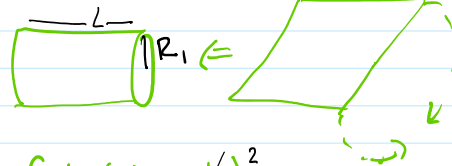
$$\left(\frac{1}{R} + \frac{1}{R} \right)$$



$$G = \frac{K_b}{2} \int da \frac{4}{R^2} \Rightarrow \frac{2}{R^2} K_b 4\pi R^2 = \underline{\underline{8\pi K_b}}$$

küresel $\Rightarrow$ zor oluşturmak için enerji

Silindirik için



bükülme yok $R = \infty$

$$G_{\text{silindirik}} = \frac{K_b}{2} \int da \left(\frac{1}{R} + \frac{1}{\infty} \right)^2$$

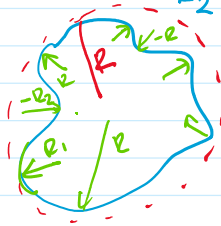
$$= \frac{K_b}{2 R^2} \underbrace{\int da}_{2\pi R L} \Rightarrow \frac{K_b 2\pi R L}{2 R^2} = \underline{\underline{K_b \pi \frac{L}{R}}}$$

$$G = \frac{K_b}{2} \int da \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^2 ;$$

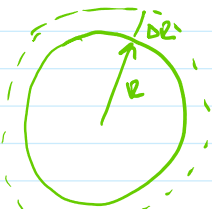


$$R_1 = R_2 \Rightarrow R = \text{küre}$$

$$R_2 \rightarrow \infty \quad R_1 \Rightarrow \text{silindirik}$$



$$\frac{K_b}{2} \int da (k_1 + k_2)$$

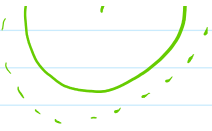


$$a_0 \rightarrow a_0 + \Delta a$$

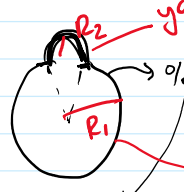
$$4\pi R^2 \quad 4\pi (R + \Delta R)^2$$

$$\Delta a = 8\pi R \Delta R$$

kur



$$\Delta a = 8\pi R \Delta R$$

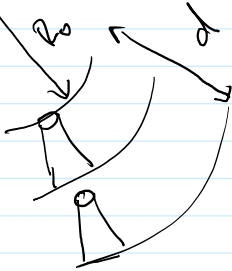
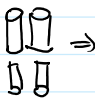
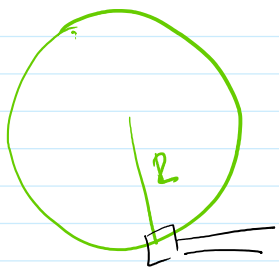
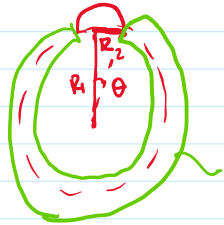


$$\theta = \tan^{-1}\left(\frac{R_2}{R_1}\right)$$

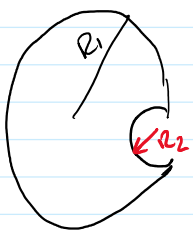
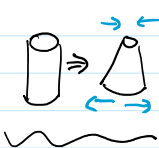
$$1 - \theta$$

$$\int da \left(\frac{1}{R} + \frac{1}{R}\right)^2$$

2πR²



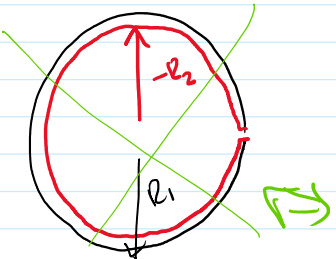
$$d \ll R_0$$



$$R_1 > 0$$

$$R_2 < 0$$

$$\int da \left(\frac{1}{R_1} - \frac{1}{R_2}\right)^2$$



$$R_2 \approx R_1$$

$$\frac{1}{R_1} - \frac{1}{R_2} \approx 0$$

