13

Sinusoidal Alternating Waveforms

13.1 INTRODUCTION

The analysis thus far has been limited to dc networks, networks in which the currents or voltages are fixed in magnitude except for transient effects. We will now turn our attention to the analysis of networks in which the magnitude of the source varies in a set manner. Of particular interest is the time-varying voltage that is commercially available in large quantities and is commonly called the *ac voltage.* (The letters *ac* are an abbreviation for *alternating current.*) To be absolutely rigorous, the terminology *ac voltage* or *ac current* is not sufficient to describe the type of signal we will be analyzing. Each waveform of Fig. 13.1 is an **alternating waveform** available from commercial supplies. The term *alternating* indicates only that the waveform alternates between two prescribed levels in a set time sequence (Fig. 13.1). To be absolutely correct, the term *sinusoidal, square wave,* or *triangular* must also be applied. The pattern of particular interest is the **sinusoidal ac waveform** for voltage of Fig. 13.1. Since this type of signal is encountered in the vast majority of instances, the abbreviated phrases *ac voltage* and *ac current* are commonly applied without confusion. For the other patterns of Fig. 13.1, the descriptive term is always present, but frequently the *ac* abbreviation is dropped, resulting in the designation *square-wave* or *triangular* waveforms.

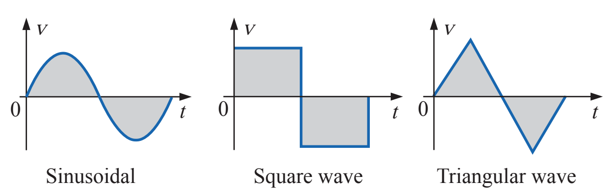


FIG. 13.1
Alternating waveforms.

One of the important reasons for concentrating on the sinusoidal ac voltage is that it is the voltage generated by utilities throughout the world. Other reasons include its application throughout electrical, electronic, communication, and industrial systems. In addition, the chapters to follow will reveal that the waveform itself has a number of characteristics that will result in a unique response when it is applied to the basic electrical elements. The wide range of theorems and methods introduced for dc networks will also be applied to sinusoidal ac systems. Although the application of sinusoidal signals will raise the required math level, once the notation given in Chapter 14 is understood, most of the concepts introduced in the dc chapters can be applied to ac networks with a minimum of added difficulty.

The increasing number of computer systems used in the industrial community requires, at the very least, a brief introduction to the terminology employed with pulse waveforms and the response of some fundamental configurations to the application of such signals. Chapter 24 will serve such a purpose.

13.2 SINUSOIDAL ac VOLTAGE CHARACTERISTICS AND DEFINITIONS

Generation

Sinusoidal ac voltages are available from a variety of sources. The most common source is the typical home outlet, which provides an ac voltage that originates at a power plant; such a power plant is most commonly fueled by water power, oil, gas, or nuclear fusion. In each case an *ac generator* (also called an *alternator*), as shown in Fig. 13.2(a), is the primary component in the energy-conversion process.

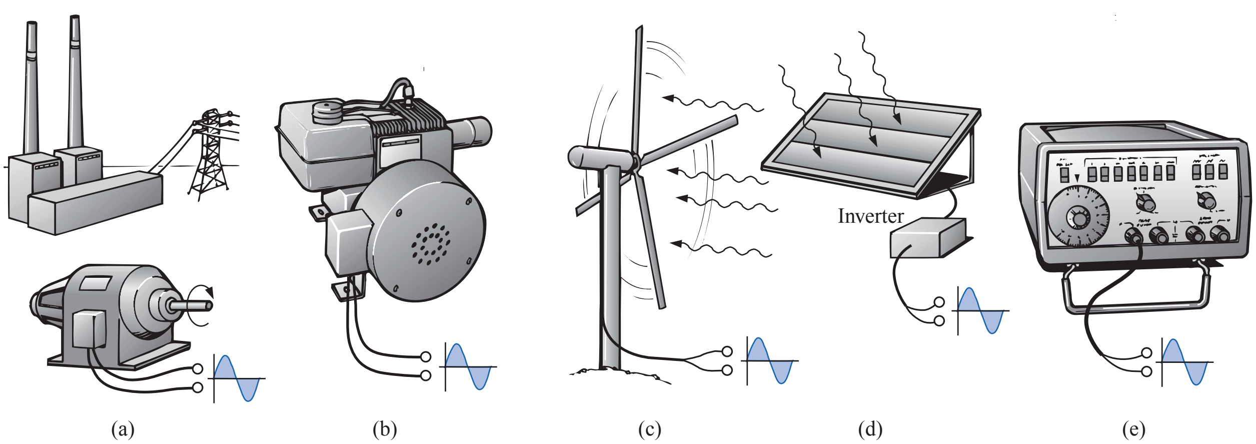


FIG. 13.2
Various sources of ac power: (a) generating plant; (b) portable ac generator; (c) wind-power station; (d) solar panel; (e) function generator.

The power to the shaft developed by one of the energy sources listed will turn a *rotor* (constructed of alternating magnetic poles) inside a set of windings housed in the *stator* (the stationary part of the dynamo) and will induce a voltage across the windings of the stator, as defined by Faraday's law,

$$e = N\frac{d\phi}{dt}$$

Through proper design of the generator, a sinusoidal ac voltage is developed that can be transformed to higher levels for distribution through the power lines to the consumer. For isolated locations where power lines have not been installed, portable ac generators [Fig. 13.2(b)] are available that run on gasoline. As in the larger power plants, however, an ac generator is an integral part of the design.

In an effort to conserve our natural resources, wind power and solar energy are receiving increasing interest from various districts of the world that have such energy sources available in level and duration that make the conversion process viable. The turning propellers of the wind-power station [Fig. 13.2(c)] are connected directly to the shaft of an ac generator to provide the ac voltage described above. Through light energy absorbed in the form of *photons,* solar cells [Fig. 13.2(d)] can generate dc voltages. Through an electronic package called an *inverter,* the dc voltage can be converted to one of a sinusoidal nature. Boats, recreational vehicles (RVs), etc., make frequent use of the inversion process in isolated areas.

Sinusoidal ac voltages with characteristics that can be controlled by the user are available from *function generators,* such as the one in Fig. 13.2(e). By setting the various switches and controlling the position of the knobs on the face of the instrument, one can make available sinusoidal voltages of different peak values and different repetition rates. The function generator plays an integral role in the investigation of the variety of theorems, methods of analysis, and topics to be introduced in the chapters that follow.

Definitions

The sinusoidal waveform of Fig. 13.3 with its additional notation will now be used as a model in defining a few basic terms. These terms, however, can

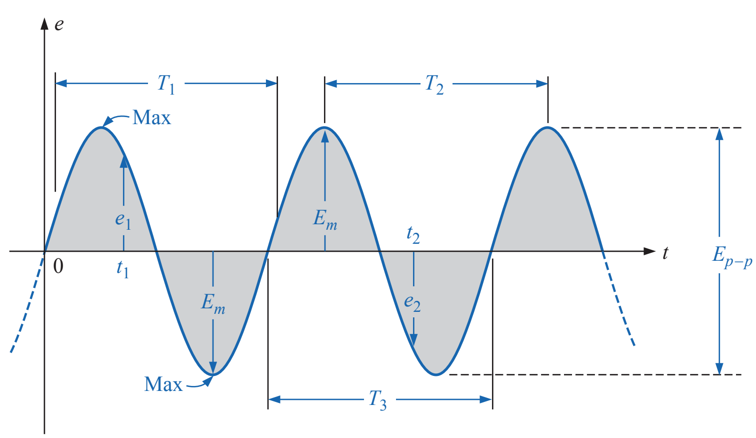


FIG. 13.3
Important parameters for a sinusoidal voltage.

be applied to any alternating waveform. It is important to remember as you proceed through the various definitions that the vertical scaling is in volts or amperes and the horizontal scaling is *always* in units of time.

Waveform: The path traced by a quantity, such as the voltage in Fig. 13.3, plotted as a function of some variable such as time (as above), position, degrees, radians, temperature, and so on.

Instantaneous value: The magnitude of a waveform at any instant of time; denoted by lowercase letters (e_1, e_2).

Peak amplitude: The maximum value of a waveform as measured from its *average,* or *mean,* value, denoted by uppercase letters (such as E_m for sources of voltage and V_m for the voltage drop across a load). For the waveform of Fig. 13.3, the average value is zero volts, and E_m is as defined by the figure.

Peak value: The maximum instantaneous value of a function as measured from the zero-volt level. For the waveform of Fig. 13.3, the peak amplitude and peak value are the same, since the average value of the function is zero volts.

Peak-to-peak value: Denoted by $E_{p\text{-}p}$ or $V_{p\text{-}p}$, the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.

Periodic waveform: A waveform that continually repeats itself after the same time interval. The waveform of Fig. 13.3 is a periodic waveform.

Period (T): The time interval between successive repetitions of a periodic waveform (the period $T_1 = T_2 = T_3$ in Fig. 13.3), as long as successive *similar points* of the periodic waveform are used in determining T.

Cycle: The portion of a waveform contained in *one period* of time. The cycles within T_1, T_2, and T_3 of Fig. 13.3 may appear different in Fig. 13.4, but they are all bounded by one period of time and therefore satisfy the definition of a cycle.

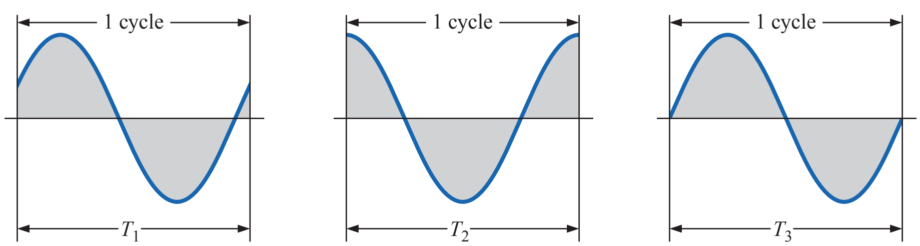


FIG. 13.4
Defining the cycle and period of a sinusoidal waveform.

Frequency (f): The number of cycles that occur in 1 s. The frequency of the waveform of Fig. 13.5(a) is 1 cycle per second, and for Fig. 13.5(b), 2½ cycles per second. If a waveform of similar shape had a period of 0.5 s [Fig. 13.5(c)], the frequency would be 2 cycles per second.

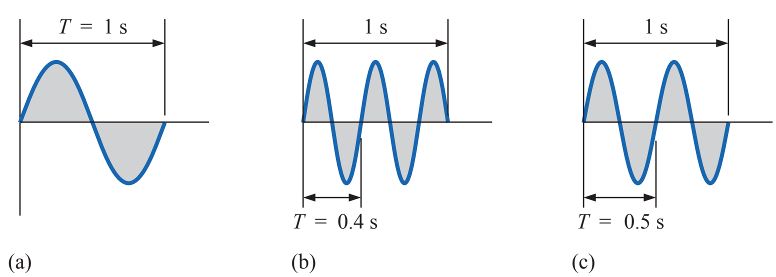


FIG. 13.5
Demonstrating the effect of a changing frequency on the period of a sinusoidal waveform.

The unit of measure for frequency is the *hertz* (Hz), where

$$\boxed{1 \text{ hertz (Hz)} = 1 \text{ cycle per second (c/s)}} \tag{13.1}$$

The unit hertz is derived from the surname of Heinrich Rudolph Hertz (Fig. 13.6), who did original research in the area of alternating currents and voltages and their effect on the basic R, L, and C elements. The frequency standard for North America is 60 Hz, whereas for Europe it is predominantly 50 Hz.

As with all standards, any variation from the norm will cause difficulties. In 1993, Berlin, Germany, received all its power from eastern plants, whose output frequency was varying between 50.03 and 51 Hz. The result was that clocks were gaining as much as 4 minutes a day. Alarms went off too soon, VCRs clicked off before the end of the program, etc., requiring that clocks be continually reset. In 1994, however, when power was linked with the rest of Europe, the precise standard of 50 Hz was reestablished and everyone was on time again.

Using a log scale (described in detail in Chapter 23), a frequency spectrum from 1 Hz to 1000 GHz can be scaled off on the same axis, as shown in Fig. 13.7. A number of terms in the various spectrums are probably familiar to the reader from everyday experiences. Note that the audio range (human ear) extends from only 15 Hz to 20 kHz, but the transmission of radio signals can occur between 3 kHz and 300 GHz. The uniform process of defining the intervals of the radio-frequency spectrum from VLF to EHF is quite evident from the length of the bars in the figure (although keep in mind that it is a log scale, so the frequencies encompassed within each segment are quite different). Other frequencies of particular interest (TV, CB, microwave, etc.) are also included for reference purposes. Although it is numerically easy to talk about frequencies in the megahertz and gigahertz range, keep in mind that a frequency of 100 MHz, for instance, represents a sinusoidal waveform that passes through 100,000,000 cycles in only 1 s—an incredible number when we compare it to the 60 Hz of our conventional power sources. The new Pentium II chip manufactured by Intel can run at speeds up to 450 MHz. Imagine a product able to handle 450,000,000 instructions per second—an incredible achievement. The new Pentium IV chip manufactured by Intel can run at a speed of 1.5 GHz. Try to imagine a product able to handle 1,500,000,000,000 instructions in just 1 s—an incredible achievement.

Since the frequency is inversely related to the period—that is, as one increases, the other decreases by an equal amount—the two can be related by the following equation:

$$\boxed{f = \frac{1}{T}} \qquad \begin{aligned} f &= \text{Hz} \\ T &= \text{seconds (s)} \end{aligned} \tag{13.2}$$

or

$$\boxed{T = \frac{1}{f}} \tag{13.3}$$

German (Hamburg, Berlin, Karlsruhe)
(1857–94)
Physicist
Professor of Physics, Karlsruhe Polytechnic and University of Bonn

Courtesy of the Smithsonian Institution Photo No. 66,606

Spurred on by the earlier predictions of the English physicist James Clerk Maxwell, Heinrich Hertz produced *electromagnetic waves* in his laboratory at the Karlsruhe Polytechnic while in his early 30s. The rudimentary *transmitter* and *receiver* were in essence the first to broadcast and receive radio waves. He was able to measure the *wavelength* of the electromagnetic waves and confirmed that the *velocity of propagation* is in the same order of magnitude as light. In addition, he demonstrated that the *reflective* and *refractive* properties of electromagnetic waves are the same as those for heat and light waves. It was indeed unfortunate that such an ingenious, industrious individual should pass away at the very early age of 37 due to a bone disease.

FIG. 13.6
Heinrich Rudolph Hertz.

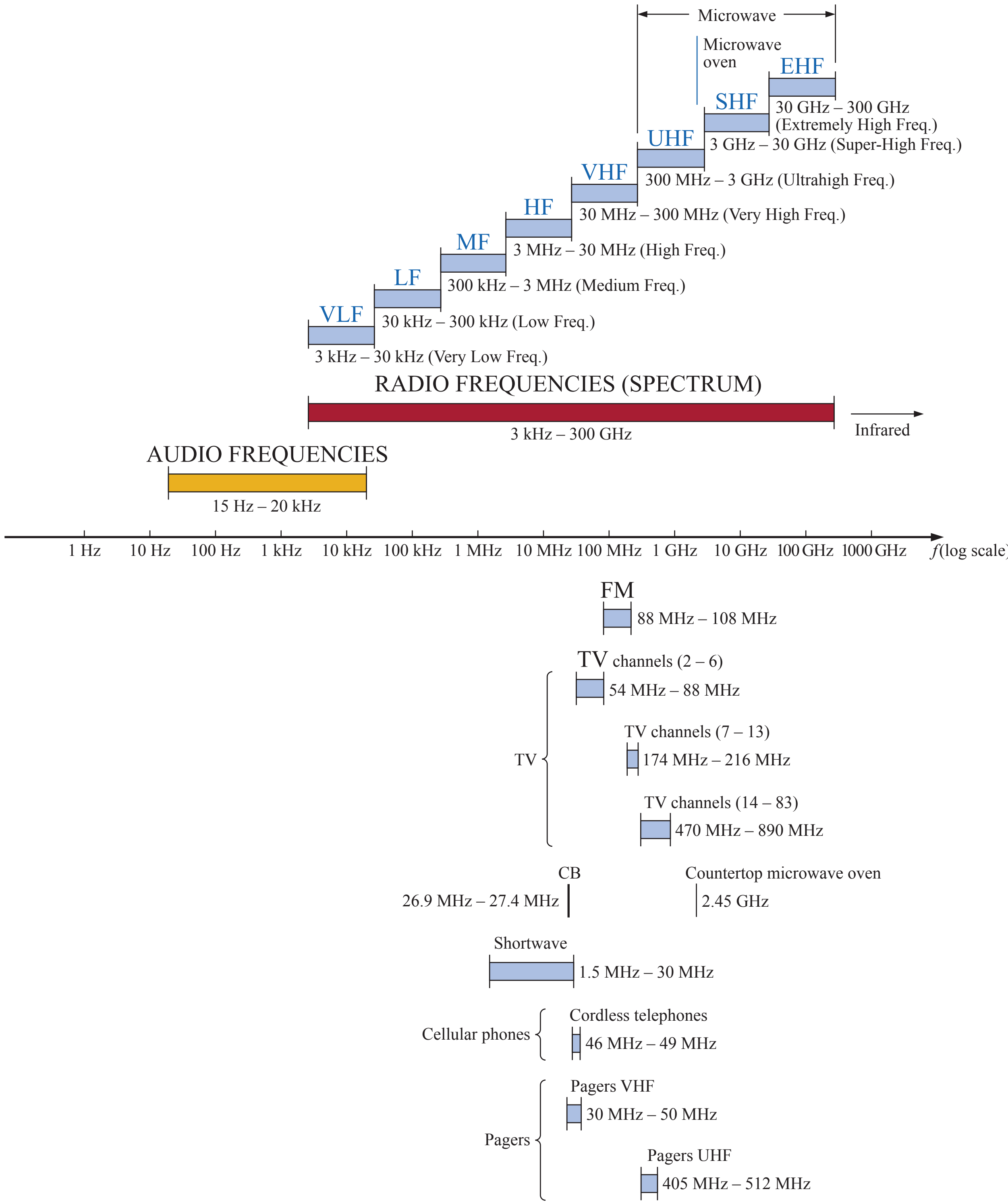


FIG. 13.7
Areas of application for specific frequency bands.

EXAMPLE 13.1 Find the period of a periodic waveform with a frequency of
a. 60 Hz.
b. 1000 Hz.

Solutions:

a. $T = \dfrac{1}{f} = \dfrac{1}{60\text{ Hz}} \cong 0.01667\text{ s or } \mathbf{16.67\text{ ms}}$

(a recurring value since 60 Hz is so prevalent)

b. $T = \dfrac{1}{f} = \dfrac{1}{1000\text{ Hz}} = 10^{-3}\text{ s} = \mathbf{1\text{ ms}}$

EXAMPLE 13.2 Determine the frequency of the waveform of Fig. 13.8.

Solution: From the figure, $T = (25\text{ ms} - 5\text{ ms}) = 20\text{ ms}$, and

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}\text{ s}} = \mathbf{50\text{ Hz}}$$

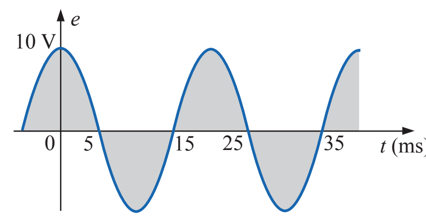
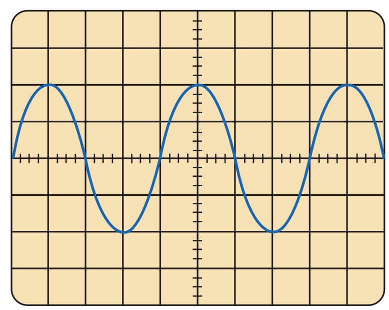


FIG. 13.8
Example 13.2.

EXAMPLE 13.3 The **oscilloscope** is an instrument that will display alternating waveforms such as those described above. A sinusoidal pattern appears on the oscilloscope of Fig. 13.9 with the indicated vertical and horizontal sensitivities. The vertical sensitivity defines the voltage associated with each vertical division of the display. Virtually all oscilloscope screens are cut into a crosshatch pattern of lines separated by 1 cm in the vertical and horizontal directions. The horizontal sensitivity defines the time period associated with each horizontal division of the display.

For the pattern of Fig. 13.9 and the indicated sensitivities, determine the period, frequency, and peak value of the waveform.



Vertical sensitivity = 0.1 V/div.
Horizontal sensitivity = 50 μs/div.

FIG. 13.9
Example 13.3.

Solution: One cycle spans 4 divisions. The period is therefore

$$T = 4\,\cancel{\text{div.}}\left(\frac{50\,\mu\text{s}}{\cancel{\text{div.}}}\right) = \mathbf{200\,\mu s}$$

and the frequency is

$$f = \frac{1}{T} = \frac{1}{200 \times 10^{-6}\text{ s}} = \mathbf{5\text{ kHz}}$$

The vertical height above the horizontal axis encompasses 2 divisions. Therefore,

$$V_m = 2\,\cancel{\text{div.}}\left(\frac{0.1\text{ V}}{\cancel{\text{div.}}}\right) = \mathbf{0.2\text{ V}}$$

Defined Polarities and Direction

In the following analysis, we will find it necessary to establish a set of polarities for the sinusoidal ac voltage and a direction for the sinusoidal ac current. In each case, the polarity and current direction will be for an instant of time in the positive portion of the sinusoidal waveform. This is shown in Fig. 13.10 with the symbols for the sinusoidal ac voltage and current. A lowercase letter is employed for each to indicate that the quantity is time dependent; that is, its magnitude will change with time.

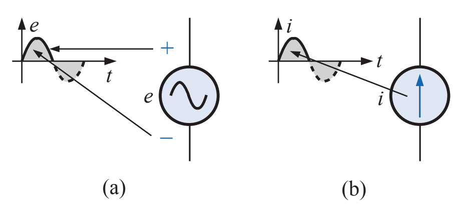


FIG. 13.10
(a) Sinusoidal ac voltage sources;
(b) sinusoidal current sources.

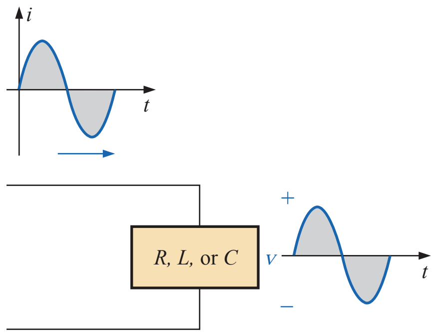


FIG. 13.11
The sine wave is the only alternating waveform whose shape is not altered by the response characteristics of a pure resistor, inductor, or capacitor.

The need for defining polarities and current direction will become quite obvious when we consider multisource ac networks. Note in the last sentence the absence of the term *sinusoidal* before the phrase *ac networks.* This phrase will be used to an increasing degree as we progress; *sinusoidal* is to be understood unless otherwise indicated.

13.3 THE SINE WAVE

The terms defined in the previous section can be applied to any type of periodic waveform, whether smooth or discontinuous. The sinusoidal waveform is of particular importance, however, since it lends itself readily to the mathematics and the physical phenomena associated with electric circuits. Consider the power of the following statement:

The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R, L, and C elements.

In other words, if the voltage across (or current through) a resistor, coil, or capacitor is sinusoidal in nature, the resulting current (or voltage, respectively) for each will also have sinusoidal characteristics, as shown in Fig. 13.11. If a square wave or a triangular wave were applied, such would not be the case.

The unit of measurement for the horizontal axis of Fig. 13.12 is the *degree.* A second unit of measurement frequently used is the **radian** (rad). It is defined by a quadrant of a circle such as in Fig. 13.13 where the distance subtended on the circumference equals the radius of the circle.

If we define x as the number of intervals of r (the radius) around the circumference of the circle, then

$$C = 2\pi r = x \cdot r$$

and we find

$$x = 2\pi$$

Therefore, there are 2π rad around a 360° circle, as shown in Fig. 13.14, and

$$\boxed{2\pi \text{ rad} = 360°} \qquad \textbf{(13.4)}$$

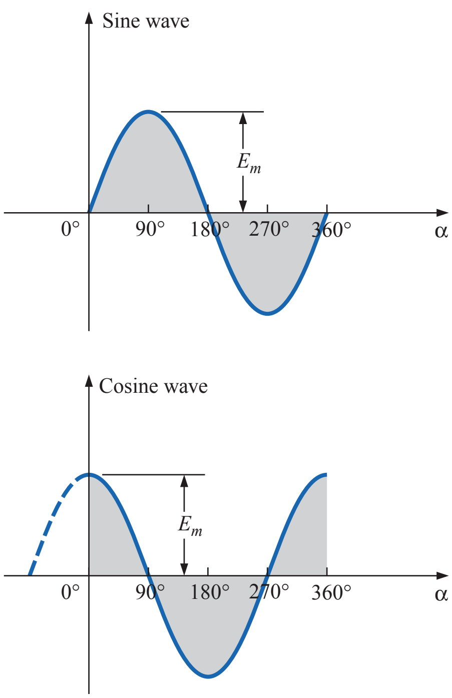


FIG. 13.12
Sine wave and cosine wave with the horizontal axis in degrees.

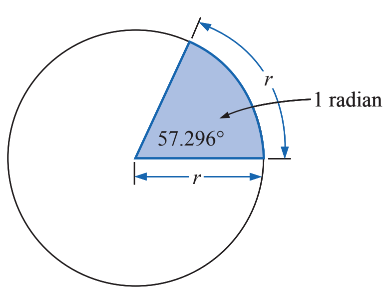


FIG. 13.13
Defining the radian.

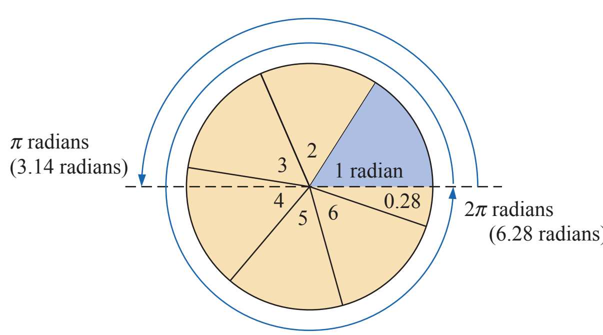


FIG. 13.14
There are 2π radians in one full circle of 360°.

with
$$\boxed{1 \text{ rad} = 57.296° \cong 57.3°} \tag{13.5}$$

A number of electrical formulas contain a multiplier of π. For this reason, it is sometimes preferable to measure angles in radians rather than in degrees.

The quantity π is the ratio of the circumference of a circle to its diameter.

π has been determined to an extended number of places primarily in an attempt to see if a repetitive sequence of numbers appears. It does not. A sampling of the effort appears below:

$$\pi = 3.14159\ 26535\ 89793\ 23846\ 26433\ldots$$

Although the approximation $\pi \cong 3.14$ is often applied, all the calculations in this text will use the π function as provided on all scientific calculators.

For 180° and 360°, the two units of measurement are related as shown in Fig. 13.14. The conversion equations between the two are the following:

$$\boxed{\text{Radians} = \left(\frac{\pi}{180°}\right) \times (\text{degrees})} \tag{13.6}$$

$$\boxed{\text{Degrees} = \left(\frac{180°}{\pi}\right) \times (\text{radians})} \tag{13.7}$$

Applying these equations, we find

$$\mathbf{90°:}\quad \text{Radians} = \frac{\pi}{180°}(90°) = \boldsymbol{\frac{\pi}{2}}\ \textbf{rad}$$

$$\mathbf{30°:}\quad \text{Radians} = \frac{\pi}{180°}(30°) = \boldsymbol{\frac{\pi}{6}}\ \textbf{rad}$$

$$\boldsymbol{\frac{\pi}{3}}\ \textbf{rad:}\quad \text{Degrees} = \frac{180°}{\pi}\left(\frac{\pi}{3}\right) = \mathbf{60°}$$

$$\boldsymbol{\frac{3\pi}{2}}\ \textbf{rad:}\quad \text{Degrees} = \frac{180°}{\pi}\left(\frac{3\pi}{2}\right) = \mathbf{270°}$$

Using the radian as the unit of measurement for the abscissa, we would obtain a sine wave, as shown in Fig. 13.15.

It is of particular interest that the sinusoidal waveform can be derived from the length of the *vertical projection* of a radius vector rotating in a uniform circular motion about a fixed point. Starting as shown in Fig. 13.16(a) and plotting the amplitude (above and below zero) on the coordinates drawn to the right [Figs. 13.16(b) through (i)], we will trace a complete sinusoidal waveform after the radius vector has completed a 360° rotation about the center.

The velocity with which the radius vector rotates about the center, called the **angular velocity,** can be determined from the following equation:

$$\boxed{\text{Angular velocity} = \frac{\text{distance (degrees or radians)}}{\text{time (seconds)}}} \tag{13.8}$$

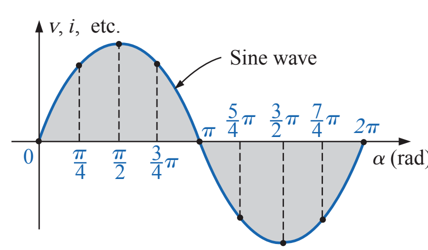


FIG. 13.15
Plotting a sine wave versus radians.

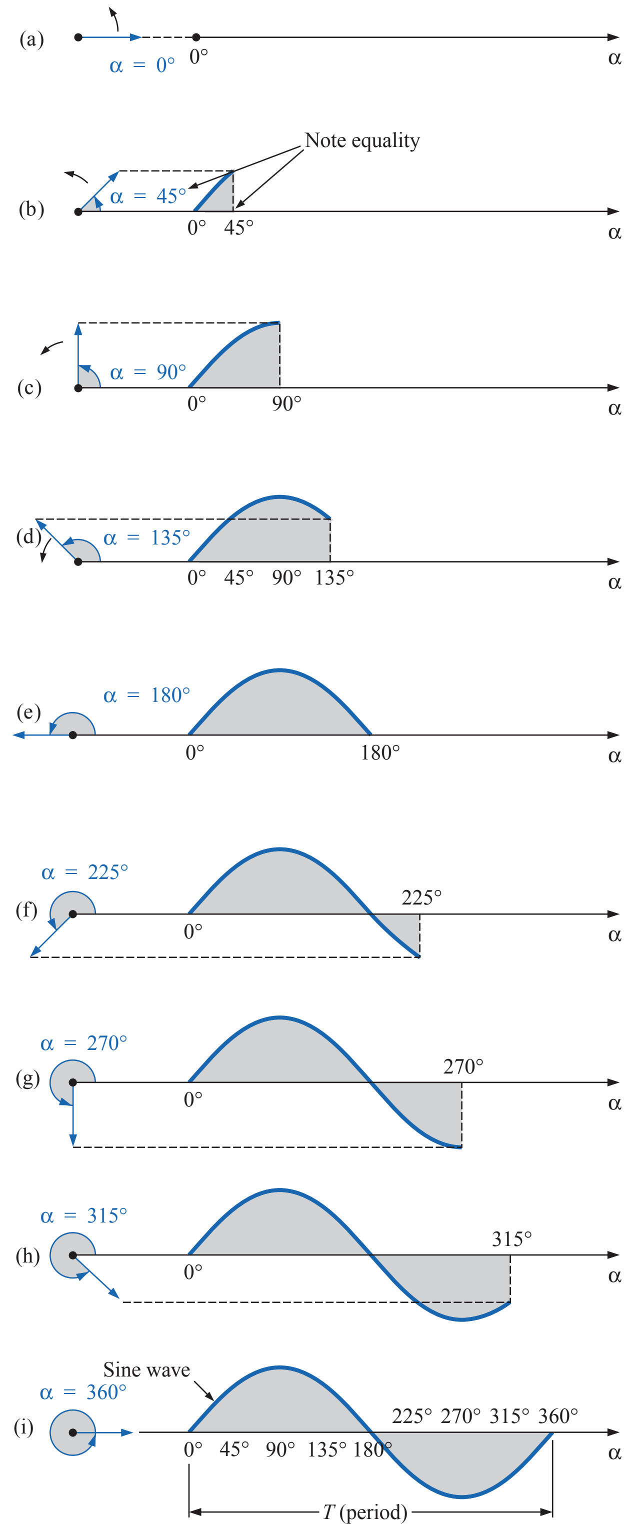


FIG. 13.16
Generating a sinusoidal waveform through the vertical projection of a rotating vector.

Substituting into Eq. (13.8) and assigning the Greek letter omega (ω) to the angular velocity, we have

$$\boxed{\omega = \frac{\alpha}{t}} \qquad \textbf{(13.9)}$$

and

$$\boxed{\alpha = \omega t} \qquad \textbf{(13.10)}$$

Since ω is typically provided in radians per second, the angle α obtained using Eq. (13.10) is usually in radians. If α is required in degrees, Equation (13.7) must be applied. The importance of remembering the above will become obvious in the examples to follow.

In Fig. 13.16, the time required to complete one revolution is equal to the period (T) of the sinusoidal waveform of Fig. 13.16(i). The radians subtended in this time interval are 2π. Substituting, we have

$$\boxed{\omega = \frac{2\pi}{T}} \quad \text{(rad/s)} \qquad \textbf{(13.11)}$$

In words, this equation states that the smaller the period of the sinusoidal waveform of Fig. 13.16(i), or the smaller the time interval before one complete cycle is generated, the greater must be the angular velocity of the rotating radius vector. Certainly this statement agrees with what we have learned thus far. We can now go one step further and apply the fact that the frequency of the generated waveform is inversely related to the period of the waveform; that is, $f = 1/T$. Thus,

$$\boxed{\omega = 2\pi f} \quad \text{(rad/s)} \qquad \textbf{(13.12)}$$

This equation states that the higher the frequency of the generated sinusoidal waveform, the higher must be the angular velocity. Equations (13.11) and (13.12) are verified somewhat by Fig. 13.17, where for the same radius vector, $\omega = 100$ rad/s and 500 rad/s.

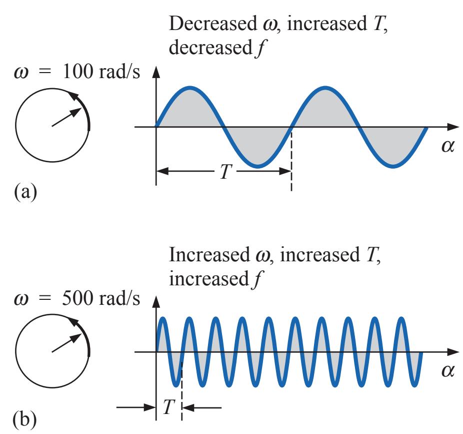


FIG. 13.17
Demonstrating the effect of ω on the frequency and period.

EXAMPLE 13.4 Determine the angular velocity of a sine wave having a frequency of 60 Hz.

Solution:

$$\omega = 2\pi f = (2\pi)(60\text{ Hz}) \cong \mathbf{377\ rad/s}$$

(a recurring value due to 60-Hz predominance)

EXAMPLE 13.5 Determine the frequency and period of the sine wave of Fig. 13.17(b).

Solution: Since $\omega = 2\pi/T$,

$$T = \frac{2\pi}{\omega} = \frac{2\pi\text{ rad}}{500\text{ rad/s}} = \frac{2\pi\text{ rad}}{500\text{ rad/s}} = \mathbf{12.57\ ms}$$

and

$$f = \frac{1}{T} = \frac{1}{12.57 \times 10^{-3}\text{ s}} = \mathbf{79.58\ Hz}$$

EXAMPLE 13.6 Given $\omega = 200$ rad/s, determine how long it will take the sinusoidal waveform to pass through an angle of 90°.

Solution: Eq. (13.10): $\alpha = \omega t$, and

$$t = \frac{\alpha}{\omega}$$

However, α must be substituted as $\pi/2$ (= 90°) since ω is in radians per second:

$$t = \frac{\alpha}{\omega} = \frac{\pi/2 \text{ rad}}{200 \text{ rad/s}} = \frac{\pi}{400} \text{ s} = \mathbf{7.85 \text{ ms}}$$

EXAMPLE 13.7 Find the angle through which a sinusoidal waveform of 60 Hz will pass in a period of 5 ms.

Solution: Eq. (13.11): $\alpha = \omega t$, or

$$\alpha = 2\pi f t = (2\pi)(60 \text{ Hz})(5 \times 10^{-3} \text{s}) = \mathbf{1.885 \text{ rad}}$$

If not careful, one might be tempted to interpret the answer as 1.885°. However,

$$\alpha\ (°) = \frac{180°}{\pi \text{ rad}}(1.885 \text{ rad}) = \mathbf{108°}$$

13.4 GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT

The basic mathematical format for the sinusoidal waveform is

$$\boxed{A_m \sin \alpha} \tag{13.13}$$

where A_m is the peak value of the waveform and α is the unit of measure for the horizontal axis, as shown in Fig. 13.18.

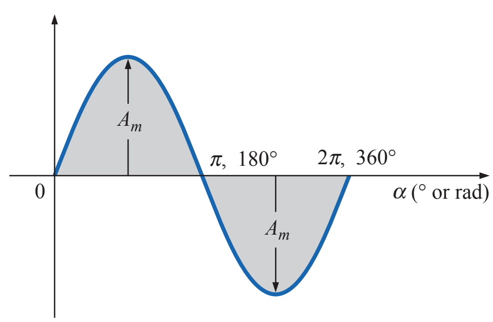


FIG. 13.18
Basic sinusoidal function.

The equation $\alpha = \omega t$ states that the angle α through which the rotating vector of Fig. 13.16 will pass is determined by the angular velocity of the rotating vector and the length of time the vector rotates. For example, for a particular angular velocity (fixed ω), the longer the radius vector is permitted to rotate (that is, the greater the value of t), the greater will be the number of degrees or radians through which the vector will pass. Relating this statement to the sinusoidal waveform, for a particular angular velocity, the longer the time, the greater the number of cycles shown. For a fixed time interval, the greater the angular velocity, the greater the number of cycles generated.

Due to Eq. (13.10), the general format of a sine wave can also be written

$$\boxed{A_m \sin \omega t} \qquad \textbf{(13.14)}$$

with ωt as the horizontal unit of measure.

For electrical quantities such as current and voltage, the general format is

$$i = I_m \sin \omega t = I_m \sin \alpha$$
$$e = E_m \sin \omega t = E_m \sin \alpha$$

where the capital letters with the subscript m represent the amplitude, and the lowercase letters i and e represent the instantaneous value of current or voltage, respectively, at any time t. This format is particularly important since it presents the sinusoidal voltage or current as a function of time, which is the horizontal scale for the oscilloscope. Recall that the horizontal sensitivity of a scope is in time per division and not degrees per centimeter.

EXAMPLE 13.8 Given $e = 5 \sin \alpha$, determine e at $\alpha = 40°$ and $\alpha = 0.8\pi$.

Solution: For $\alpha = 40°$,

$$e = 5 \sin 40° = 5(0.6428) = \mathbf{3.214\ V}$$

For $\alpha = 0.8\pi$,

$$\alpha\ (°) = \frac{180°}{\pi}(0.8\pi) = 144°$$

and $$e = 5 \sin 144° = 5(0.5878) = \mathbf{2.939\ V}$$

The conversion to degrees will not be required for most modern-day scientific calculators since they can perform the function directly. First, be sure that the calculator is in the RAD mode. Then simply enter the radian measure and use the appropriate trigonometric key (sin, cos, tan, etc.).

The angle at which a particular voltage level is attained can be determined by rearranging the equation

$$e = E_m \sin \alpha$$

in the following manner:

$$\sin \alpha = \frac{e}{E_m}$$

which can be written

$$\boxed{\alpha = \sin^{-1} \frac{e}{E_m}} \qquad \textbf{(13.15)}$$

Similarly, for a particular current level,

$$\boxed{\alpha = \sin^{-1} \frac{i}{I_m}} \qquad \textbf{(13.16)}$$

The function $\sin^{-1}$ is available on all scientific calculators.

EXAMPLE 13.9

a. Determine the angle at which the magnitude of the sinusoidal function $v = 10 \sin 377t$ is 4 V.
b. Determine the time at which the magnitude is attained.

Solutions:

a. Eq. (13.15):

$$\alpha_1 = \sin^{-1}\frac{v}{E_m} = \sin^{-1}\frac{4\text{ V}}{10\text{ V}} = \sin^{-1} 0.4 = \mathbf{23.578^\circ}$$

However, Figure 13.19 reveals that the magnitude of 4 V (positive) will be attained at two points between 0° and 180°. The second intersection is determined by

$$\alpha_2 = 180^\circ - 23.578^\circ = \mathbf{156.422^\circ}$$

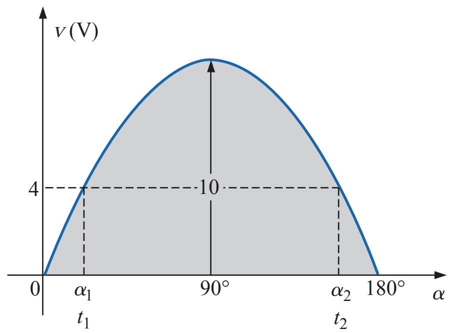


FIG. 13.19
Example 13.9.

In general, therefore, keep in mind that Equations (13.15) and (13.16) will provide an angle with a magnitude between 0° and 90°.

b. Eq. (13.10): $\alpha = \omega t$, and so $t = \alpha/\omega$. However, α must be in radians. Thus,

$$\alpha\text{ (rad)} = \frac{\pi}{180^\circ}(23.578^\circ) = 0.411\text{ rad}$$

and $$t_1 = \frac{\alpha}{\omega} = \frac{0.411\text{ rad}}{377\text{ rad/s}} = \mathbf{1.09\text{ ms}}$$

For the second intersection,

$$\alpha\text{ (rad)} = \frac{\pi}{180^\circ}(156.422^\circ) = 2.73\text{ rad}$$

$$t_2 = \frac{\alpha}{\omega} = \frac{2.73\text{ rad}}{377\text{ rad/s}} = \mathbf{7.24\text{ ms}}$$

The sine wave can also be plotted against *time* on the horizontal axis. The time period for each interval can be determined from $t = \alpha/\omega$, but the most direct route is simply to find the period T from $T = 1/f$ and break it up into the required intervals. This latter technique will be demonstrated in Example 13.10.

Before reviewing the example, take special note of the relative simplicity of the mathematical equation that can represent a sinusoidal waveform. Any alternating waveform whose characteristics differ from those of the sine wave cannot be represented by a single term, but may require two, four, six, or perhaps an infinite number of terms to be represented accurately. Additional description of nonsinusoidal waveforms can be found in Chapter 25.

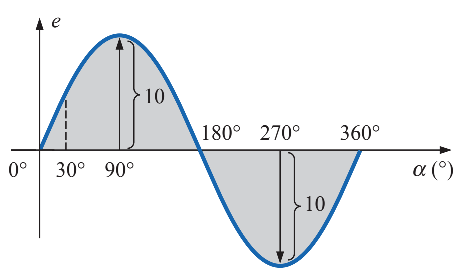


FIG. 13.20
Example 13.10, horizontal axis in degrees.

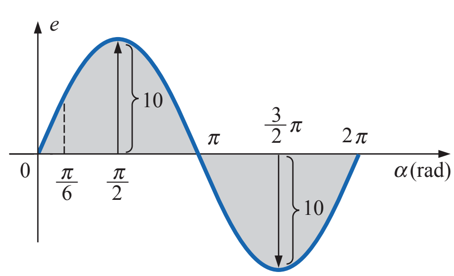


FIG. 13.21
Example 13.10, horizontal axis in radians.

EXAMPLE 13.10

Sketch $e = 10 \sin 314t$ with the abscissa

a. angle (α) in degrees.
b. angle (α) in radians.
c. time (t) in seconds.

Solutions:

a. See Fig 13.20. (Note that no calculations are required.)
b. See Fig. 13.21. (Once the relationship between degrees and radians is understood, there is again no need for calculations.)

c. 360°: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{314} = 20\text{ ms}$

180°: $\dfrac{T}{2} = \dfrac{20\text{ ms}}{2} = 10\text{ ms}$

90°: $\dfrac{T}{4} = \dfrac{20\text{ ms}}{4} = 5\text{ ms}$

30°: $\dfrac{T}{12} = \dfrac{20\text{ ms}}{12} = 1.67\text{ ms}$

See Fig. 13.22.

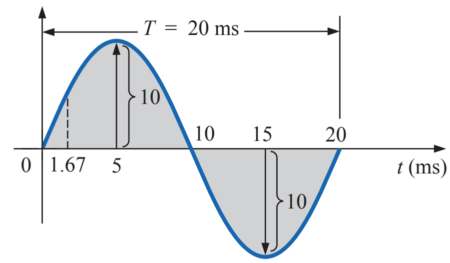


FIG. 13.22
Example 13.10, horizontal axis in milliseconds.

EXAMPLE 13.11 Given $i = 6 \times 10^{-3} \sin 1000t$, determine i at $t =$ 2 ms.

Solution:

$$\alpha = \omega t = 1000t = (1000\text{ rad/s})(2 \times 10^{-3}\text{ s}) = 2\text{ rad}$$

$$\alpha\,(°) = \frac{180°}{\pi\text{ rad}}(2\text{ rad}) = 114.59°$$

$$\begin{aligned} i &= (6 \times 10^{-3})(\sin 114.59°) \\ &= (6\text{ mA})(0.9093) = \mathbf{5.46\text{ mA}} \end{aligned}$$

13.5 PHASE RELATIONS

Thus far, we have considered only sine waves that have maxima at $\pi/2$ and $3\pi/2$, with a zero value at 0, π, and 2π, as shown in Fig. 13.21. If the waveform is shifted to the right or left of 0°, the expression becomes

$$\boxed{A_m \sin(\omega t \pm \theta)} \qquad \textbf{(13.17)}$$

where θ is the angle in degrees or radians that the waveform has been shifted.

If the waveform passes through the horizontal axis with a *positive-going* (increasing with time) slope *before* 0°, as shown in Fig. 13.23, the expression is

$$\boxed{A_m \sin(\omega t + \theta)} \qquad \textbf{(13.18)}$$

At $\omega t = \alpha = 0°$, the magnitude is determined by $A_m \sin \theta$. If the waveform passes through the horizontal axis with a positive-going slope *after* 0°, as shown in Fig. 13.24, the expression is

$$\boxed{A_m \sin(\omega t - \theta)} \qquad \textbf{(13.19)}$$

And at $\omega t = \alpha = 0°$, the magnitude is $A_m \sin(-\theta)$, which, by a trigonometric identity, is $-A_m \sin \theta$.

If the waveform crosses the horizontal axis with a positive-going slope 90° ($\pi/2$) sooner, as shown in Fig. 13.25, it is called a *cosine wave;* that is,

$$\boxed{\sin(\omega t + 90°) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t} \qquad \textbf{(13.20)}$$

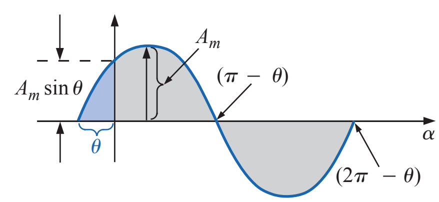


FIG. 13.23
Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope before 0°.

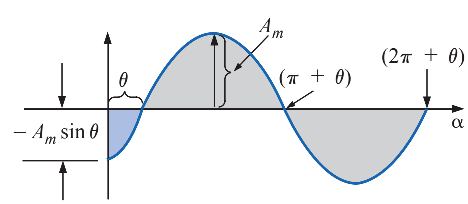


FIG. 13.24
Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope after 0°.

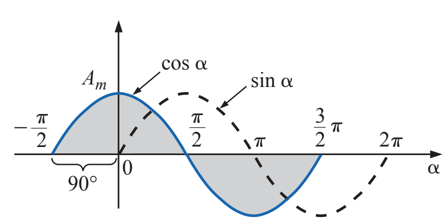


FIG. 13.25
Phase relationship between a sine wave and a cosine wave.

or

$$\boxed{\sin \omega t = \cos(\omega t - 90°) = \cos\left(\omega t - \frac{\pi}{2}\right)} \qquad \textbf{(13.21)}$$

The terms *lead* and *lag* are used to indicate the relationship between two sinusoidal waveforms of the *same frequency* plotted on the same set of axes. In Fig. 13.25, the cosine curve is said to *lead* the sine curve by 90°, and the sine curve is said to *lag* the cosine curve by 90°. The 90° is referred to as the phase angle between the two waveforms. In language commonly applied, the waveforms are *out of phase* by 90°. Note that the phase angle between the two waveforms is measured between those two points on the horizontal axis through which each passes with the *same slope.* If both waveforms cross the axis at the same point with the same slope, they are *in phase.*

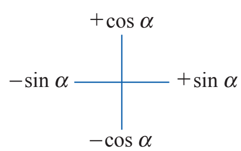


FIG. 13.26
Graphic tool for finding the relationship between specific sine and cosine functions.

The geometric relationship between various forms of the sine and cosine functions can be derived from Fig. 13.26. For instance, starting at the $\sin \alpha$ position, we find that $\cos \alpha$ is an additional 90° in the counterclockwise direction. Therefore, $\cos \alpha = \sin(\alpha + 90°)$. For $-\sin \alpha$ we must travel 180° in the counterclockwise (or clockwise) direction so that $-\sin \alpha = \sin(\alpha \pm 180°)$, and so on, as listed below:

$$\boxed{\begin{aligned} \cos \alpha &= \sin(\alpha + 90°) \\ \sin \alpha &= \cos(\alpha - 90°) \\ -\sin \alpha &= \sin(\alpha \pm 180°) \\ -\cos \alpha &= \sin(\alpha + 270°) = \sin(\alpha - 90°) \\ &\text{etc.} \end{aligned}} \qquad \textbf{(13.22)}$$

In addition, one should be aware that

$$\boxed{\begin{aligned} \sin(-\alpha) &= -\sin \alpha \\ \cos(-\alpha) &= \cos \alpha \end{aligned}} \qquad \textbf{(13.23)}$$

If a sinusoidal expression should appear as

$$e = -E_m \sin \omega t$$

the negative sign is associated with the sine portion of the expression, not the peak value E_m. In other words, the expression, if not for convenience, would be written

$$e = E_m(-\sin \omega t)$$

Since

$$-\sin \omega t = \sin(\omega t \pm 180°)$$

the expression can also be written

$$e = E_m \sin(\omega t \pm 180°)$$

revealing that a negative sign can be replaced by a 180° change in phase angle (+ or −); that is,

$$\begin{aligned} e = E_m \sin \omega t &= E_m \sin(\omega t + 180°) \\ &= E_m \sin(\omega t - 180°) \end{aligned}$$

A plot of each will clearly show their equivalence. There are, therefore, two correct mathematical representations for the functions.

The **phase relationship** between two waveforms indicates which one leads or lags, and by how many degrees or radians.

EXAMPLE 13.12 What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10 \sin(\omega t + 30°)$
   $i = 5 \sin(\omega t + 70°)$
b. $i = 15 \sin(\omega t + 60°)$
   $v = 10 \sin(\omega t - 20°)$
c. $i = 2 \cos(\omega t + 10°)$
   $v = 3 \sin(\omega t - 10°)$
d. $i = -\sin(\omega t + 30°)$
   $v = 2 \sin(\omega t + 10°)$
e. $i = -2 \cos(\omega t - 60°)$
   $v = 3 \sin(\omega t - 150°)$

Solutions:

a. See Fig. 13.27.
   ***i* leads *v* by 40°, or *v* lags *i* by 40°.**

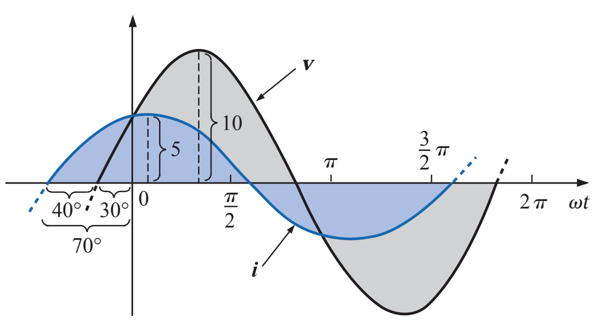


FIG. 13.27
Example 13.12; i leads v by 40°.

b. See Fig. 13.28.
   ***i* leads *v* by 80°, or *v* lags *i* by 80°.**

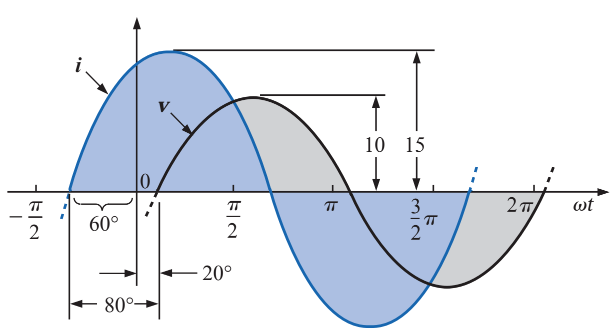


FIG. 13.28
Example 13.12; i leads v by 80°.

c. See Fig. 13.29.

$$i = 2\cos(\omega t + 10°) = 2\sin(\omega t + 10° + 90°)$$
$$= 2\sin(\omega t + 100°)$$

***i* leads *v* by 110°, or *v* lags *i* by 110°.**

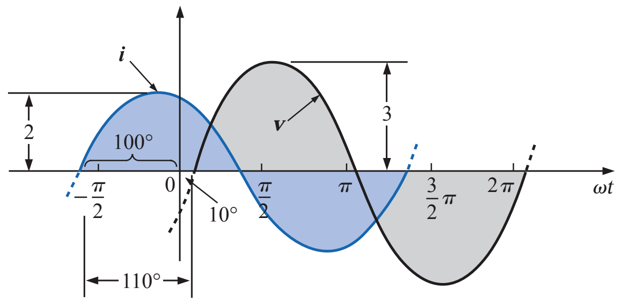


FIG. 13.29

Example 13.12; i leads v by 110°.

d. See Fig. 13.30.

$$-\sin(\omega t + 30°) = \sin(\omega t + 30° \overset{\text{Note}}{-} 180°)$$
$$= \sin(\omega t - 150°)$$

***v* leads *i* by 160°, or *i* lags *v* by 160°.**

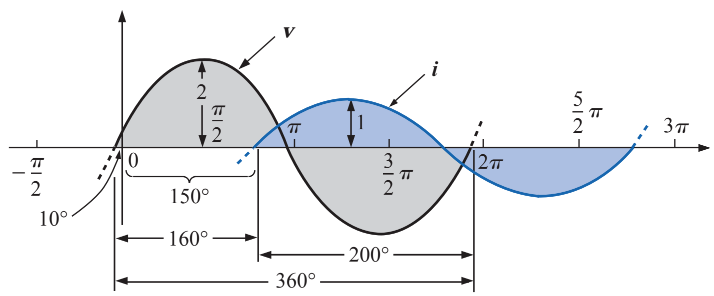


FIG. 13.30

Example 13.12; v leads i by 160°.

Or using

$$-\sin(\omega t + 30°) = \sin(\omega t + 30° \overset{\text{Note}}{+} 180°)$$
$$= \sin(\omega t + 210°)$$

***i* leads *v* by 200°, or *v* lags *i* by 200°.**

e. See Fig. 13.31.

$$i = -2\cos(\omega t - 60°) = 2\cos(\omega t - 60° \overset{\text{By choice}}{-} 180°)$$
$$= 2\cos(\omega t - 240°)$$

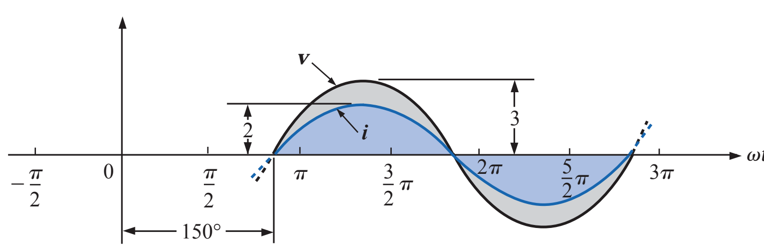


FIG. 13.31

Example 13.12; v and i are in phase.

However, $\cos \alpha = \sin(\alpha + 90°)$

so that
$$2 \cos(\omega t - 240°) = 2 \sin(\omega t - 240° + 90°)$$
$$= 2 \sin(\omega t - 150°)$$

***v* and *i* are in phase.**

Phase Measurements

The hookup procedure for using an oscilloscope to measure phase angles is covered in detail in Section 15.13. However, the equation for determining the phase angle can be introduced using Fig. 13.32. First, note that each sinusoidal function has the same frequency, permitting the use of either waveform to determine the period. For the waveform chosen in Fig. 13.32, the period encompasses 5 divisions at 0.2 ms/div. The phase shift between the waveforms (irrespective of which is leading or lagging) is 2 divisions. Since the full period represents a cycle of 360°, the following ratio [from which Equation (13.24) can be derived] can be formed:

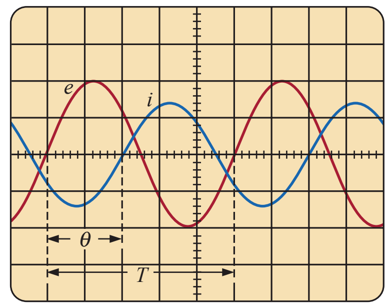
$$\frac{360°}{T\,(\text{no. of div.})} = \frac{\theta}{\text{phase shift (no. of div.)}}$$

and
$$\boxed{\theta = \frac{\text{phase shift (no. of div.)}}{T\,(\text{no. of div.})} \times 360°} \qquad \textbf{(13.24)}$$

Substituting into Eq. (13.24) will result in

$$\theta = \frac{(2\text{ div.})}{(5\text{ div.})} \times 360° = 144°$$

and e leads i by 144°.



Vertical sensitivity = 2 V/div.
Horizontal sensitivity = 0.2 ms/div.

FIG. 13.32
Finding the phase angle between waveforms using a dual-trace oscilloscope.

13.6 AVERAGE VALUE

Even though the concept of the **average value** is an important one in most technical fields, its true meaning is often misunderstood. In Fig. 13.33(a), for example, the average height of the sand may be required to determine the volume of sand available. The average height of the sand is that height obtained if the distance from one end to the other is maintained while the sand is leveled off, as shown in Fig. 13.33(b). The area under the mound of Fig. 13.33(a) will then equal the area under the rectangular shape of Fig. 13.33(b) as determined by $A = b \times h$. Of course, the depth (into the page) of the sand must be the same for Fig. 13.33(a) and (b) for the preceding conclusions to have any meaning.

In Fig. 13.33 the distance was measured from one end to the other. In Fig. 13.34(a) the distance extends beyond the end of the original pile of Fig. 13.33. The situation could be one where a landscaper would like to know the average height of the sand if spread out over a distance such as defined in Fig. 13.34(a). The result of an increased distance is as shown in Fig. 13.34(b). The average height has decreased compared to Fig. 13.33. Quite obviously, therefore, the longer the distance, the lower is the average value.

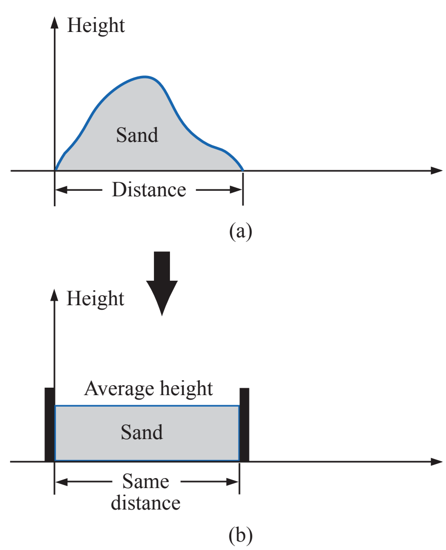


FIG. 13.33
Defining average value.

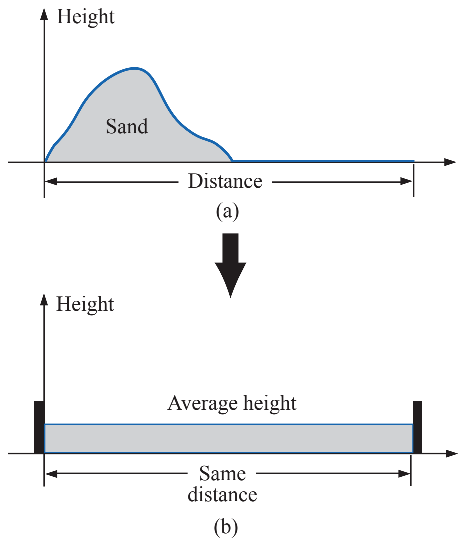


FIG. 13.34
Effect of distance (length) on average value.

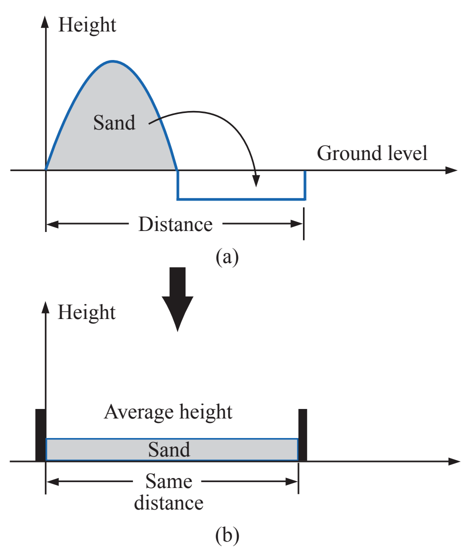


FIG. 13.35
Effect of depressions (negative excursions) on average value.

If the distance parameter includes a depression, as shown in Fig. 13.35(a), some of the sand will be used to fill the depression, resulting in an even lower average value for the landscaper, as shown in Fig. 13.35(b). For a sinusoidal waveform, the depression would have the same shape as the mound of sand (over one full cycle), resulting in an average value at ground level (or zero volts for a sinusoidal voltage over one full period).

After traveling a considerable distance by car, some drivers like to calculate their average speed for the entire trip. This is usually done by dividing the miles traveled by the hours required to drive that distance. For example, if a person traveled 225 mi in 5 h, the average speed was 225 mi/5 h, or 45 mi/h. This same distance may have been traveled at various speeds for various intervals of time, as shown in Fig. 13.36.

By finding the total area under the curve for the 5 h and then dividing the area by 5 h (the total time for the trip), we obtain the same result of 45 mi/h; that is,

$$\boxed{\text{Average speed} = \frac{\text{area under curve}}{\text{length of curve}}} \tag{13.25}$$

$$\begin{aligned}\text{Average speed} &= \frac{A_1 + A_2}{5\text{ h}}\\ &= \frac{(60\text{ mi/h})(2\text{ h}) + (50\text{ mi/h})(2.5\text{ h})}{5\text{ h}}\\ &= \frac{225}{5}\text{ mi/h}\\ &= \mathbf{45\ mi/h}\end{aligned}$$

Equation (13.25) can be extended to include any variable quantity, such as current or voltage, if we let G denote the average value, as follows:

$$\boxed{G\text{ (average value)} = \frac{\text{algebraic sum of areas}}{\text{length of curve}}} \tag{13.26}$$

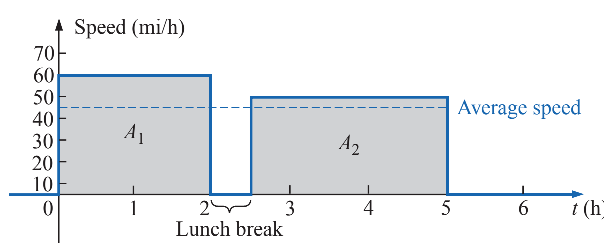


FIG. 13.36
Plotting speed versus time for an automobile excursion.

The *algebraic* sum of the areas must be determined, since some area contributions will be from below the horizontal axis. Areas above the axis will be assigned a positive sign, and those below, a negative sign. A positive average value will then be above the axis, and a negative value, below.

The average value of *any* current or voltage is the value indicated on a dc meter. In other words, over a complete cycle, the average value is the equivalent dc value. In the analysis of electronic circuits to be considered in a later course, both dc and ac sources of voltage will be applied to the same network. It will then be necessary to know or determine the dc (or average value) and ac components of the voltage or current in various parts of the system.

EXAMPLE 13.13 Determine the average value of the waveforms of Fig. 13.37.

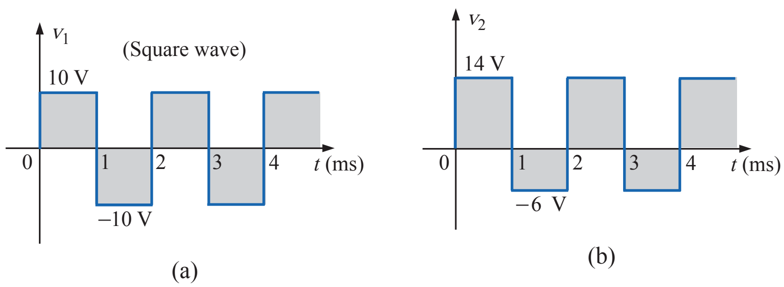


FIG. 13.37
Example 13.13.

Solutions:

a. By inspection, the area above the axis equals the area below over one cycle, resulting in an average value of zero volts. Using Eq. (13.26):

$$G = \frac{(10\text{ V})(1\text{ ms}) - (10\text{ V})(1\text{ ms})}{2\text{ ms}}$$

$$= \frac{0}{2\text{ ms}} = \mathbf{0\text{ V}}$$

b. Using Eq. (13.26):

$$G = \frac{(14\text{ V})(1\text{ ms}) - (6\text{ V})(1\text{ ms})}{2\text{ ms}}$$

$$= \frac{14\text{ V} - 6\text{ V}}{2} = \frac{8\text{ V}}{2} = \mathbf{4\text{ V}}$$

as shown in Fig. 13.38.

In reality, the waveform of Fig. 13.37(b) is simply the square wave of Fig. 13.37(a) with a dc shift of 4 V; that is,

$$v_2 = v_1 + 4\text{ V}$$

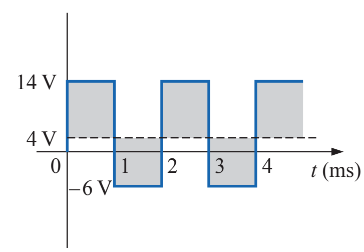


FIG. 13.38
Defining the average value for the waveform of Fig. 13.37(b).

EXAMPLE 13.14 Find the average values of the following waveforms over one full cycle:

a. Fig. 13.39.
b. Fig. 13.40.

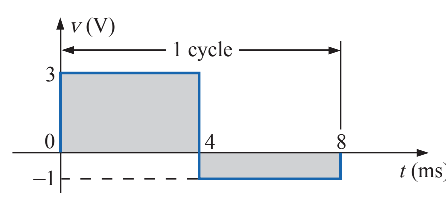


FIG. 13.39
Example 13.14, part (a).

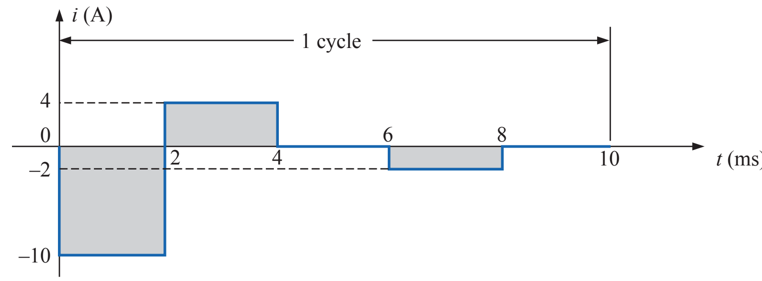


FIG. 13.40
Example 13.14, part (b).

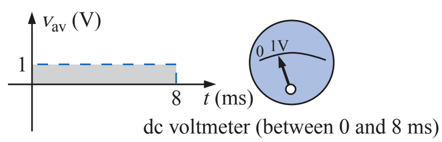


FIG. 13.41
The response of a dc meter to the waveform of Fig. 13.39.

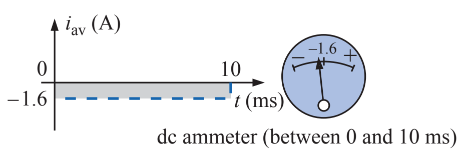


FIG. 13.42
The response of a dc meter to the waveform of Fig. 13.40.

Solutions:

a. $$G = \frac{+(3\text{ V})(4\text{ ms}) - (1\text{ V})(4\text{ ms})}{8\text{ ms}} = \frac{12\text{ V} - 4\text{ V}}{8} = \mathbf{1\text{ V}}$$

Note Fig. 13.41.

b. $$G = \frac{-(10\text{ V})(2\text{ ms}) + (4\text{ V})(2\text{ ms}) - (2\text{ V})(2\text{ ms})}{10\text{ ms}}$$

$$= \frac{-20\text{ V} + 8\text{ V} - 4\text{ V}}{10} = -\frac{16\text{ V}}{10} = \mathbf{-1.6\text{ V}}$$

Note Fig. 13.42.

We found the areas under the curves in the preceding example by using a simple geometric formula. If we should encounter a sine wave or any other unusual shape, however, we must find the area by some other means. We can obtain a good approximation of the area by attempting to reproduce the original wave shape using a number of small rectangles or other familiar shapes, the area of which we already know through simple geometric formulas. For example,

the area of the positive (or negative) pulse of a sine wave is $2A_m$.

Approximating this waveform by two triangles (Fig. 13.43), we obtain (using *area* = 1/2 *base* × *height* for the area of a triangle) a rough idea of the actual area:

$$\text{Area shaded} = 2\left(\frac{1}{2}bh\right) = 2\left[\left(\frac{1}{2}\right)\overbrace{\left(\frac{\pi}{2}\right)}^{b}\overbrace{(A_m)}^{h}\right] = \frac{\pi}{2}A_m$$

$$\cong 1.58A_m$$

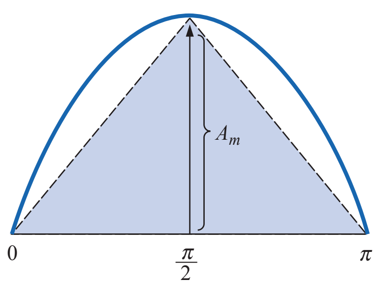


FIG. 13.43
Approximating the shape of the positive pulse of a sinusoidal waveform with two right triangles.

A closer approximation might be a rectangle with two similar triangles (Fig. 13.44):

$$\text{Area} = A_m\frac{\pi}{3} + 2\left(\frac{1}{2}\,bh\right) = A_m\frac{\pi}{3} + \frac{\pi}{3}\,A_m = \frac{2}{3}\pi A_m$$

$$= 2.094A_m$$

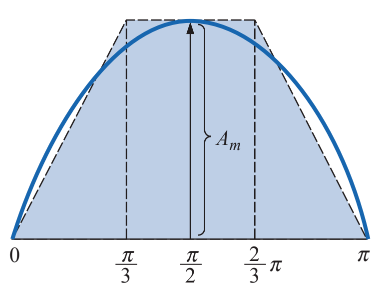


FIG. 13.44
A better approximation for the shape of the positive pulse of a sinusoidal waveform.

which is certainly close to the actual area. If an infinite number of forms were used, an exact answer of $2A_m$ could be obtained. For irregular waveforms, this method can be especially useful if data such as the average value are desired.

The procedure of calculus that gives the exact solution $2A_m$ is known as *integration.* Integration is presented here only to make the method recognizable to the reader; it is not necessary to be proficient in its use to continue with this text. It is a useful mathematical tool, however, and should be learned. Finding the area under the positive pulse of a sine wave using integration, we have

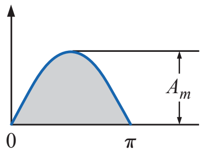
$$\text{Area} = \int_0^{\pi} A_m \sin \alpha \, d\alpha$$

where $\int$ is the sign of integration, 0 and π are the limits of integration, $A_m \sin \alpha$ is the function to be integrated, and $d\alpha$ indicates that we are integrating with respect to α.

Integrating, we obtain

$$\begin{aligned}\text{Area} &= A_m[-\cos \alpha]_0^{\pi}\\ &= -A_m(\cos \pi - \cos 0°)\\ &= -A_m[-1 - (+1)] = -A_m(-2)\end{aligned}$$

$$\boxed{\text{Area} = 2A_m} \qquad (13.27)$$

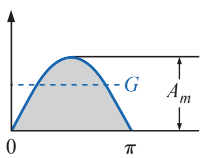


Since we know the area under the positive (or negative) pulse, we can easily determine the average value of the positive (or negative) region of a sine wave pulse by applying Eq. (13.26):

$$G = \frac{2A_m}{\pi}$$

and

$$\boxed{G = 0.637A_m} \qquad (13.28)$$



For the waveform of Fig. 13.45,

$$G = \frac{(2A_m/2)}{\pi/2} = \frac{2A_m}{\pi} \qquad \text{(average the same as for a full pulse)}$$

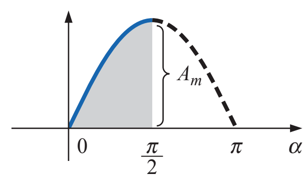


FIG. 13.45
Finding the average value of one-half the positive pulse of a sinusoidal waveform.

EXAMPLE 13.15 Determine the average value of the sinusoidal waveform of Fig. 13.46.

Solution: By inspection it is fairly obvious that

the average value of a pure sinusoidal waveform over one full cycle is zero.

Eq. (13.26):

$$G = \frac{+2A_m - 2A_m}{2\pi} = \mathbf{0\ V}$$

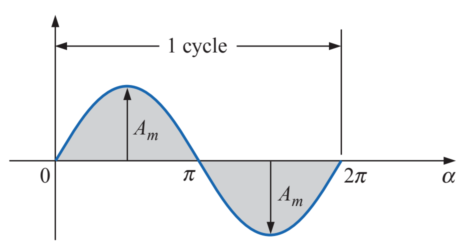


FIG. 13.46
Example 13.15.

EXAMPLE 13.16 Determine the average value of the waveform of Fig. 13.47.

Solution: The peak-to-peak value of the sinusoidal function is 16 mV + 2 mV = 18 mV. The peak amplitude of the sinusoidal waveform is, therefore, 18 mV/2 = 9 mV. Counting down 9 mV from 2 mV (or 9 mV up from −16 mV) results in an average or dc level of −7 mV, as noted by the dashed line of Fig. 13.47.

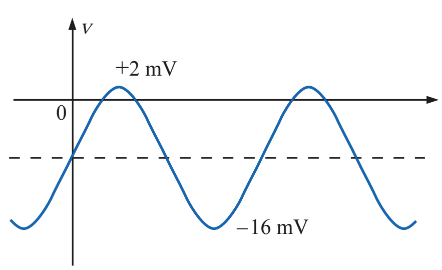


FIG. 13.47
Example 13.16.

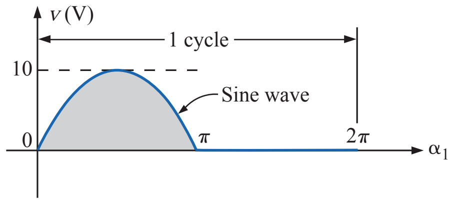


FIG. 13.48
Example 13.17.

EXAMPLE 13.17 Determine the average value of the waveform of Fig. 13.48.

Solution:

$$G = \frac{2A_m + 0}{2\pi} = \frac{2(10\text{ V})}{2\pi} \cong \mathbf{3.18\ V}$$

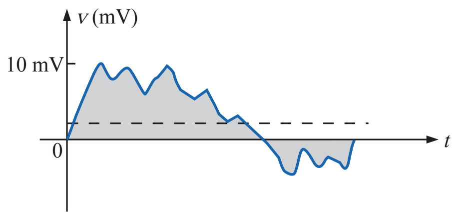


FIG. 13.49
Example 13.18.

EXAMPLE 13.18 For the waveform of Fig. 13.49, determine whether the average value is positive or negative, and determine its approximate value.

Solution: From the appearance of the waveform, the average value is positive and in the vicinity of 2 mV. Occasionally, judgments of this type will have to be made.

Instrumentation

The dc level or average value of any waveform can be found using a digital multimeter (DMM) or an **oscilloscope.** For purely dc circuits, simply set the DMM on dc, and read the voltage or current levels. Oscilloscopes are limited to voltage levels using the sequence of steps listed below:

1. First choose GND from the DC-GND-AC option list associated with each vertical channel. The GND option blocks any signal to which the oscilloscope probe may be connected from entering the oscilloscope and responds with just a horizontal line. Set the resulting line in the middle of the vertical axis on the horizontal axis, as shown in Fig. 13.50(a).

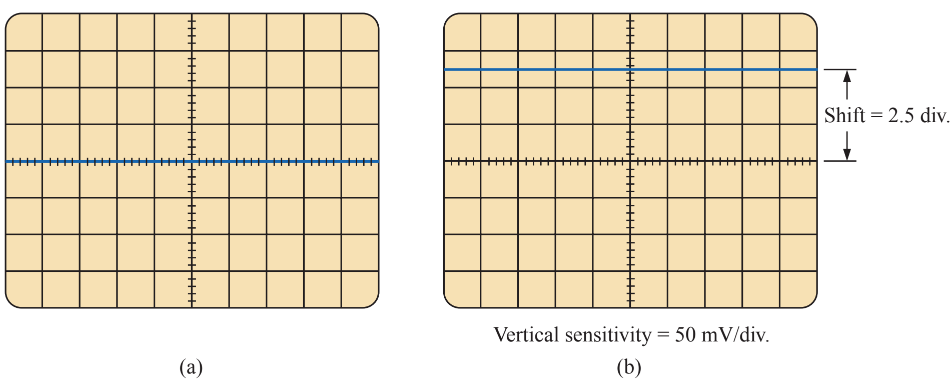


FIG. 13.50
Using the oscilloscope to measure dc voltages: (a) setting the GND condition; (b) the vertical shift resulting from a dc voltage when shifted to the DC option.

2. Apply the oscilloscope probe to the voltage to be measured (if not already connected), and switch to the DC option. If a dc voltage is present, the horizontal line will shift up or down, as demonstrated in Fig. 13.50(b). Multiplying the shift by the vertical sensitivity will result in the dc voltage. An upward shift is a positive voltage (higher potential at the red or positive lead of the oscilloscope), while a downward shift is a negative voltage (lower potential at the red or positive lead of the oscilloscope).

In general,

$$\boxed{V_{dc} = (\text{vertical shift in div.}) \times (\text{vertical sensitivity in V/div.})} \qquad \textbf{(13.29)}$$

For the waveform of Fig. 13.50(b),

$$V_{dc} = (2.5 \text{ div.})(50 \text{ mV/div.}) = \mathbf{125\ mV}$$

The oscilloscope can also be used to measure the dc or average level of any waveform using the following sequence:

1. Using the GND option, reset the horizontal line to the middle of the screen.
2. Switch to AC (all dc components of the signal to which the probe is connected will be blocked from entering the oscilloscope—only the alternating, or changing, components will be displayed). Note the location of some definitive point on the waveform, such as the bottom of the half-wave rectified waveform of Fig. 13.51(a); that is, note its position on the vertical scale. For the future, whenever you use the AC option, keep in mind that the computer will distribute the waveform above and below the horizontal axis such that the average value is zero; that is, the area above the axis will equal the area below.
3. Then switch to DC (to permit both the dc and the ac components of the waveform to enter the oscilloscope), and note the shift in the chosen level of part 2, as shown in Fig. 13.51(b). Equation (13.29) can then be used to determine the dc or average value of the waveform. For the waveform of Fig. 13.51(b), the average value is about

$$V_{av} = V_{dc} = (0.9 \text{ div.})(5 \text{ V/div.}) = \mathbf{4.5\ V}$$

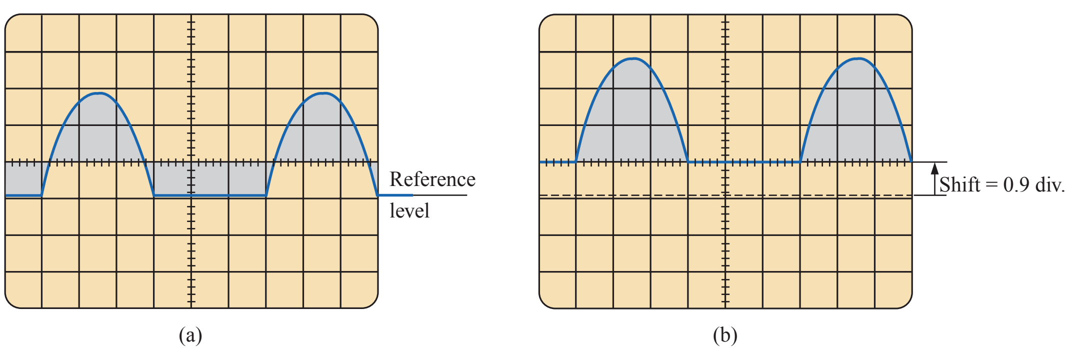


FIG. 13.51
Determining the average value of a nonsinusoidal waveform using the oscilloscope: (a) vertical channel on the ac mode; (b) vertical channel on the dc mode.

The procedure outlined above can be applied to any alternating waveform such as the one in Fig. 13.49. In some cases the average value may require moving the starting position of the waveform under the AC option to a different region of the screen or choosing a higher voltage scale. DMMs can read the average or dc level of any waveform by simply choosing the appropriate scale.

13.7 EFFECTIVE (rms) VALUES

This section will begin to relate dc and ac quantities with respect to the power delivered to a load. It will help us determine the amplitude of a sinusoidal ac current required to deliver the same power as a particular dc current. The question frequently arises, How is it possible for a sinusoidal ac quantity to deliver a net power if, over a full cycle, the net current in any one direction is zero (average value = 0)? It would almost appear that the power delivered during the positive portion of the sinusoidal waveform is withdrawn during the negative portion, and since the two are equal in magnitude, the net power delivered is zero. However, understand that *irrespective of direction,* current of any magnitude through a resistor will deliver power *to that resistor.* In other words, during the positive or negative portions of a sinusoidal ac current, power is being delivered at *each instant of time* to the resistor. The power delivered at each instant will, of course, vary with the magnitude of the sinusoidal ac current, but there will be a net flow during either the positive or the negative pulses with a net flow over the full cycle. The net power flow will equal twice that delivered by either the positive or the negative regions of sinusoidal quantity.

A fixed relationship between ac and dc voltages and currents can be derived from the experimental setup shown in Fig. 13.52. A resistor in a water bath is connected by switches to a dc and an ac supply. If switch 1 is closed, a dc current *I,* determined by the resistance *R* and battery voltage *E,* will be established through the resistor *R.* The temperature reached by the water is determined by the dc power dissipated in the form of heat by the resistor.

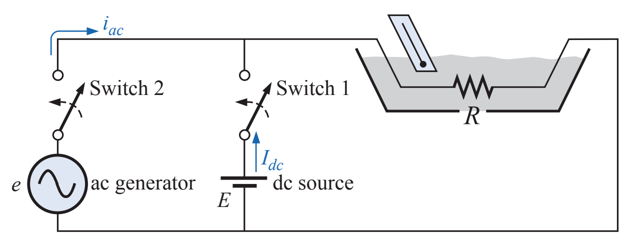


FIG. 13.52
An experimental setup to establish a relationship between dc and ac quantities.

If switch 2 is closed and switch 1 left open, the ac current through the resistor will have a peak value of I_m. The temperature reached by the water is now determined by the ac power dissipated in the form of heat by the resistor. The ac input is varied until the temperature is the same as that reached with the dc input. When this is accomplished, the average electrical power delivered to the resistor *R* by the ac source is the same as that delivered by the dc source.

The power delivered by the ac supply at any instant of time is

$$P_{ac} = (i_{ac})^2R = (I_m \sin \omega t)^2R = (I_m^2 \sin^2 \omega t)R$$

but

$$\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t) \qquad \text{(trigonometric identity)}$$

Therefore,

$$P_{ac} = I_m^2\left[\frac{1}{2}(1 - \cos 2\omega t)\right]R$$

and

$$\boxed{P_{ac} = \frac{I_m^2 R}{2} - \frac{I_m^2 R}{2}\cos 2\omega t} \tag{13.30}$$

The *average power* delivered by the ac source is just the first term, since the average value of a cosine wave is zero even though the wave may have twice the frequency of the original input current waveform. Equating the average power delivered by the ac generator to that delivered by the dc source,

$$P_{av(ac)} = P_{dc}$$

$$\frac{I_m^2 R}{2} = I_{dc}^2 R \quad \text{and} \quad I_m = \sqrt{2}I_{dc}$$

or

$$I_{dc} = \frac{I_m}{\sqrt{2}} = 0.707I_m$$

which, in words, states that

the equivalent dc value of a sinusoidal current or voltage is $1/\sqrt{2}$ or 0.707 of its maximum value.

The equivalent dc value is called the **effective value** of the sinusoidal quantity.

In summary,

$$\boxed{I_{eq(dc)} = I_{eff} = 0.707I_m} \tag{13.31}$$

or

$$\boxed{I_m = \sqrt{2}I_{eff} = 1.414I_{eff}} \tag{13.32}$$

and

$$\boxed{E_{eff} = 0.707E_m} \tag{13.33}$$

or

$$\boxed{E_m = \sqrt{2}E_{eff} = 1.414E_{eff}} \tag{13.34}$$

As a simple numerical example, it would require an ac current with a peak value of $\sqrt{2}(10) = 14.14$ A to deliver the same power to the resistor in Fig. 13.52 as a dc current of 10 A. The effective value of any quantity plotted as a function of time can be found by using the following equation derived from the experiment just described:

$$\boxed{I_{eff} = \sqrt{\frac{\int_0^T i^2(t)\,dt}{T}}} \tag{13.35}$$

or

$$\boxed{I_{eff} = \sqrt{\frac{\text{area } (i^2(t))}{T}}} \tag{13.36}$$

which, in words, states that to find the effective value, the function $i(t)$ must first be squared. After $i(t)$ is squared, the area under the curve is found by integration. It is then divided by T, the length of the cycle or the period of the waveform, to obtain the average or *mean* value of the squared waveform. The final step is to take the *square root* of the mean value. This procedure gives us another designation for the effective value, the **root-mean-square (rms) value.** In fact, since the rms term is the most commonly used in the educational and industrial communities, it will used throughout this text.

EXAMPLE 13.19 Find the rms values of the sinusoidal waveform in each part of Fig. 13.53.

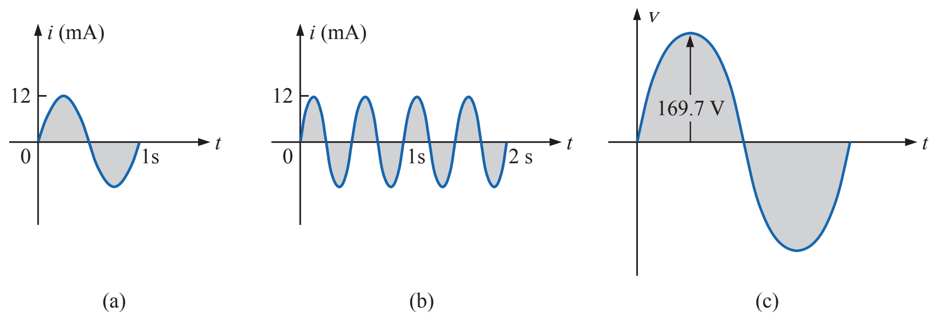


FIG. 13.53
Example 13.19.

Solution: For part (a), $I_{\text{rms}} = 0.707(12 \times 10^{-3}\text{ A}) =$ **8.484 mA.** For part (b), again $I_{\text{rms}} =$ **8.484 mA.** Note that frequency did not change the effective value in (b) above compared to (a). For part (c), $V_{\text{rms}} = 0.707(169.73\text{ V}) \cong$ **120 V,** the same as available from a home outlet.

EXAMPLE 13.20 The 120-V dc source of Fig. 13.54(a) delivers 3.6 W to the load. Determine the peak value of the applied voltage (E_m) and the current (I_m) if the ac source [Fig. 13.54(b)] is to deliver the same power to the load.

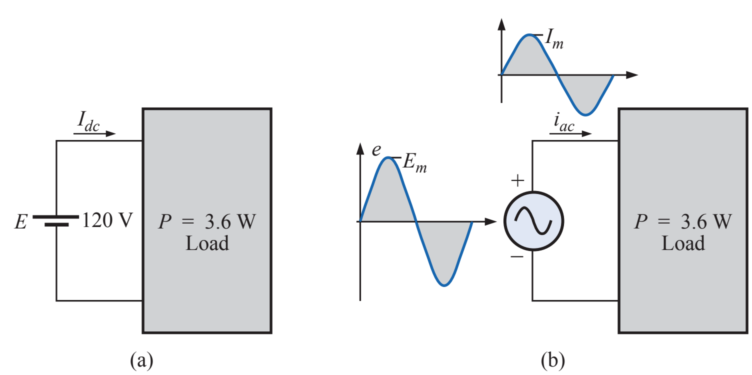


FIG. 13.54
Example 13.20.

Solution:

$$P_{dc} = V_{dc}I_{dc}$$

and
$$I_{dc} = \frac{P_{dc}}{V_{dc}} = \frac{3.6\text{ W}}{120\text{ V}} = 30\text{ mA}$$
$$I_m = \sqrt{2}I_{dc} = (1.414)(30\text{ mA}) = \mathbf{42.42\text{ mA}}$$
$$E_m = \sqrt{2}E_{dc} = (1.414)(120\text{ V}) = \mathbf{169.68\text{ V}}$$

EXAMPLE 13.21 Find the effective or rms value of the waveform of Fig. 13.55.

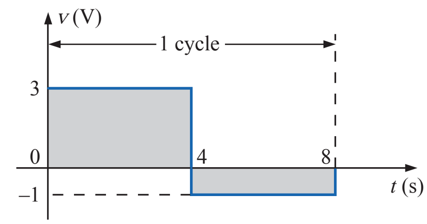


FIG. 13.55
Example 13.21.

Solution:
v^2 (Fig. 13.56):

$$V_{rms} = \sqrt{\frac{(9)(4) + (1)(4)}{8}} = \sqrt{\frac{40}{8}} = \mathbf{2.236\text{ V}}$$

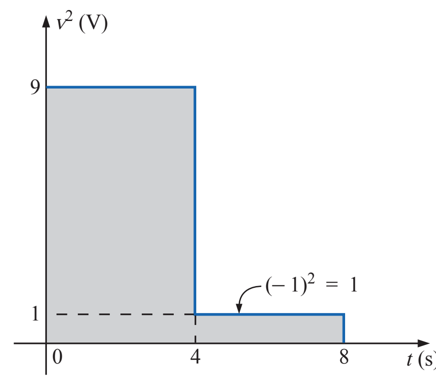


FIG. 13.56
The squared waveform of Fig. 13.55.

EXAMPLE 13.22 Calculate the rms value of the voltage of Fig. 13.57.

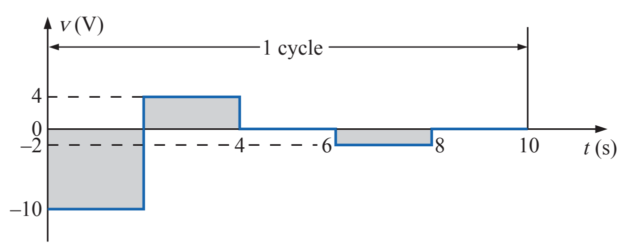


FIG. 13.57
Example 13.22.

Solution:
v^2 (Fig. 13.58):

$$V_{rms} = \sqrt{\frac{(100)(2) + (16)(2) + (4)(2)}{10}} = \sqrt{\frac{240}{10}}$$
$$= \mathbf{4.899\text{ V}}$$

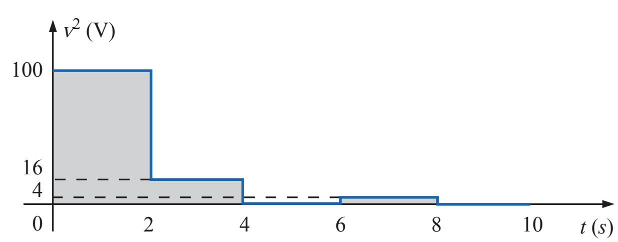


FIG. 13.58
The squared waveform of Fig. 13.57.

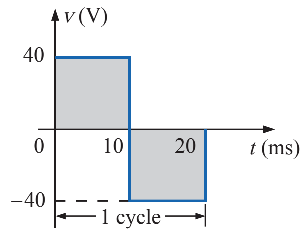


FIG. 13.59
Example 13.23.

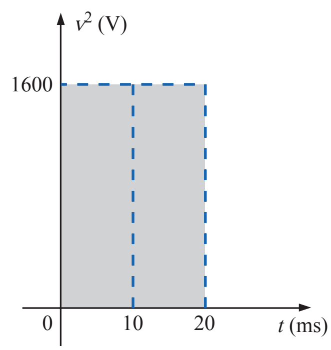


FIG. 13.60
The squared waveform of Fig. 13.59.

EXAMPLE 13.23 Determine the average and rms values of the square wave of Fig. 13.59.

Solution: By inspection, the average value is zero.

v^2 (Fig. 13.60):

$$V_{\text{rms}} = \sqrt{\frac{(1600)(10 \times 10^{-3}) + (1600)(10 \times 10^{-3})}{20 \times 10^{-3}}}$$

$$= \sqrt{\frac{32{,}000 \times 10^{-3}}{20 \times 10^{-3}}} = \sqrt{1600}$$

$$V_{\text{rms}} = \mathbf{40\ V}$$

(the maximum value of the waveform of Fig. 13.60)

The waveforms appearing in these examples are the same as those used in the examples on the average value. It might prove interesting to compare the rms and average values of these waveforms.

The rms values of sinusoidal quantities such as voltage or current will be represented by E and I. These symbols are the same as those used for dc voltages and currents. To avoid confusion, the peak value of a waveform will always have a subscript m associated with it: $I_m \sin \omega t$. *Caution:* When finding the rms value of the positive pulse of a sine wave, note that the squared area is *not* simply $(2A_m)^2 = 4A_m^2$; it must be found by a completely new integration. This will always be the case for any waveform that is not rectangular.

A unique situation arises if a waveform has both a dc and an ac component that may be due to a source such as the one in Fig. 13.61. The combination appears frequently in the analysis of electronic networks where both dc and ac levels are present in the same system.

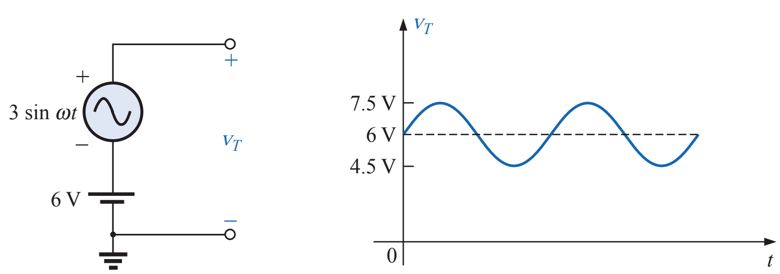


FIG. 13.61
Generation and display of a waveform having a dc and an ac component.

The question arises, What is the rms value of the voltage v_T? One might be tempted to simply assume that it is the sum of the rms values of each component of the waveform; that is, $V_{T_{\text{rms}}} = 0.7071(1.5\ \text{V}) + 6\ \text{V} = 1.06\ \text{V} + 6\ \text{V} = 7.06\ \text{V}$. However, the rms value is actually determined by

$$V_{\text{rms}} = \sqrt{V_{\text{dc}}^2 + V_{\text{ac(rms)}}^2} \qquad \textbf{(13.37)}$$

which for the above example is

$$V_{\text{rms}} = \sqrt{(6\ \text{V})^2 + (1.06\ \text{V})^2}$$
$$= \sqrt{37.124}\ \text{V}$$
$$\cong \mathbf{6.1\ V}$$

This result is noticeably less than the above solution. The development of Eq. (13.37) can be found in Chapter 25.

Instrumentation

It is important to note whether the DMM in use is a *true rms* meter or simply a meter where the average value is calibrated (as described in the next section) to indicate the rms level. A *true rms* meter will read the effective value of any waveform (such as Figs. 13.49 and 13.61) and is not limited to only sinusoidal waveforms. Since the label *true rms* is normally not placed on the face of the meter, it is prudent to check the manual if waveforms other than purely sinusoidal are to be encountered. For any type of rms meter, be sure to check the manual for its frequency range of application. For most it is less than 1 kHz.

13.8 ac METERS AND INSTRUMENTS

The d'Arsonval movement employed in dc meters can also be used to measure sinusoidal voltages and currents if the *bridge rectifier* of Fig. 13.62 is placed between the signal to be measured and the average reading movement.

The bridge rectifier, composed of four diodes (electronic switches), will convert the input signal of zero average value to one having an average value sensitive to the peak value of the input signal. The conversion process is well described in most basic electronics texts. Fundamentally, conduction is permitted through the diodes in such a manner as to convert the sinusoidal input of Fig. 13.63(a) to one having the appearance of Fig. 13.63(b). The negative portion of the input has been effectively "flipped over" by the bridge configuration. The resulting waveform of Fig. 13.63(b) is called a *full-wave rectified waveform.*

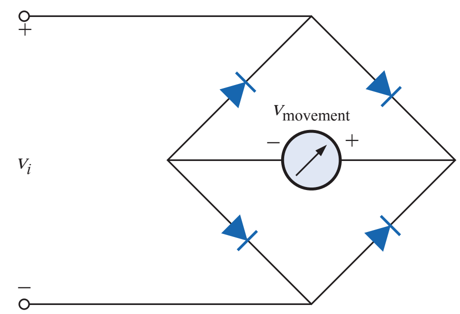


FIG. 13.62
Full-wave bridge rectifier.

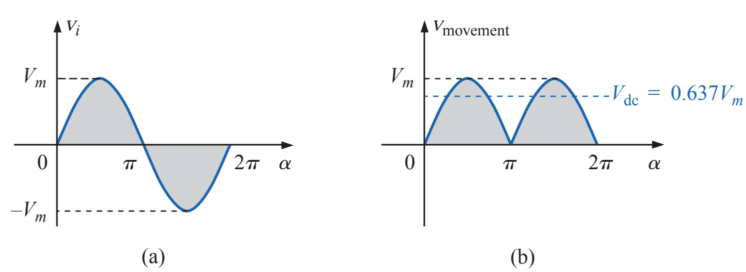


FIG. 13.63
(a) Sinusoidal input; (b) full-wave rectified signal.

The zero average value of Fig. 13.63(a) has been replaced by a pattern having an average value determined by

$$G = \frac{2V_m + 2V_m}{2\pi} = \frac{4V_m}{2\pi} = \frac{2V_m}{\pi} = 0.637V_m$$

The movement of the pointer will therefore be directly related to the peak value of the signal by the factor 0.637.

Forming the ratio between the rms and dc levels will result in

$$\frac{V_{rms}}{V_{dc}} = \frac{0.707V_m}{0.637V_m} \cong 1.11$$

revealing that the scale indication is 1.11 times the dc level measured by the movement; that is,

$$\boxed{\text{Meter indication} = 1.11\ (\text{dc or average value})} \quad \textit{full-wave} \qquad \textbf{(13.38)}$$

Some ac meters use a half-wave rectifier arrangement that results in the waveform of Fig. 13.64, which has half the average value of Fig. 13.63(b) over one full cycle. The result is

$$\boxed{\text{Meter indication} = 2.22\ (\text{dc or average value})} \quad \textit{half-wave} \qquad \textbf{(13.39)}$$

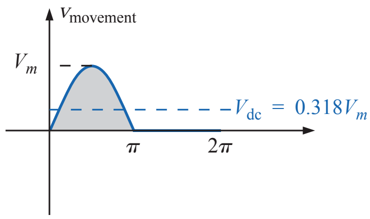


FIG. 13.64
Half-wave rectified signal.

A second movement, called the **electrodynamometer movement** (Fig. 13.65), can measure both ac and dc quantities without a change in internal circuitry. The movement can, in fact, read the effective value of any periodic or nonperiodic waveform because a reversal in current direction reverses the fields of both the stationary and the movable coils, so the deflection of the pointer is always up-scale.

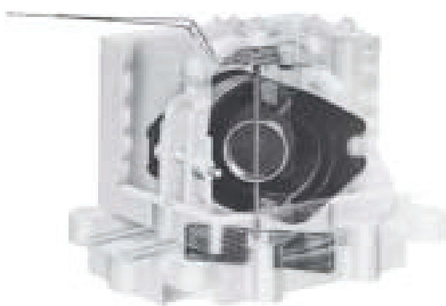


FIG. 13.65
Electrodynamometer movement. (Courtesy of Weston Instruments, Inc.)

The **VOM,** introduced in Chapter 2, can be used to measure both dc and ac voltages using a d'Arsonval movement and the proper switching networks. That is, when the meter is used for dc measurements, the dial setting will establish the proper series resistance for the chosen scale and will permit the appropriate dc level to pass directly to the movement. For ac measurements, the dial setting will introduce a network that employs a full- or half-wave rectifier to establish a dc level. As discussed above, each setting is properly calibrated to indicate the desired quantity on the face of the instrument.

EXAMPLE 13.24 Determine the reading of each meter for each situation of Fig. 13.66(a) and (b).

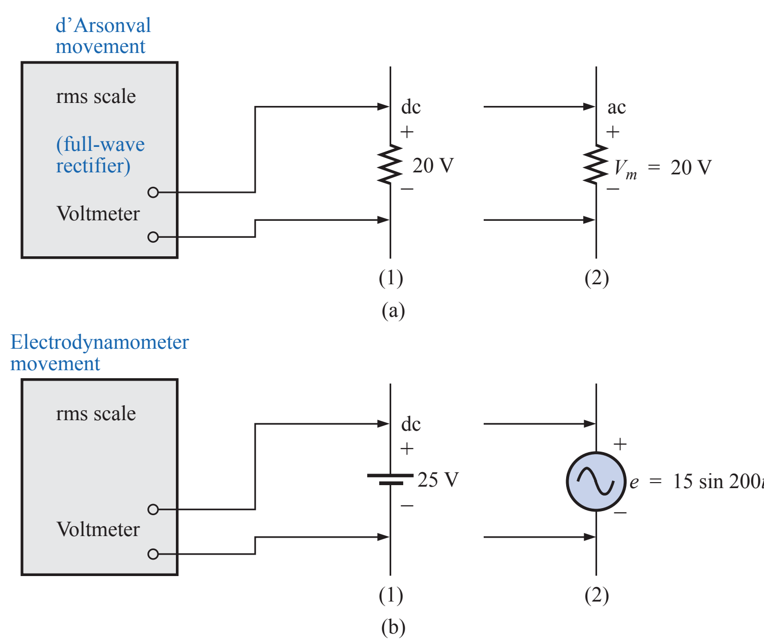


FIG. 13.66
Example 13.24.

Solution: For Fig. 13.66(a), situation (1): By Eq. (13.38),

$$\text{Meter indication} = 1.11(20\text{ V}) = \mathbf{22.2\text{ V}}$$

For Fig. 13.66(a), situation (2):

$$V_{\text{rms}} = 0.707V_m = 0.707(20\text{ V}) = \mathbf{14.14\text{ V}}$$

For Fig. 13.66(b), situation (1):

$$V_{\text{rms}} = V_{\text{dc}} = \mathbf{25\text{ V}}$$

For Fig. 13.66(b), situation (2):

$$V_{\text{rms}} = 0.707V_m = 0.707(15\text{ V}) \cong \mathbf{10.6\text{ V}}$$

Most DMMs employ a full-wave rectification system to convert the input ac signal to one with an average value. In fact, for the DMM of Fig. 2.27, the same scale factor of Eq. (13.38) is employed; that is, the average value is scaled up by a factor of 1.11 to obtain the rms value. In digital meters, however, there are no moving parts such as in the d'Arsonval or electrodynamometer movements to display the signal level. Rather, the average value is sensed by a multiprocessor integrated circuit (IC), which in turn determines which digits should appear on the digital display.

Digital meters can also be used to measure nonsinusoidal signals, but the scale factor of each input waveform must first be known (normally provided by the manufacturer in the operator's manual). For instance, the scale factor for an average responding DMM on the ac rms scale will produce an indication for a square-wave input that is 1.11 times the peak value. For a triangular input, the response is 0.555 times the peak value. Obviously, for a sine wave input, the response is 0.707 times the peak value.

For any instrument, it is always good practice to read (if only briefly) the operator's manual if it appears that you will use the instrument on a regular basis.

For frequency measurements, the **frequency counter** of Fig. 13.67 provides a digital readout of sine, square, and triangular waves from 5 Hz to 100 MHz at input levels from 30 mV to 42 V. Note the relative simplicity of the panel and the high degree of accuracy available.

FIG. 13.67
Frequency counter. (Courtesy of Tektronix, Inc.)

The **Amp-Clamp®** of Fig. 13.68 is an instrument that can measure alternating current in the ampere range without having to open the circuit. The loop is opened by squeezing the "trigger"; then it is placed around the current-carrying conductor. Through transformer action, the level of current in rms units will appear on the appropriate scale. The accuracy of this instrument is ±3% of full scale at 60 Hz, and its scales have maximum values ranging from 6 A to 300 A. The addition of two leads, as indicated in the figure, permits its use as both a voltmeter and an ohmmeter.

FIG. 13.68
Amp-Clamp®. (Courtesy of Simpson Instruments, Inc.)

One of the most versatile and important instruments in the electronics industry is the **oscilloscope,** which has already been introduced in this chapter. It provides a display of the waveform on a cathode-ray tube to permit the detection of irregularities and the determination of quantities such as magnitude, frequency, period, dc component, and so on. The analog oscilloscope of Fig. 13.69 can display two waveforms at the same time (dual-channel) using an innovative interface (front panel). It employs menu buttons to set the vertical and horizontal scales by choosing from selections appearing on the screen. One can also store up to four measurement setups for future use.

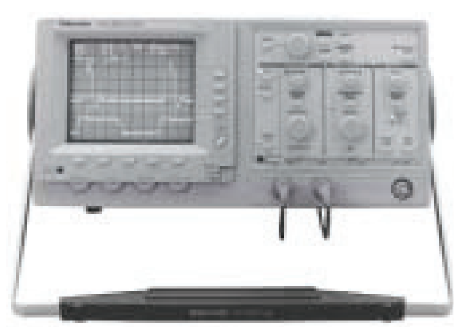


FIG. 13.69
Dual-channel oscilloscope. (Courtesy of Tektronix, Inc.)

A student accustomed to watching TV might be confused when first introduced to an oscilloscope. There is, at least initially, an assumption that the oscilloscope is generating the waveform on the screen—much like a TV broadcast. However, it is important to clearly understand that

an oscilloscope displays only those signals generated elsewhere and connected to the input terminals of the oscilloscope. The absence of an external signal will simply result in a horizontal line on the screen of the scope.

On most modern-day oscilloscopes, there is a switch or knob with the choice DC/GND/AC, as shown in Fig. 13.70(a), that is often ignored or treated too lightly in the early stages of scope utilization. The effect of each position is fundamentally as shown in Fig. 13.70(b). In the DC mode the dc and ac components of the input signal can pass directly to the display. In the AC position the dc input is blocked by the capacitor, but the ac portion of the signal can pass through to the screen. In the GND position the input signal is prevented from reaching the scope display by a direct ground connection, which reduces the scope display to a single horizontal line.

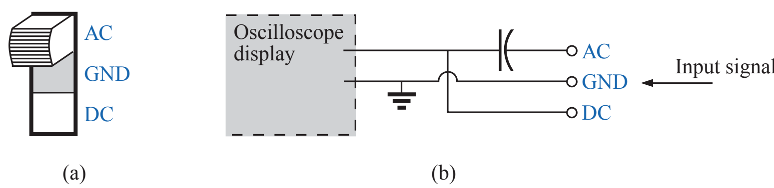


FIG. 13.70
AC-GND-DC switch for the vertical channel of an oscilloscope.

13.9 APPLICATIONS

(120 V at 60 Hz) versus (220 V at 50 Hz)

In North and South America the most common available ac supply is 120 V at 60 Hz, while in Europe and the Eastern countries it is 220 V at 50 Hz. The choices of rms value and frequency were obviously made carefully because they have such an important impact on the design and operation of so many systems.

The fact that the frequency difference is only 10 Hz reveals that there was agreement on the general frequency range that should be used for power generation and distribution. History suggests that the question of frequency selection was originally focused on that frequency that would not exhibit *flicker in the incandescent lamps* available in those days. Technically, however, there really wouldn't be a noticeable difference between 50 and 60 cycles per second based on this criterion. Another important factor in the early design stages was the effect of frequency on the size of transformers, which play a major role in power generation and distribution. Working through the fundamental equations for transformer design, you will find that *the size of a transformer is inversely proportional to frequency.* The result is that transformers operating at 50 Hz must be larger (on a purely mathematical basis about 17% larger) than those operating at 60 Hz. You will therefore find that transformers designed for the international market where they can operate on 50 Hz or 60 Hz are designed around the 50-Hz frequency. On the other side of the coin, however, higher frequencies result in increased concerns about arcing, increased losses in the transformer core due to eddy current and hysteresis losses (Chapter 19), and skin effect phenomena (Chapter 19). Somewhere in the discussion we must consider the fact that 60 Hz is an exact multiple of 60 seconds in a minute and 60 minutes in an hour. Since accurate timing is such a critical part of our technological design, was this a significant motive in the final choice? There is also the question about whether the 50 Hz is a result of the close affinity of this value to the metric system. Keep in mind that powers of 10 are all powerful in the metric system, with 100 cm in a meter, 100°C the boiling point of water, and so on. Note that 50 Hz is exactly half of this special number. All in all, it would seem that both sides have an argument that would be worth defending. However, in the final analysis, we must also wonder whether the difference is simply political in nature.

The difference in voltage between North America and Europe is a different matter entirely in the sense that the difference is close to 100%. Again, however, there are valid arguments for both sides. There is no question that larger voltages such as 220 V *raise safety issues* beyond those raised by voltages of 120 V. However, when higher voltages are supplied, there is less current in the wire for the same power demand, permitting the use of smaller conductors—a real money saver. In addition, motors, compressors, and so on, found in common home appliances and throughout the industrial community *can be smaller in size.* Higher voltages, however, also bring back the concern about arcing effects, insulation requirements, and, due to real safety concerns, higher installation costs. In general, however, international travelers are prepared for most situations if they have a transformer that can convert from their home level to that of the country they plan to visit. Most equipment (not clocks, of course) can run quite well on 50 Hz or 60 Hz for most travel periods. For any unit not operating at its design frequency, it will simply have to “work a little harder” to perform the given task. The major problem for the traveler is not the transformer itself but the wide variety of plugs used from one country to another. Each country has its own design for the “female” plug in the wall. For the three-week tour, this could mean as many as 6 to 10 different plugs of the type shown in Fig. 13.71. For a 120-V, 60-Hz supply, the plug is quite standard in appearance with its two spade leads (and possible ground connection).

In any event, both the 120 V at 60 Hz and the 220 V at 50 Hz are obviously meeting the needs of the consumer. It is a debate that could go on at length without an ultimate victor.

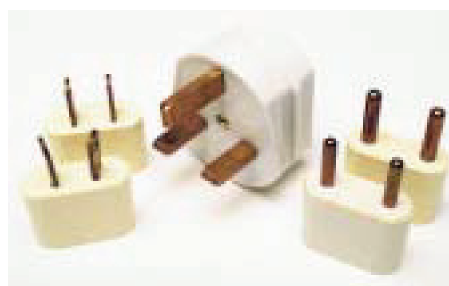


FIG. 13.71
Variety of plugs for a 220-V, 50-Hz connection.

Safety Concerns (High Voltages and dc versus ac)

Be aware that any “live” network should be treated with a calculated level of respect. Electricity in its various forms is not to be feared but should be employed with some awareness of its potentially dangerous side effects. It is common knowledge that electricity and water do not mix (never use extension cords or plug in TVs or radios in the bathroom) because a full 120 V in a layer of water of any height (from a shallow puddle to a full bath) can be *lethal.* However, other effects of dc and ac voltages are less known. In general, as the voltage and current increase, your concern about safety should increase exponentially.

For instance, under dry conditions, most human beings can survive a 120-V ac shock such as obtained when changing a light bulb, turning on a switch, and so on. Most electricians have experienced such a jolt many times in their careers. However, ask an electrician to relate how it feels to hit 220 V, and the response (if he or she has been unfortunate to have had such an experience) will be totally different. How often have you heard of a back-hoe operator hitting a 220-V line and having a fatal heart attack? Remember, the operator is sitting in a metal container on a damp ground which provides an excellent path for the resulting current to flow from the line to ground. If only for a short period of time, with the best environment (rubber-sole shoes, etc.), in a situation where you can quickly escape the situation, most human beings can also survive a 220-V shock. However, as mentioned above, it is one you will not quickly forget. For voltages beyond 220 V rms, the chances of survival go down exponentially with increase in voltage. It takes only about 10 mA of steady current through the heart to put it in defibrillation. In general, therefore, always be sure that the power is disconnected when working on the repair of electrical equipment. Don't assume that throwing a wall switch will disconnect the power. Throw the main circuit breaker and test the lines with a voltmeter before working on the system. Since voltage is a two-point phenomenon, don't be a hero and work with one line at at time—accidents happen!

You should also be aware that the reaction to dc voltages is quite different from that to ac voltages. You have probably seen in movies or comic strips that people are often unable to let go of a *hot* wire. This is evidence of the most important difference between the two types of voltages. As mentioned above, if you happen to touch a "hot" 120-V ac line, you will probably get a good sting, but *you can let go.* If it happens to be a "hot" 120-V dc line, you will probably not be able to let go, and a fatality could occur. Time plays an important role when this happens, because the longer you are subjected to the dc voltage, the more the resistance in the body decreases until a fatal current can be established. The reason that we can let go of an ac line is best demonstrated by carefully examining the 120-V rms, 60-Hz voltage in Fig. 13.72. Since the voltage is oscillating, there is a period of time when the voltage is near zero or less than, say, 20 V, and is reversing in direction. Although this time interval is very short, it appears every 8.3 ms and provides a window to *let go.*

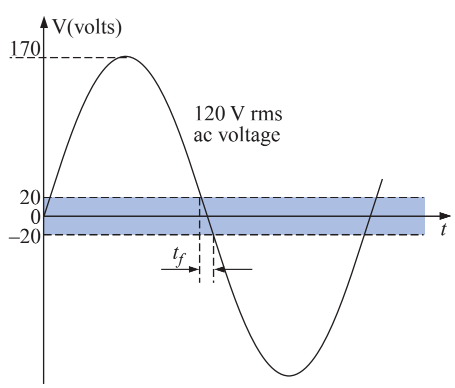


FIG 13.72
Interval of time when sinusoidal voltage is near zero volts.

Now that we are aware of the additional dangers of dc voltages, it is important to mention that under the wrong conditions, dc voltages as low as 12 V such as from a car battery can be quite dangerous. If you happen to be working on a car under wet conditions, or if you are sweating badly for some reason or, worse yet, wearing a wedding ring that may have moisture and body salt underneath, touching the positive terminal may initiate the process whereby the body resistance begins to drop and serious injury could take place. It is one of the reasons you seldom see a professional electrician wearing any rings or jewelry—it is just not worth the risk.

Before leaving this topic of safety concerns, you should also be aware of the dangers of high-frequency supplies. We are all aware of what 2.45 GHz at 120 V can do to a meat product in a microwave oven. As discussed in Chapter 5, it is therefore very important that the seal around the oven be as tight as possible. However, don't ever assume that anything is absolutely perfect in design—so don't make it a habit to view the cooking process in the microwave 6 in. from the door on a continuing basis. Find something else to do, and check the food only when the cooking process is complete. If you ever visit the Empire State Building, you will notice that you are unable to get close to the antenna on the dome due to the high-frequency signals being emitted with a great deal of power. Also note the large KEEP OUT signs near radio transmission towers for local radio stations. Standing within 10 ft of an AM transmitter working at 540 kHz would bring on disaster. Simply holding (not to be tried!) a fluorescent bulb near the tower could make it light up due to the excitation of the molecules inside the bulb.

In total, therefore, treat any situation with high ac voltages or currents, high-energy dc levels, and high frequencies with added care.

Bulb Savers

Ever since the invention of the light bulb, consumers have clamored for ways to extend the life of a bulb. I can remember the days when I was taught to always turn a light off when leaving a room and not to play with a light switch because it cost us a penny (at a time when a penny had some real value) every time I turned the switch on and off. Through advanced design we now have bulbs that are guaranteed to last a number of years. They cost more, but there is no need to replace the bulb as often, and over time there is a financial savings. For some of us it is simply a matter of having to pay so much for a single bulb.

For interest sake, I measured the cold dc resistance of a standard 60-W bulb and found it to be about 14 Ω. Forgetting any inductive effects due to the filament and wire, this would mean a current of 120 V/14 Ω = 8.6 A when the light is first turned on. This is a fairly heavy current for the filament to absorb when you consider that the normal operating current is 60 W/120 V = 0.5 A. Fortunately, it lasts for only a few milliseconds, as shown in Fig. 13.73(a), before the bulb heats up, causing the filament resistance to quickly increase and cut the current down to reasonable levels. However, over time, hitting the bulb with 8.6 A every time you turn the switch on will take its toll on the filament, and eventually the filament will simply surrender its natural characteristics and open up. You can easily tell if a bulb is bad by simply shaking it and listening for the clinking sound of the broken filament hitting the bulb. Assuming an initial current of 8.6 A for a single bulb, if the light switch controlled four 60-W bulbs in the same room, the surge current through the switch could be as high as 4(8.6 A) = 34.4 A as shown in Fig. 13.73(b), which probably exceeds the rating of the breaker (typically 20 A) for the circuit. However, the saving grace is that it lasts for only a few milliseconds, and circuit breakers are not designed to react that quickly. Even the GFI safety breakers in the bathroom are typically rated at a 5-ms response time. However, when you look at the big picture and imagine all these spikes on the line generated throughout a residential community, it is certainly a problem that the power company has to deal with on a continuing basis.

One way to suppress this surge current is to place an inductor in series with the bulb to choke out the spikes down the line. This method, in fact, leads to one way of extending the life of a light bulb through the use of dimmers. Any well-designed dimmer (such as the one described in Chapter 12) has an inductor in the line to suppress current surges. The results are both an extended life for the bulb and the ability to control the power level. Left on in the full voltage position, the switch could be used as a regular switch and the life of the bulb could be

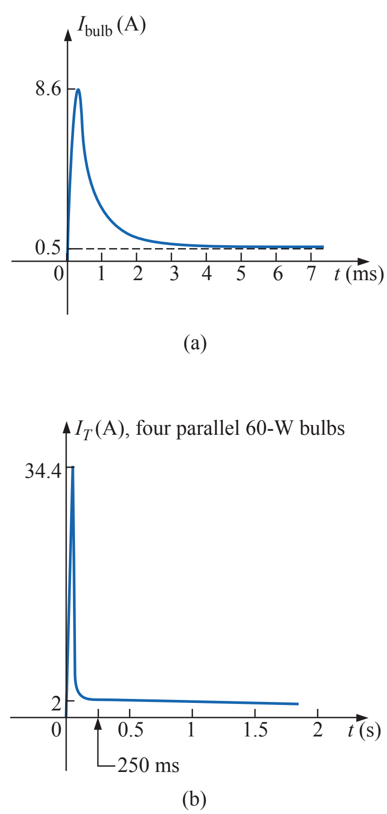


FIG 13.73
Surge currents: (a) single 60-W bulb; (b) four parallel 60-W bulbs.

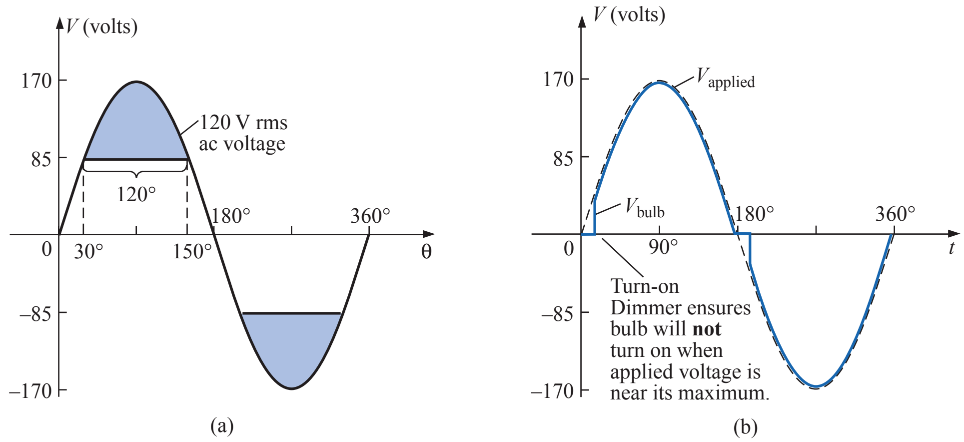


FIG. 13.74

Turn-on voltage: (a) equal to or greater than one-half the peak value; (b) when a dimmer is used.

extended. In fact, many dimmers now use triacs designed to turn on only when the applied voltage passes through zero. If we look at the full sine wave of Fig. 13.74(a), we find that the voltage is at least half of its maximum value of 85 V for a full two-thirds of each cycle, or about 67% of the time. The chances, therefore, of your turning on a light bulb with at least 85 V on the line is far better than 2 to 1, so you can expect the current for a 60-W light bulb to be at least 85 V/14 Ω = 6 A 67% of the time, which exceeds the rated 0.5-A rated value by 1100%. If we use a dimmer with a triac designed to turn on only when the applied voltage passes through zero or shortly thereafter, as shown in Fig.

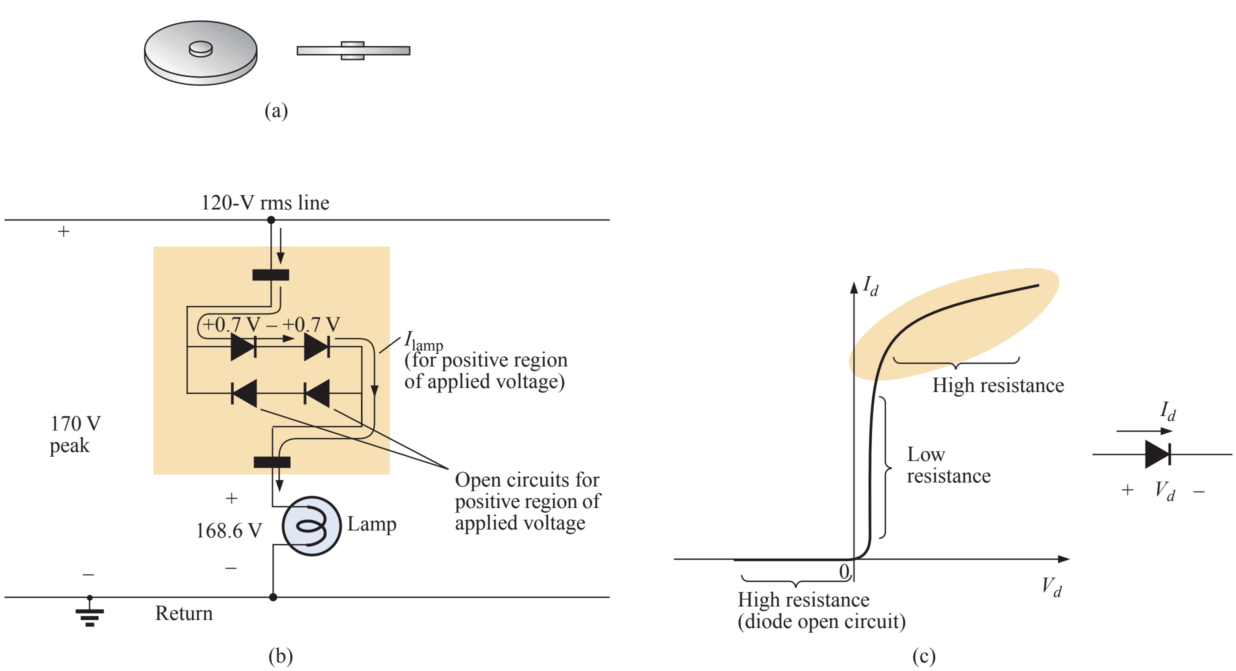


FIG. 13.75

Bulb saver: (a) external appearance; (b) basic operation; (c) diode characteristics at high current levels.

13.74(b), the applied voltage will increase from about zero volts, giving the bulb time to warm up before the full voltage is applied.

Another commercial offering to extend the life of light bulbs is the smaller circular disc shown in Fig. 13.75(a) which is inserted between the bulb and the holder. Contacts are provided on both sides to permit conduction through the simple diode network shown in Fig. 13.75(b). You may recall from an earlier chapter that the voltage across diodes in the on state is 0.7 V as shown for each diode in Fig. 13.75(b) for the positive portion of the input voltage. The result is that the voltage to the bulb is reduced by about 1.4 V throughout the cycle, reducing the power delivered to the bulb. For most situations the reduced lighting is not a problem, and the bulb will last longer simply because it is not pressed to work at full output. However, the real saving in the device is the manner in which it could help suppress the surge currents through the light bulb. The true characteristics of a diode are shown in Fig. 13.75(c) for the full range of currents through the diode. For most applications in electronic circuits, the vertical region is employed. For excessive currents the diode characteristics flatten out as shown. This region is characterized as having a large resistance (compared to very small resistance of the vertical region) which will come into play when the bulb is first turned on. In other words, when the bulb is first turned on, the current will be so high that the diode will enter its high-resistance region and by Ohm's law will limit the surge current—thereby extending the life of the bulb. The two diodes facing the other way are for the negative portion of the supply voltage.

New methods of extending the life of bulbs hit the marketplace every day. All in all, however, one guaranteed way to extend the life of your bulbs is to return to the old philosophy of turning lights off when you leave a room, and "Don't play with the light switch!"

13.10 COMPUTER ANALYSIS

PSpice

OrCAD Capture offers a variety of ac voltage and current sources. However, for the purposes of this text, the voltage source **VSIN** and the current source **ISIN** are the most appropriate because they have a list of attributes that will cover current areas of interest. Under the library **SOURCE,** a number of others are listed, but they don't have the full range of the above, or they are dedicated to only one type of analysis. On occasion, **ISRC** will be used because it has an arrow symbol like that appearing in the text, and it can be used for dc, ac, and some transient analyses. The symbol for **ISIN** is simply a sine wave which utilizes the plus-and-minus sign to indicate direction. The sources **VAC, IAC, VSRC,** and **ISRC** are fine if the magnitude and the phase of a specific quantity are desired or if a transient plot against frequency is desired. However, they will not provide a transient response against time even if the frequency and the transient information are provided for the simulation.

For all of the sinusoidal sources, the magnitude (**VAMPL**) is the peak value of the waveform and not the rms value. This will become clear when a plot of a quantity is desired and the magnitude calculated by PSpice is the peak value of the transient response. However, for a purely steady-state ac response, the magnitude provided can be the rms value, and the output read as the rms value. Only when a plot is desired will it be clear that PSpice is accepting every ac magnitude as the peak value of the waveform. Of course, the phase angle is the same whether the magnitude is the peak or the rms value.

Before examining the mechanics of getting the various sources, remember that

***Transient Analysis** provides an ac or a dc output versus time, while **AC Sweep** is used to obtain a plot versus frequency.*

To obtain any of the sources listed above, apply the following sequence: **Place part** key-**Place Part** dialog box-**Source**-(enter type of source). Once selected the ac source **VSIN** will appear on the schematic with **VOFF, VAMPL,** and **FREQ.** Always specify **VOFF** as 0 V (unless a specific value is part of the analysis), and provide a value for the amplitude and frequency. The remaining quantities of **PHASE, AC, DC, DF,** and **TD** can be entered by double-clicking on the source symbol to obtain the **Property Editor,** although **PHASE, DF** (damping factor), and **TD** (time delay) do have a default of 0 s. To add a phase angle, simply click on **PHASE,** enter the phase angle in the box below, and then select **Apply.** If you want to display a factor such as a phase angle of 60°, simply click on **PHASE** followed by **Display** to obtain the **Display Properties** dialog box. Then choose **Name and Value** followed by **OK** and **Apply,** and leave the **Properties Editor** dialog box (**X**) to see **PHASE=60** next to the **VSIN** source. The next chapter will include the use of the ac source in a simple circuit.

Electronics Workbench

For EWB, the ac voltage source is available from two sources—the **Sources** parts bin and the **Function Generator.** The major difference between the two is that the phase angle can be set when using the **Sources** parts bin, whereas it cannot be set using the **Function Generator.**

Under **Sources,** the ac voltage source is the fourth option down on the left column of the toolbar. When selected and placed, it will display the default values for the amplitude, frequency, and phase. All the parameters of the source can be changed by double-clicking on the source symbol to obtain the **AC Voltage** dialog box. The **Voltage Amplitude** and **Voltage RMS** are interlinked so that when you change one, the other will change accordingly. For the **1V** default value, the rms value is automatically listed as **0.71** (not 0.7071 because of the hundredths-place accuracy). Note that the unit of measurement is controlled by the scrolls to the right of the default label and cannot be set by typing in the desired unit of measurement. The label can be changed by simply switching the **Label** heading and inserting the desired label. After all the changes have been made in the **AC Voltage** dialog box, click **OK,** and all the changes will appear next to the ac voltage source symbol. In Fig. 13.76 the label was changed to **Vs** and the amplitude to 10 V while the frequency and phase angle were left with their default values. It is particularly important to realize that

*for any frequency analysis (that is, where the frequency will change), the **AC Magnitude** of the ac source must be set under **Analysis Setup** in the **AC Voltage** dialog box. Failure to do so will create results linked to the default values rather than the value set under the **Value** heading.*

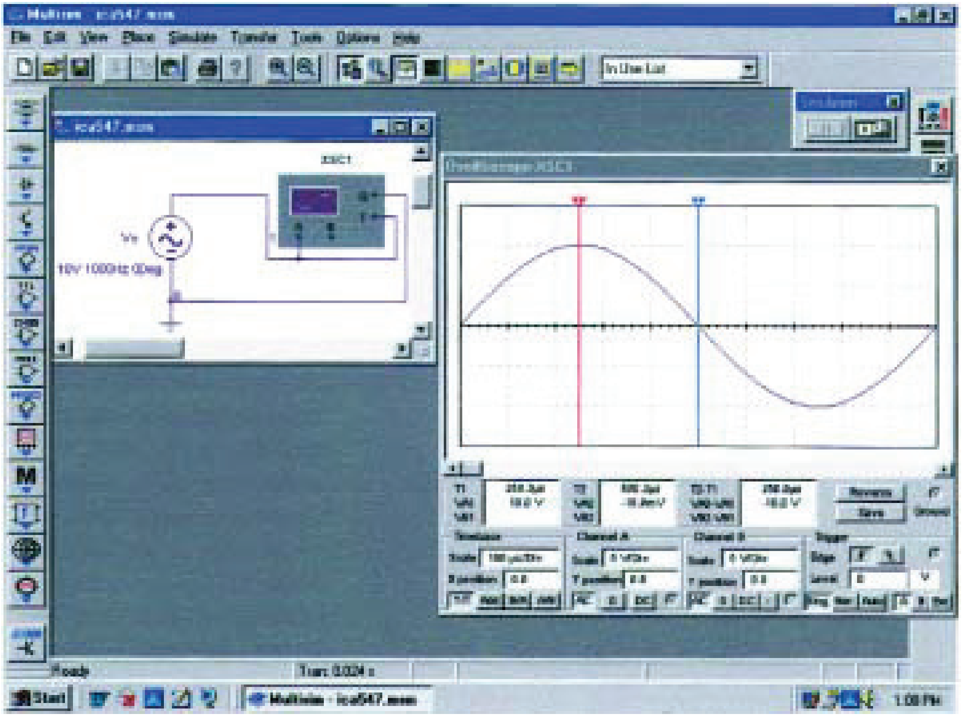


FIG. 13.76
*Using the oscilloscope to display the sinusoidal ac voltage source available in the Electronics Workbench **Sources** tool bin.*

To view the sinusoidal voltage set in Fig. 13.76, an oscilloscope can be selected from the **Instrument** toolbar at the right of the screen. It is the fourth option down and has the appearance shown in Fig. 13.76 when selected. Note that it is a dual-channel oscilloscope with an **A** channel and a **B** channel. It has a ground (**G**) connection and a trigger (**T**) connection. The connections for viewing the ac voltage source on the **A** channel are provided in Fig. 13.76. Note that the trigger control is also connected to the **A** channel for sync control. The screen appearing in Fig. 13.76 can be displayed by double-clicking on the oscilloscope symbol on the screen. It has all the major controls of a typical laboratory oscilloscope. When you select **Simulate-Run** or select **1** on the **Simulate Switch,** the ac voltage will appear on the screen. Changing the **Time base** to 100 μs/div. will result in the display of Fig. 13.76 since there are 10 divisions across the screen and $10(100\mu s) = 1$ ms (the period of the applied signal). Changes in the **Time base** are made by simply clicking on the default value to obtain the scrolls in the same box. It is important to remember, however, that

changes in the oscilloscope setting or any network should not be made until the simulation is ended by disabling the* Simulate-Run *option or placing the* Simulate *switch in the 0 mode.

The options within the time base are set by the scroll bars and cannot be changed—again they match those typically available on a laboratory oscilloscope. The vertical sensitivity of the **A** channel was automatically set by the program at 5 V/div. to result in two vertical boxes for the peak value as shown in Fig. 13.76. Note the **AC** and **DC** key pads below Channel **A.** Since there is no dc component in the applied signal, either one will result in the same display. The **Trigger** control is set on the positive transition at a level of 0 V. The **T1** and **T2** refer to the cursor positions on the horizontal time axis. By simply clicking on the small red triangle at the top of the red line at the far left edge of the screen and dragging the triangle, you can move the vertical red line to any position along the axis. In Fig.13.76 it was moved to the peak value of the waveform at ¼ of the total period or 0.25 ms = 250 μs. Note the value of **T1** (250.3 μs) and the corresponding value of **VA1** (10.0V). By moving the other cursor with a blue triangle at the top to ½ the total period or 0.5 ms = 500 μs, we find that the value at **T2** (500.3 μs) is −18.9 mV (**VA2**), which is approximately 0 V for a waveform with a peak value of 10 V. The accuracy is controlled by the number of data points called for in the simulation setup. The more data points, the higher the likelihood of a higher degree of accuracy for the desired quantity. However, an increased number of data points will also extend the running time of the simulation. The third display box to the right gives the difference between **T2** and **T1** as 250 μs and difference between their magnitudes (**VA2-VA1**) as −10 V, with the negative sign appearing because **VA1** is greater than **VA2.**

As mentioned above, an ac voltage can also be obtained from the **Function Generator** appearing as the second option down on the **Instrument** toolbar. Its symbol appears in Fig. 13.77 with positive, negative, and ground connections. Double-click on the generator graphic symbol, and the **Function Generator-XFG1** dialog box will appear in which selections can be made. For this example, the sinusoidal waveform is chosen. The **Frequency** is set at 1 kHz, the **Amplitude** is set at 10 V, and the **Offset** is left at 0 V. Note that there is no option to set the phase angle as was possible for the source above. Double-clicking on the

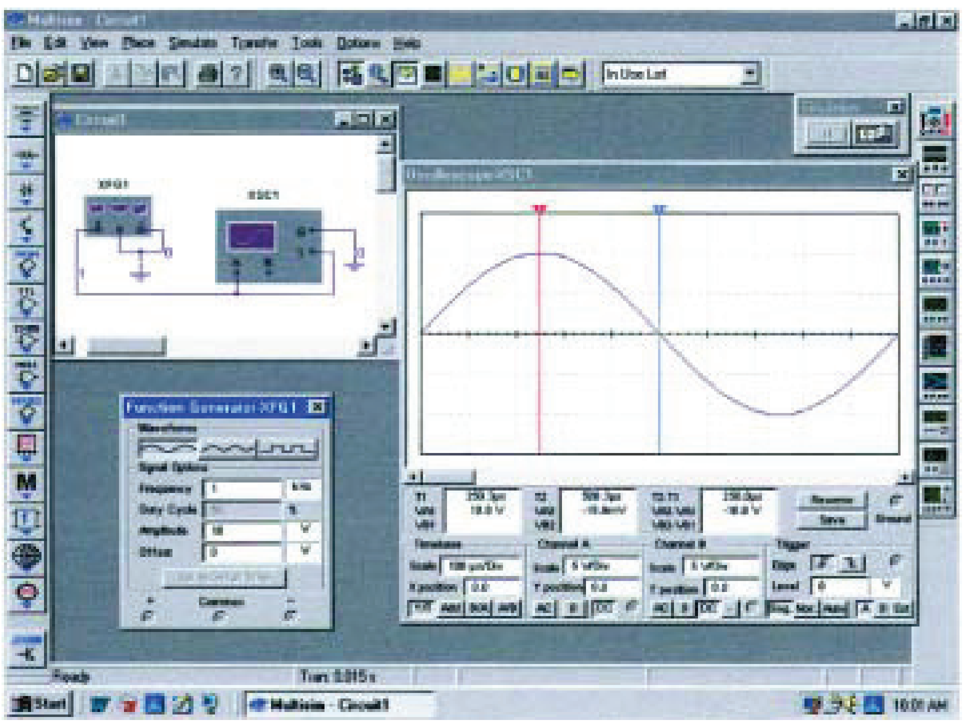


FIG. 13.77

Using the function generator to place a sinusoidal ac voltage waveform on the screen of the oscilloscope.

oscilloscope will generate the oscilloscope on which a **Timebase** of 100 μs/div. can be set again with a vertical sensitivity of 5 V/div. Select **1** on the **Simulate** switch, and the waveform of Fig. 13.77 will appear. Choosing **Singular** under **Trigger** will result in a fixed display; then set the **Simulate** switch on **0** to the end the simulation. Placing the cursors in the same position shows that the waveforms for Figs. 13.76 and 13.77 are the same.

For most of the EWB analyses to appear in this text, the **AC_VOLTAGE_SOURCE** under **Sources** will be employed. However, with such a limited introduction to EWB, it seemed appropriate to introduce the use of the **Function Generator** because of its close linkage to the laboratory experience.

C++

Calculating the Average Value of a Waveform The absence of any network configurations to analyze in this chapter severely limits the content with respect to packaged computer programs. However, the door is still wide open for the application of a language to write programs that can be helpful in the application of some of the concepts introduced in the chapter. In particular, let us examine the C++ program of Fig. 13.78, designed to calculate the average value of a pulse waveform having up to 5 different levels.

The program begins with a heading and preprocessor directive. Recall that the *iostream.h* header file sets up the input-output path

```
Heading                  //C++ Average Waveform Voltage Calculation

Preprocessor directive   #include <iostream.h>         //needed for input/output

Body of program          main()
                         {
Define form and name             float Vave;            //average value of waveform
of variables                     float Vlevel;          //voltage level during time Tlevel
                                 float VTsum = 0;       //used for adding voltage-time products
                                 float T = 0;           //total waveform time
                                 float Tlevel;          //time duration of Vlevel
                                 int levels;            //the number of levels in the waveform
                                 int count;             //loop counter

Obtain # of levels               cout << "How many levels do you wish to enter (1..5) ? ";
                                 cin >> levels;         //get number of levels from user

Iterative for statement          for(count = 1; count <= levels; count++)    //begin loop
                                 {
                                         cout << "\n";
                                         cout << "Enter voltage level " << count << ": ";
                                         cin >> Vlevel;     //get voltage from user
                                         cout << "Enter time for level " << count << ": ";
                                         cin >> Tlevel;     //get time from user
                                         VTsum += Vlevel * Tlevel;     //add product to VTsum
                                         T += Tlevel;       //add Tlevel to total waveform time
                                 }
Calculate Vave                   Vave = VTsum / T;          //calculate average value
                                 cout << "\n";
Display results                  cout << "The average value of the waveform is ";
                                 cout << Vave << " volts.\n";
                         }
```

FIG. 13.78

C++ program designed to calculate the average value of a waveform with up to five positive or negative pulses.

between the program and the disk operating system. Note that the *main* ( ) part of the program extends all the way down to the bottom, as identified by the braces { }. Within this region all the calculations will be performed, and the results will be displayed.

Within the *main* ( ) part of the program, all the variables to be employed in the calculations are defined as floating point (decimal values) or integer (whole numbers). The comments on the right identify each variable. This is followed by a display of the question about how many levels will be encountered in the waveform using *cout* (comment out). The *cin* (comment in) statement permits a response from the user.

Next, the loop statement *for* is employed to establish a fixed number of repetitions of the sequence appearing within the parentheses ( ) for a number of loops defined by the variable *levels.* The format of this *for* statement is such that the first entry within the parentheses ( ) is the initial value of the variable *count* (1 in this case), followed by a semicolon and then a test expression determining how many times the sequence to follow will be repeated. In other words, if *levels* is 5, then the first pass through the *for* statement will result in 1 being compared to 5, and the test expression will be satisfied because 5 is greater than or equal to 1 (< =). On the next pass, *count* will be increased to 2, and the same test will be performed. Eventually *count* will equal 5, the test expression will not be satisfied, and the program will move to its next statement, which is $Vave = VT\ sum/T$. The last entry *count++* of the *for* statement simply increments the variable *count* after each iteration. The first line within the *for* statement calls for a line to be skipped, followed by a question on the display about the level of voltage for the first time interval. The question will include the current state of the *count* variable followed by a colon. In C++ all character outputs must be displayed in quotes (not required for numerical values). However, note the absence of the quotes for *count* since it will be a numerical value. Next the user enters the first voltage level through *cin,* followed by a request for the time interval. In this case units are not provided but simply measured as an increment of the whole; that is, if the total period is 5 μs and the first interval is 2 μs, then just a 2 is entered.

The area under the pulse is then calculated to establish the variable *VTsum,* which was initially set at 0. On the next pass the value of *VTsum* will be the value obtained by the first run plus the new area. In other words, *VTsum* is a storage for the total accumulated area. Similarly, T is the accumulated sum of the time intervals.

Following a FALSE response from the test expression of the *for* statement, the program will move to calculate the average value of the waveform using the accumulated values of the area and time. A line is then skipped; and the average value is displayed with the remaining *cout* statements. Brackets have been added along the edge of the program to help identify the various components of the program.

A program is now available that can find the average value of any pulse waveform having up to five positive or negative pulses. It can be placed in storage and simply called for when needed. Operations such as the above are not available in either form of PSpice or in any commercially available software package. It took the knowledge of a language and a few minutes of time to generate a short program of lifetime value.

Two runs will clearly reveal what will be displayed and how the output will appear. The waveform of Fig. 13.79 has five levels, entered as shown in the output file of Fig. 13.80. As indicated the average value is 1.6 V. The waveform of Fig. 13.81 has only three pulses, and the time

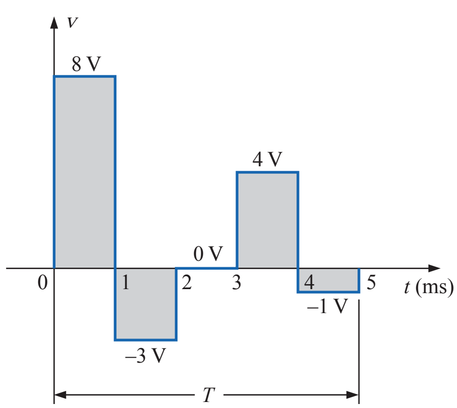


FIG. 13.79
Waveform with five pulses to be analyzed by the C++ program of Fig. 13.78.

```
How many levels do you wish to enter (1..5) ? 5

Enter voltage level 1: 8
Enter time for level 1: 1

Enter voltage level 2: -3
Enter time for level 2: 1

Enter voltage level 3: 0
Enter time for level 3: 1

Enter voltage level 4: 4
Enter time for level 4: 1

Enter voltage level 5: -1
Enter time for level 5: 1

The average value of the waveform is 1.6 volts.
```

FIG. 13.80
Output results for the waveform of Fig. 13.79.

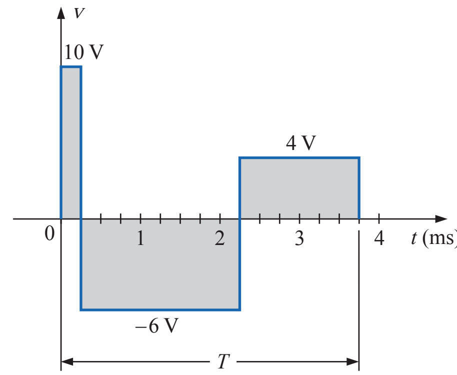


FIG. 13.81
Waveform with three pulses to be analyzed by the C++ program of Fig. 13.78.

```
How many levels do you wish to enter (1..5) ? 3

Enter voltage level 1: 10
Enter time for level 1: .25

Enter voltage level 2: -6
Enter time for level 2: 2

Enter voltage level 3: 4
Enter time for level 3: 1.5

The average value of the waveform is -0.933333 volts.
```

FIG. 13.82
Output results for the waveform of Fig. 13.81.

interval for each is different. Note the manner in which the time intervals were entered. Each is entered as a multiplier of the standard unit of measure for the horizontal axis. The variable *levels* will be only 3, requiring only three iterations of the *for* statement. The result is a negative value of −0.933 V, as shown in the output file of Fig. 13.82.

PROBLEMS

SECTION 13.2 Sinusoidal ac Voltage Characteristics and Definitions

1. For the periodic waveform of Fig. 13.83:
   a. Find the period *T*.
   b. How many cycles are shown?
   c. What is the frequency?
   ***d.** Determine the positive amplitude and peak-to-peak value (think!).

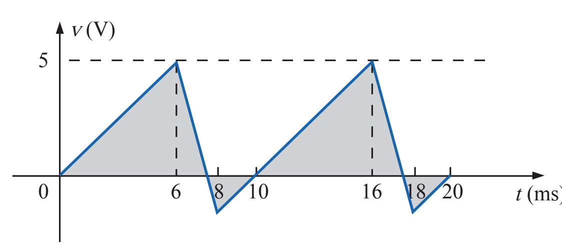


FIG. 13.83
Problem 1.

2. Repeat Problem 1 for the periodic waveform of Fig. 13.84.

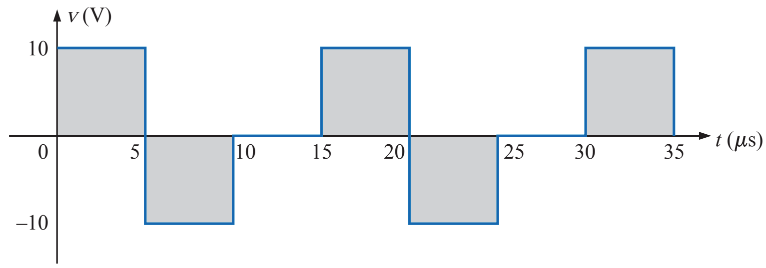


FIG. 13.84
Problems 2, 9, and 47.

3. Determine the period and frequency of the sawtooth waveform of Fig. 13.85.

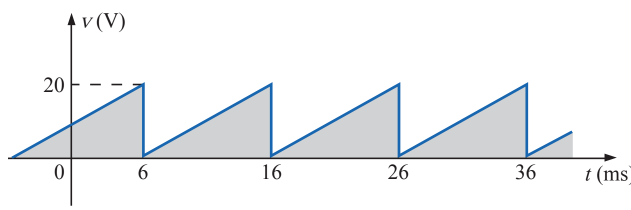
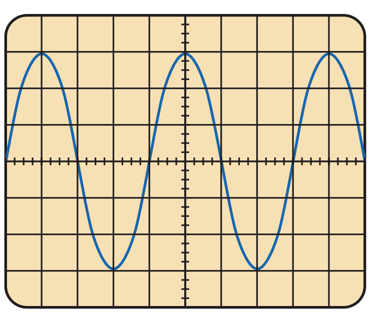


FIG. 13.85
Problems 3 and 48.

4. Find the period of a periodic waveform whose frequency is
   a. 25 Hz. **b.** 35 MHz.
   c. 55 kHz. **d.** 1 Hz.
5. Find the frequency of a repeating waveform whose period is
   a. 1/60 s. **b.** 0.01 s.
   c. 34 ms. **d.** 25 μs.
6. Find the period of a sinusoidal waveform that completes 80 cycles in 24 ms.
7. If a periodic waveform has a frequency of 20 Hz, how long (in seconds) will it take to complete five cycles?
8. What is the frequency of a periodic waveform that completes 42 cycles in 6 s?
9. Sketch a periodic square wave like that appearing in Fig. 13.84 with a frequency of 20,000 Hz and a peak value of 10 mV.
10. For the oscilloscope pattern of Fig. 13.86:
   a. Determine the peak amplitude.
   b. Find the period.
   c. Calculate the frequency.
   Redraw the oscilloscope pattern if a +25-mV dc level were added to the input waveform.



Vertical sensitivity = 50 mV/div.
Horizontal sensitivity = 10 μs/div.

FIG. 13.86
Problem 10.

SECTION 13.3 The Sine Wave

11. Convert the following degrees to radians:

a. 45° **b.** 60°
c. 120° **d.** 270°
e. 178° **f.** 221°

12. Convert the following radians to degrees:

a. $\pi/4$ **b.** $\pi/6$
c. $\frac{1}{10}\pi$ **d.** $\frac{7}{6}\pi$
e. 3π **f.** 0.55π

13. Find the angular velocity of a waveform with a period of

a. 2 s. **b.** 0.3 ms.
c. 4 μs. **d.** $\frac{1}{26}$ s.

14. Find the angular velocity of a waveform with a frequency of

a. 50 Hz. **b.** 600 Hz.
c. 2 kHz. **d.** 0.004 MHz.

15. Find the frequency and period of sine waves having an angular velocity of

a. 754 rad/s. **b.** 8.4 rad/s.
c. 6000 rad/s. **d.** $\frac{1}{16}$ rad/s.

16. Given $f = 60$ Hz, determine how long it will take the sinusoidal waveform to pass through an angle of 45°.

17. If a sinusoidal waveform passes through an angle of 30° in 5 ms, determine the angular velocity of the waveform.

SECTION 13.4 General Format for the Sinusoidal Voltage or Current

18. Find the amplitude and frequency of the following waves:

a. $20 \sin 377t$ **b.** $5 \sin 754t$
c. $10^6 \sin 10{,}000t$ **d.** $0.001 \sin 942t$
e. $-7.6 \sin 43.6t$ **f.** $(\frac{1}{42}) \sin 6.283t$

19. Sketch $5 \sin 754t$ with the abscissa

a. angle in degrees.
b. angle in radians.
c. time in seconds.

20. Sketch $10^6 \sin 10{,}000t$ with the abscissa

a. angle in degrees.
b. angle in radians.
c. time in seconds.

21. Sketch $-7.6 \sin 43.6\text{t}$ with the abscissa

a. angle in degrees.
b. angle in radians.
c. time in seconds.

22. If $e = 300 \sin 157t$, how long (in seconds) does it take this waveform to complete 1/2 cycle?

23. Given $i = 0.5 \sin \alpha$, determine i at $\alpha = 72°$.

24. Given $v = 20 \sin \alpha$, determine v at $\alpha = 1.2\pi$.

***25.** Given $v = 30 \times 10^{-3} \sin \alpha$, determine the angles at which v will be 6 mV.

***26.** If $v = 40$ V at $\alpha = 30°$ and $t = 1$ ms, determine the mathematical expression for the sinusoidal voltage.

SECTION 13.5 Phase Relations

27. Sketch $\sin(377t + 60°)$ with the abscissa
  a. angle in degrees.
  b. angle in radians.
  c. time in seconds.

28. Sketch the following waveforms:
  a. $50 \sin(\omega t + 0°)$ **b.** $-20 \sin(\omega t + 2°)$
  c. $5 \sin(\omega t + 60°)$ **d.** $4 \cos \omega t$
  e. $2 \cos(\omega t + 10°)$ **f.** $-5 \cos(\omega t + 20°)$

29. Find the phase relationship between the waveforms of each set:
  a. $v = 4 \sin(\omega t + 50°)$
  $i = 6 \sin(\omega t + 40°)$
  b. $v = 25 \sin(\omega t - 80°)$
  $i = 5 \times 10^{-3} \sin(\omega t - 10°)$
  c. $v = 0.2 \sin(\omega t - 60°)$
  $i = 0.1 \sin(\omega t + 20°)$
  d. $v = 200 \sin(\omega t - 210°)$
  $i = 25 \sin(\omega t - 60°)$

***30.** Repeat Problem 29 for the following sets:
  a. $v = 2 \cos(\omega t - 30°)$
  $i = 5 \sin(\omega t + 60°)$
  b. $v = -1 \sin(\omega t + 20°)$
  $i = 10 \sin(\omega t - 70°)$
  c. $v = -4 \cos(\omega t + 90°)$
  $i = -2 \sin(\omega t + 10°)$

31. Write the analytical expression for the waveforms of Fig. 13.87 with the phase angle in degrees.

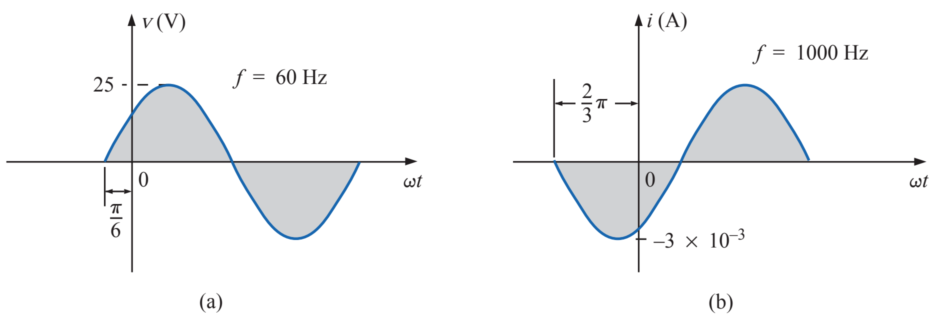


FIG. 13.87
Problem 31.

32. Repeat Problem 31 for the waveforms of Fig. 13.88.

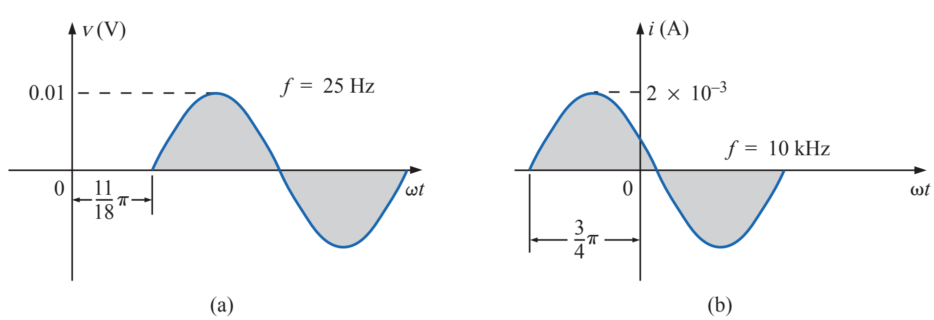


FIG. 13.88
Problem 32.

***33.** The sinusoidal voltage $v = 200 \sin(2\pi 1000t + 60°)$ is plotted in Fig. 13.89. Determine the time t_1.

***34.** The sinusoidal current $i = 4 \sin(50{,}000t - 40°)$ is plotted in Fig. 13.90. Determine the time t_1.

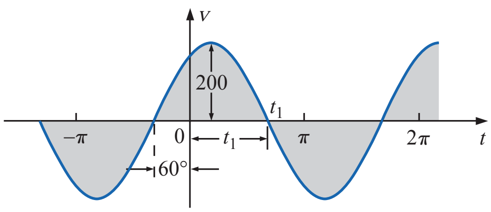


FIG. 13.89
Problem 33.

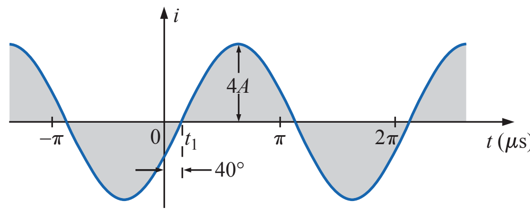
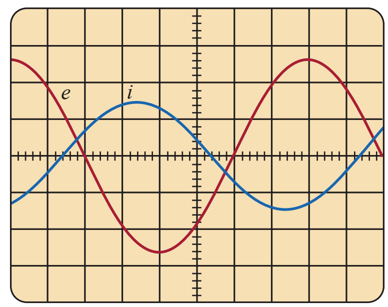


FIG. 13.90
Problem 34.

***35.** Determine the phase delay in milliseconds between the following two waveforms:

$$v = 60 \sin(1800t + 20°)$$
$$i = 1.2 \sin(1800t - 20°)$$

36. For the oscilloscope display of Fig. 13.91:
- **a.** Determine the period of each waveform.
- **b.** Determine the frequency of each waveform.
- **c.** Find the rms value of each waveform.
- **d.** Determine the phase shift between the two waveforms and which leads or lags.

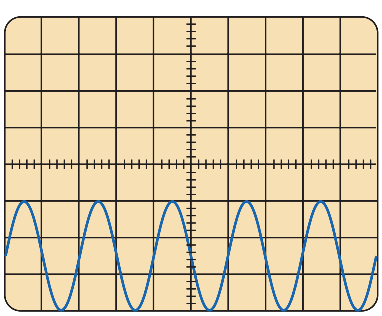


Vertical sensitivity = 0.5 V/div.
Horizontal sensitivity = 1 ms/div.

FIG. 13.91
Problem 36.

SECTION 13.6 Average Value

37. For the waveform of Fig. 13.92:
- **a.** Determine the period.
- **b.** Find the frequency.
- **c.** Determine the average value.
- **d.** Sketch the resulting oscilloscope display if the vertical channel is switched from DC to AC.



Vertical sensitivity = 10 mV/div.
Horizontal sensitivity = 0.2 ms/div.

FIG. 13.92
Problem 37.

38. Find the average value of the periodic waveforms of Fig. 13.93 over one full cycle.

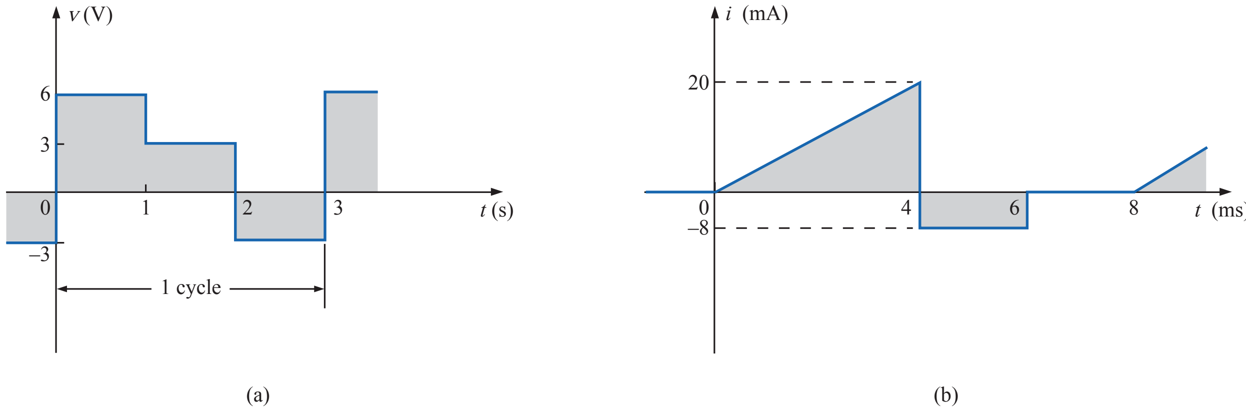


FIG. 13.93
Problem 38.

39. Find the average value of the periodic waveforms of Fig. 13.94 over one full cycle.

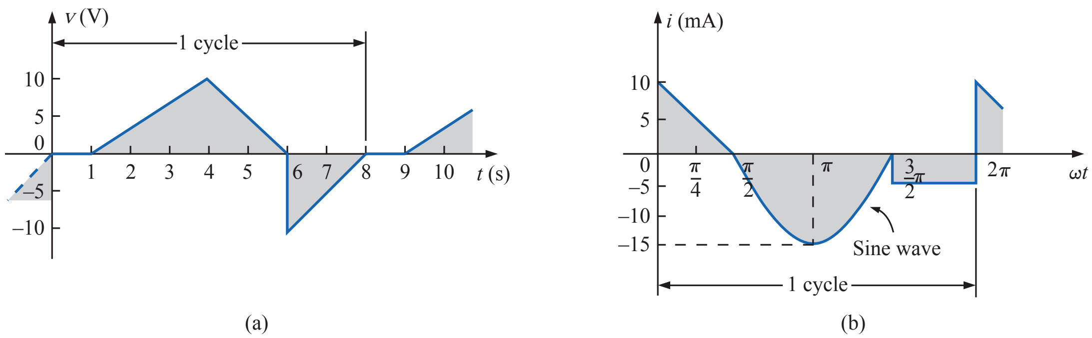


FIG. 13.94
Problem 39.

***40. a.** By the method of approximation, using familiar geometric shapes, find the area under the curve of Fig. 13.95 from zero to 10 s. Compare your solution with the actual area of 5 volt-seconds (V·s).

b. Find the average value of the waveform from zero to 10 s.

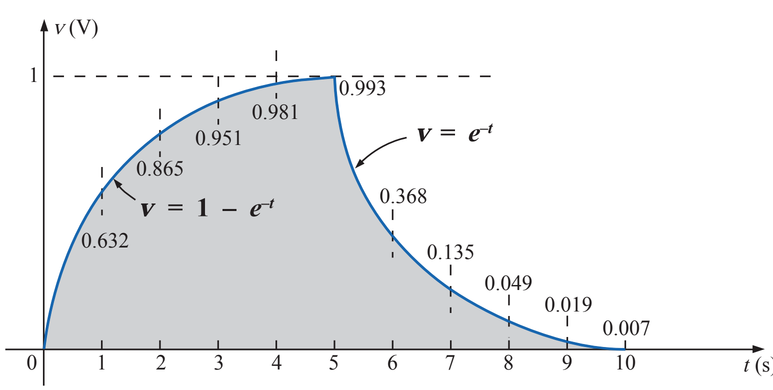
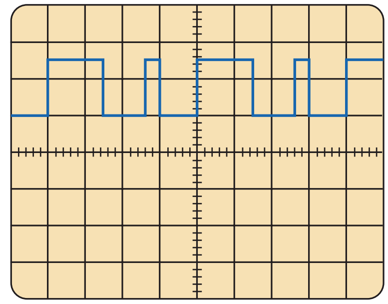


FIG. 13.95
Problem 40.

***41.** For the waveform of Fig. 13.96:
- **a.** Determine the period.
- **b.** Find the frequency.
- **c.** Determine the average value.
- **d.** Sketch the resulting oscilloscope display if the vertical channel is switched from DC to AC.



Vertical sensitivity = 10 mV/div.
Horizontal sensitivity = 10 μs/div.

FIG. 13.96
Problem 41.

SECTION 13.7 Effective (rms) Values

42. Find the rms values of the following sinusoidal waveforms:
- **a.** $v = 20 \sin 754t$
- **b.** $v = 7.07 \sin 377t$
- **c.** $i = 0.006 \sin(400t + 20°)$
- **d.** $i = 16 \times 10^{-3} \sin(377t - 10°)$

43. Write the sinusoidal expressions for voltages and currents having the following rms values at a frequency of 60 Hz with zero phase shift:
- **a.** 1.414 V   **b.** 70.7 V
- **c.** 0.06 A   **d.** 24 μA

44. Find the rms value of the periodic waveform of Fig. 13.97 over one full cycle.

45. Find the rms value of the periodic waveform of Fig. 13.98 over one full cycle.

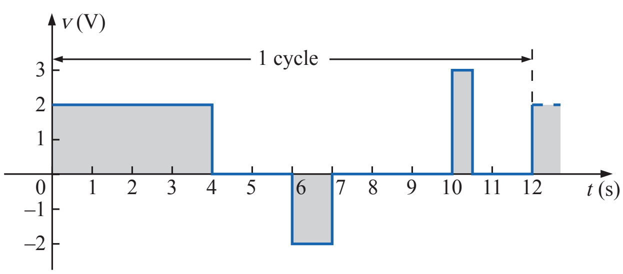


FIG. 13.97
Problem 44.

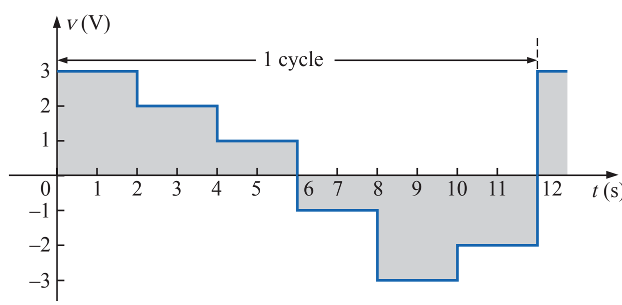


FIG. 13.98
Problem 45.

46. What are the average and rms values of the square wave of Fig. 13.99?

47. What are the average and rms values of the waveform of Fig. 13.84?

48. What is the average value of the waveform of Fig. 13.85?

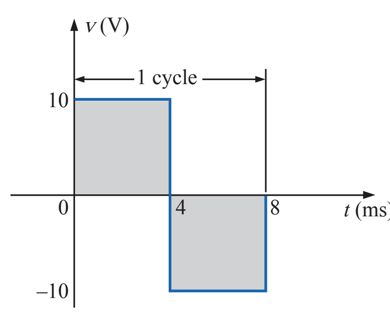
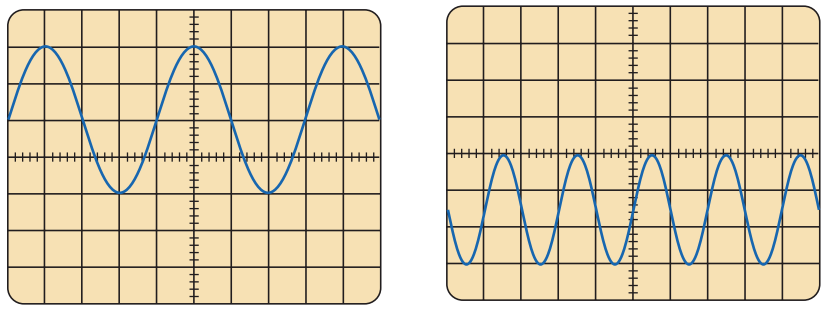


FIG. 13.99
Problem 46.

49. For each waveform of Fig. 13.100, determine the period, frequency, average value, and rms value.



Vertical sensitivity = 20 mV/div.
Horizontal sensitivity = 10 μs/div.

(a)

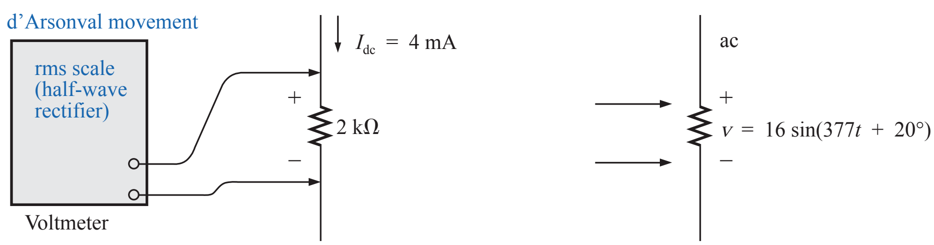
Vertical sensitivity = 0.2 V/div.
Horizontal sensitivity = 50 μs/div.

(b)

FIG. 13.100
Problem 49.

SECTION 13.8 ac Meters and Instruments

50. Determine the reading of the meter for each situation of Fig. 13.101.



(a)

(b)

FIG. 13.101
Problem 50.

SECTION 13.10 Computer Analysis

Programming Language (C++, QBASIC, Pascal, etc.)

51. Given a sinusoidal function, write a program to determine the rms value, frequency, and period.

52. Given two sinusoidal functions, write a program to determine the phase shift between the two waveforms, and indicate which is leading or lagging.

53. Given an alternating pulse waveform, write a program to determine the average and rms values of the waveform over one complete cycle.

GLOSSARY

Alternating waveform A waveform that oscillates above and below a defined reference level.

Amp-Clamp® A clamp-type instrument that will permit noninvasive current measurements and that can be used as a conventional voltmeter or ohmmeter.

Angular velocity The velocity with which a radius vector projecting a sinusoidal function rotates about its center.

Average value The level of a waveform defined by the condition that the area enclosed by the curve above this level is exactly equal to the area enclosed by the curve below this level.

Cycle A portion of a waveform contained in one period of time.

Effective value The equivalent dc value of any alternating voltage or current.

Electrodynamometer meters Instruments that can measure both ac and dc quantities without a change in internal circuitry.

Frequency (f) The number of cycles of a periodic waveform that occur in 1 second.

Frequency counter An instrument that will provide a digital display of the frequency or period of a periodic time-varying signal.

Instantaneous value The magnitude of a waveform at any instant of time, denoted by lowercase letters.

Oscilloscope An instrument that will display, through the use of a cathode-ray tube, the characteristics of a time-varying signal.

Peak amplitude The maximum value of a waveform as measured from its average, or mean, value, denoted by uppercase letters.

Peak-to-peak value The magnitude of the total swing of a signal from positive to negative peaks. The sum of the absolute values of the positive and negative peak values.

Peak value The maximum value of a waveform, denoted by uppercase letters.

Period (T) The time interval between successive repetitions of a periodic waveform.

Periodic waveform A waveform that continually repeats itself after a defined time interval.

Phase relationship An indication of which of two waveforms leads or lags the other, and by how many degrees or radians.

Radian (rad) A unit of measure used to define a particular segment of a circle. One radian is approximately equal to 57.3°; 2π rad are equal to 360°.

Root-mean-square (rms) value The root-mean-square or effective value of a waveform.

Sinusoidal ac waveform An alternating waveform of unique characteristics that oscillates with equal amplitude above and below a given axis.

VOM A multimeter with the capability to measure resistance and both ac and dc levels of current and voltage.

Waveform The path traced by a quantity, plotted as a function of some variable such as position, time, degrees, temperature, and so on.

14

The Basic Elements and Phasors

14.1 INTRODUCTION

The response of the basic R, L, and C elements to a sinusoidal voltage and current will be examined in this chapter, with special note of how frequency will affect the "opposing" characteristic of each element. Phasor notation will then be introduced to establish a method of analysis that permits a direct correspondence with a number of the methods, theorems, and concepts introduced in the dc chapters.

14.2 THE DERIVATIVE

In order to understand the response of the basic R, L, and C elements to a sinusoidal signal, you need to examine the concept of the **derivative** in some detail. It will not be necessary that you become proficient in the mathematical technique, but simply that you understand the impact of a relationship defined by a derivative.

Recall from Section 10.11 that the derivative dx/dt is defined as the rate of change of x with respect to time. If x fails to change at a particular instant, $dx = 0$, and the derivative is zero. For the sinusoidal waveform, dx/dt is zero only at the positive and negative peaks ($\omega t = \pi/2$ and $\frac{3}{2}\pi$ in Fig. 14.1), since x fails to change at these instants of time. The derivative dx/dt is actually the slope of the graph at any instant of time.

A close examination of the sinusoidal waveform will also indicate that the greatest change in x will occur at the instants $\omega t = 0$, π, and 2π. The derivative is therefore a maximum at these points. At 0 and 2π, x increases at its greatest rate, and the derivative is given a positive sign since x increases with time. At π, dx/dt decreases at the same rate as it increases at 0 and 2π, but the derivative is given a negative sign since x decreases with time. Since the rate of change at 0, π, and 2π is the same, the magnitude of the derivative at these points is the same also. For various values of ωt between these maxima and minima, the derivative will exist and will have values from the minimum to the maximum inclusive. A plot of the derivative in Fig. 14.2 shows that

the derivative of a sine wave is a cosine wave.

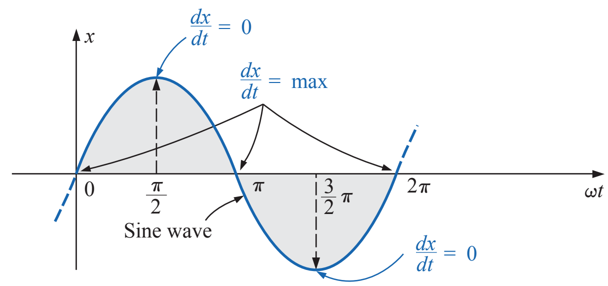


FIG. 14.1
Defining those points in a sinusoidal waveform that have maximum and minimum derivatives.

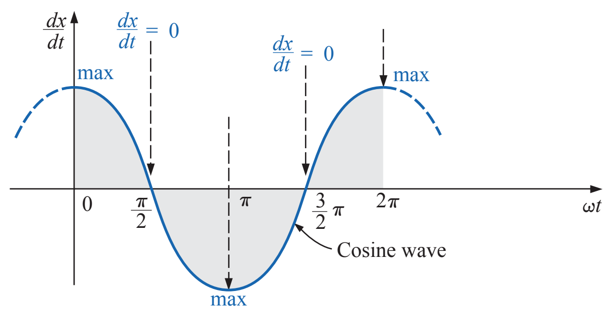


FIG. 14.2
Derivative of the sine wave of Fig. 14.1.

The peak value of the cosine wave is directly related to the frequency of the original waveform. The higher the frequency, the steeper the slope at the horizontal axis and the greater the value of dx/dt, as shown in Fig. 14.3 for two different frequencies.

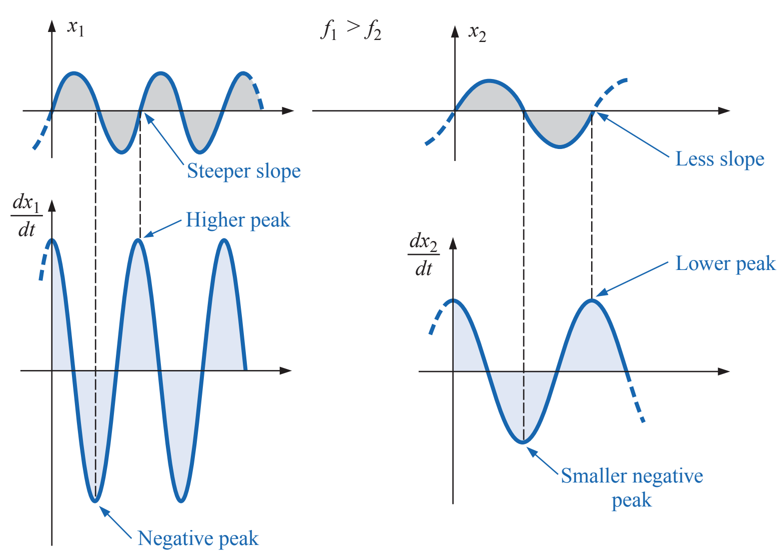


FIG. 14.3
Effect of frequency on the peak value of the derivative.

Note in Fig. 14.3 that even though both waveforms (x_1 and x_2) have the same peak value, the sinusoidal function with the higher frequency produces the larger peak value for the derivative. In addition, note that

the derivative of a sine wave has the same period and frequency as the original sinusoidal waveform.

For the sinusoidal voltage

$$e(t) = E_m \sin(\omega t \pm \theta)$$

the derivative can be found directly by differentiation (calculus) to produce the following:

$$\boxed{\begin{aligned}\frac{d}{dt}\,e(t) &= \omega E_m \cos(\omega t \pm \theta)\\ &= 2\pi f E_m \cos(\omega t \pm \theta)\end{aligned}} \qquad \textbf{(14.1)}$$

The mechanics of the differentiation process will not be discussed or investigated here; nor will they be required to continue with the text. Note, however, that the peak value of the derivative, $2\pi f E_m$, is a function of the frequency of $e(t)$, and the derivative of a sine wave is a cosine wave.

14.3 RESPONSE OF BASIC *R, L,* AND *C* ELEMENTS TO A SINUSOIDAL VOLTAGE OR CURRENT

Now that we are familiar with the characteristics of the derivative of a sinusoidal function, we can investigate the response of the basic elements R, L, and C to a sinusoidal voltage or current.

Resistor

For power-line frequencies and frequencies up to a few hundred kilohertz, resistance is, for all practical purposes, unaffected by the frequency of the applied sinusoidal voltage or current. For this frequency region, the resistor R of Fig. 14.4 can be treated as a constant, and Ohm's law can be applied as follows. For $v = V_m \sin \omega t$,

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R}\sin \omega t = I_m \sin \omega t$$

where

$$\boxed{I_m = \frac{V_m}{R}} \qquad \textbf{(14.2)}$$

In addition, for a given i,

$$v = iR = (I_m \sin \omega t)R = I_m R \sin \omega t = V_m \sin \omega t$$

where

$$\boxed{V_m = I_m R} \qquad \textbf{(14.3)}$$

A plot of v and i in Fig. 14.5 reveals that

for a purely resistive element, the voltage across and the current through the element are in phase, with their peak values related by Ohm's law.

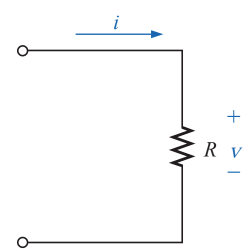


FIG. 14.4
Determining the sinusoidal response for a resistive element.

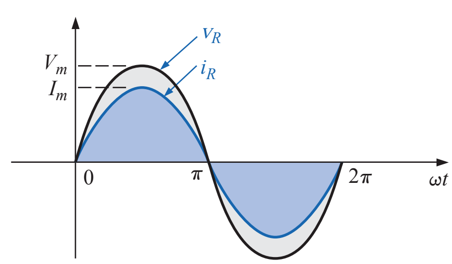


FIG. 14.5
The voltage and current of a resistive element are in phase.

Inductor

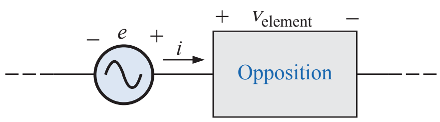


FIG. 14.6
Defining the opposition of an element to the flow of charge through the element.

For the series configuration of Fig. 14.6, the voltage v_{element} of the boxed-in element opposes the source e and thereby reduces the magnitude of the current i. The magnitude of the voltage across the element is determined by the opposition of the element to the flow of charge, or current i. For a resistive element, we have found that the opposition is its resistance and that v_{element} and i are determined by $v_{\text{element}} = iR$.

We found in Chapter 12 that the voltage across an inductor is directly related to the rate of change of current through the coil. Consequently, the higher the frequency, the greater will be the rate of change of current through the coil, and the greater the magnitude of the voltage. In addition, we found in the same chapter that the inductance of a coil will determine the rate of change of the flux linking a coil for a particular change in current through the coil. The higher the inductance, the greater the rate of change of the flux linkages, and the greater the resulting voltage across the coil.

The inductive voltage, therefore, is directly related to the frequency (or, more specifically, the angular velocity of the sinusoidal ac current through the coil) and the inductance of the coil. For increasing values of f and L in Fig. 14.7, the magnitude of v_L will increase as described above.

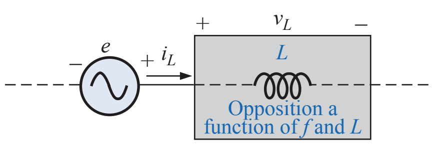


FIG. 14.7
Defining the parameters that determine the opposition of an inductive element to the flow of charge.

Utilizing the similarities between Figs. 14.6 and 14.7, we find that increasing levels of v_L are directly related to increasing levels of opposition in Fig. 14.6. Since v_L will increase with both ω $(= 2\pi f)$ and L, the opposition of an inductive element is as defined in Fig. 14.7.

We will now verify some of the preceding conclusions using a more mathematical approach and then define a few important quantities to be employed in the sections and chapters to follow.

For the inductor of Fig. 14.8, we recall from Chapter 12 that

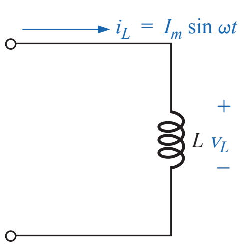


FIG. 14.8
Investigating the sinusoidal response of an inductive element.

$$v_L = L\frac{di_L}{dt}$$

and, applying differentiation,

$$\frac{di_L}{dt} = \frac{d}{dt}(I_m \sin \omega t) = \omega I_m \cos \omega t$$

Therefore, $$v_L = L\frac{di_L}{dt} = L(\omega I_m \cos \omega t) = \omega L I_m \cos \omega t$$

or $$v_L = V_m \sin(\omega t + 90^\circ)$$

where $$V_m = \omega L I_m$$

Note that the peak value of v_L is directly related to ω $(= 2\pi f)$ and L as predicted in the discussion above.

A plot of v_L and i_L in Fig. 14.9 reveals that

for an inductor, v_L leads i_L by 90°, or i_L lags v_L by 90°.

If a phase angle is included in the sinusoidal expression for i_L, such as

$$i_L = I_m \sin(\omega t \pm \theta)$$

then $$v_L = \omega L I_m \sin(\omega t \pm \theta + 90^\circ)$$

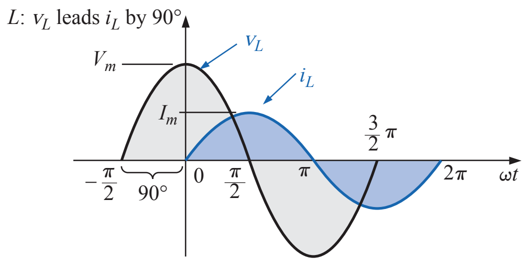


FIG. 14.9
For a pure inductor, the voltage across the coil leads the current through the coil by 90°.

The opposition established by an inductor in a sinusoidal ac network can now be found by applying Eq. (4.1):

$$\text{Effect} = \frac{\text{cause}}{\text{opposition}}$$

which, for our purposes, can be written

$$\text{Opposition} = \frac{\text{cause}}{\text{effect}}$$

Substituting values, we have

$$\text{Opposition} = \frac{V_m}{I_m} = \frac{\omega L I_m}{I_m} = \omega L$$

revealing that the opposition established by an inductor in an ac sinusoidal network is directly related to the product of the angular velocity ($\omega = 2\pi f$) and the inductance, verifying our earlier conclusions.

The quantity ωL, called the **reactance** (from the word *reaction*) of an inductor, is symbolically represented by X_L and is measured in ohms; that is,

$$\boxed{X_L = \omega L} \qquad \text{(ohms, } \Omega\text{)} \qquad \textbf{(14.4)}$$

In an Ohm's law format, its magnitude can be determined from

$$\boxed{X_L = \frac{V_m}{I_m}} \qquad \text{(ohms, } \Omega\text{)} \qquad \textbf{(14.5)}$$

Inductive reactance is the opposition to the flow of current, which results in the continual interchange of energy between the source and the magnetic field of the inductor. In other words, inductive reactance, unlike resistance (which dissipates energy in the form of heat), does not dissipate electrical energy (ignoring the effects of the internal resistance of the inductor).

Capacitor

Let us now return to the series configuration of Fig. 14.6 and insert the capacitor as the element of interest. For the capacitor, however, we will determine i for a particular voltage across the element. When this approach reaches its conclusion, the relationship between the voltage and current will be known, and the opposing voltage (v_{element}) can be determined for any sinusoidal current i.

Our investigation of the inductor revealed that the inductive voltage across a coil opposes the instantaneous change in current through the coil. For capacitive networks, the voltage across the capacitor is limited by the rate at which charge can be deposited on, or released by, the plates of the capacitor during the charging and discharging phases, respectively. In other words, an instantaneous change in voltage across a capacitor is opposed by the fact that there is an element of time required to deposit charge on (or release charge from) the plates of a capacitor, and $V = Q/C$.

Since capacitance is a measure of the rate at which a capacitor will store charge on its plates,

for a particular change in voltage across the capacitor, the greater the value of capacitance, the greater will be the resulting capacitive current.

In addition, the fundamental equation relating the voltage across a capacitor to the current of a capacitor [$i = C(dv/dt)$] indicates that

for a particular capacitance, the greater the rate of change of voltage across the capacitor, the greater the capacitive current.

Certainly, an increase in frequency corresponds to an increase in the rate of change of voltage across the capacitor and to an increase in the current of the capacitor.

The current of a capacitor is therefore directly related to the frequency (or, again more specifically, the angular velocity) and the capacitance of the capacitor. An increase in either quantity will result in an increase in the current of the capacitor. For the basic configuration of Fig. 14.10, however, we are interested in determining the opposition of the capacitor as related to the resistance of a resistor and ωL for the inductor. Since an increase in current corresponds to a decrease in opposition, and i_C is proportional to ω and C, the opposition of a capacitor is inversely related to ω ($= 2\pi f$) and C.

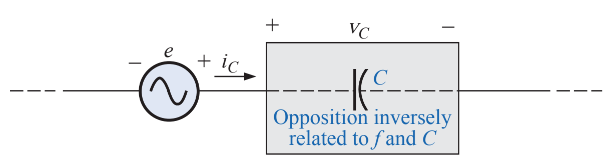


FIG. 14.10
Defining the parameters that determine the opposition of a capacitive element to the flow of the charge.

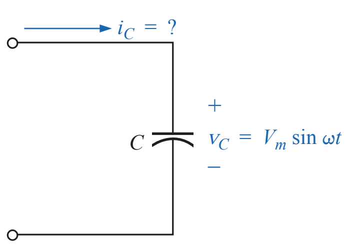


FIG. 14.11
Investigating the sinusoidal response of a capacitive element.

We will now verify, as we did for the inductor, some of the above conclusions using a more mathematical approach.

For the capacitor of Fig. 14.11, we recall from Chapter 10 that

$$i_C = C\,\frac{dv_C}{dt}$$

and, applying differentiation,

$$\frac{dv_C}{dt} = \frac{d}{dt}(V_m \sin \omega t) = \omega V_m \cos \omega t$$

Therefore,

$$i_C = C\frac{dv_C}{dt} = C(\omega V_m \cos \omega t) = \omega C V_m \cos \omega t$$

or $$i_C = I_m \sin(\omega t + 90°)$$

where $$I_m = \omega C V_m$$

Note that the peak value of i_C is directly related to ω $(= 2\pi f)$ and C, as predicted in the discussion above.

A plot of v_C and i_C in Fig. 14.12 reveals that

for a capacitor, i_C leads v_C by 90°, or v_C lags i_C by 90°.*

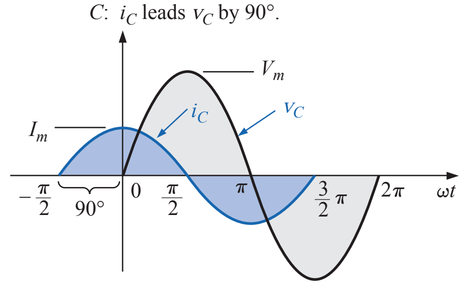


FIG. 14.12
The current of a purely capacitive element leads the voltage across the element by 90°.

If a phase angle is included in the sinusoidal expression for v_C, such as

$$v_C = V_m \sin(\omega t \pm \theta)$$

then $$i_C = \omega C V_m \sin(\omega t \pm \theta + 90°)$$

Applying

$$\text{Opposition} = \frac{\text{cause}}{\text{effect}}$$

and substituting values, we obtain

$$\text{Opposition} = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

which agrees with the results obtained above.

The quantity $1/\omega C$, called the **reactance** of a capacitor, is symbolically represented by X_C and is measured in ohms; that is,

$$\boxed{X_C = \frac{1}{\omega C}} \quad \text{(ohms, } \Omega) \qquad \textbf{(14.6)}$$

In an Ohm's law format, its magnitude can be determined from

$$\boxed{X_C = \frac{V_m}{I_m}} \quad \text{(ohms, } \Omega) \qquad \textbf{(14.7)}$$

Capacitive reactance is the opposition to the flow of charge, which results in the continual interchange of energy between the source and the electric field of the capacitor. Like the inductor, the capacitor does *not* dissipate energy in any form (ignoring the effects of the leakage resistance).

In the circuits just considered, the current was given in the inductive circuit, and the voltage in the capacitive circuit. This was done to avoid the use of integration in finding the unknown quantities. In the inductive circuit,

$$v_L = L\frac{di_L}{dt}$$

*A mnemonic phrase sometimes used to remember the phase relationship between the voltage and current of a coil and capacitor is "*ELI* the *ICE* man." Note that the *L* (inductor) has the *E* before the *I* (*e* leads *i* by 90°), and the *C* (capacitor) has the *I* before the *E* (*i* leads *e* by 90°).

but
$$\boxed{i_L = \frac{1}{L}\int v_L\,dt} \tag{14.8}$$

In the capacitive circuit,

$$i_C = C\frac{dv_C}{dt}$$

but
$$\boxed{v_C = \frac{1}{C}\int i_C\,dt} \tag{14.9}$$

Shortly, we shall consider a method of analyzing ac circuits that will permit us to solve for an unknown quantity with sinusoidal input without having to use direct integration or differentiation.

It is possible to determine whether a network with one or more elements is predominantly capacitive or inductive by noting the phase relationship between the input voltage and current.

If the source current leads the applied voltage, the network is predominantly capacitive, and if the applied voltage leads the source current, it is predominantly inductive.

Since we now have an equation for the reactance of an inductor or capacitor, we do not need to use derivatives or integration in the examples to be considered. Simply applying Ohm's law, $I_m = E_m/X_L$ (or X_C), and keeping in mind the phase relationship between the voltage and current for each element, will be sufficient to complete the examples.

EXAMPLE 14.1 The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10 Ω. Sketch the curves for v and i.

a. $v = 100 \sin 377t$
b. $v = 25 \sin(377t + 60°)$

Solutions:

a. Eq. (14.2): $I_m = \dfrac{V_m}{R} = \dfrac{100\text{ V}}{10\ \Omega} = 10\text{ A}$

(v and i are in phase), resulting in

$$i = \mathbf{10 \sin 377}\boldsymbol{t}$$

The curves are sketched in Fig. 14.13.

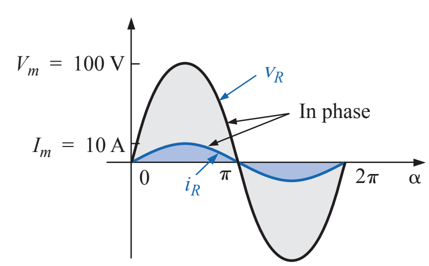


FIG. 14.13
Example 14.1(a).

b. Eq. (14.2): $I_m = \dfrac{V_m}{R} = \dfrac{25 \text{ V}}{10 \ \Omega} = 2.5 \text{ A}$

(v and i are in phase), resulting in

$$i = \mathbf{2.5 \sin(377}t \mathbf{+ 60°)}$$

The curves are sketched in Fig. 14.14.

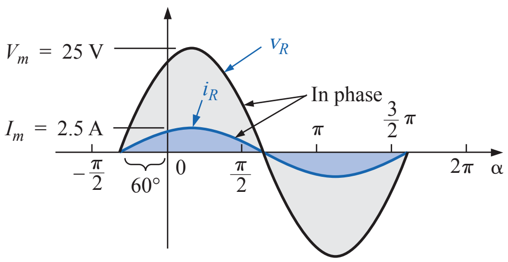


FIG. 14.14
Example 14.1(b).

EXAMPLE 14.2 The current through a 5-Ω resistor is given. Find the sinusoidal expression for the voltage across the resistor for $i = 40 \sin(377t + 30°)$.

Solution: Eq. (14.3): $V_m = I_m R = (40 \text{ A})(5 \ \Omega) = 200 \text{ V}$

(v and i are in phase), resulting in

$$v = \mathbf{200 \sin(377}t \mathbf{+ 30°)}$$

EXAMPLE 14.3 The current through a 0.1-H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the v and i curves.

a. $i = 10 \sin 377t$
b. $i = 7 \sin(377t - 70°)$

Solutions:

a. Eq. (14.4): $X_L = \omega L = (377 \text{ rad/s})(0.1 \text{ H}) = 37.7 \ \Omega$
Eq. (14.5): $V_m = I_m X_L = (10 \text{ A})(37.7 \ \Omega) = 377 \text{ V}$

and we know that for a coil v leads i by 90°. Therefore,

$$v = \mathbf{377 \sin(377}t \mathbf{+ 90°)}$$

The curves are sketched in Fig. 14.15.

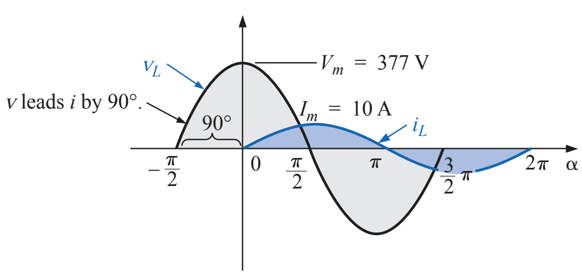


FIG. 14.15
Example 14.3(a).

b. X_L remains at 37.7 Ω.

$$V_m = I_m X_L = (7\text{ A})(37.7\ \Omega) = 263.9\text{ V}$$

and we know that for a coil v leads i by 90°. Therefore,

$$v = 263.9 \sin(377t - 70° + 90°)$$

and

$$v = \mathbf{263.9 \sin(377}\boldsymbol{t}\mathbf{ + 20°)}$$

The curves are sketched in Fig. 14.16.

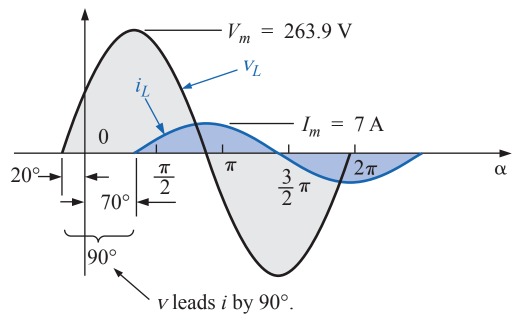


FIG. 14.16
Example 14.3(b).

EXAMPLE 14.4 The voltage across a 0.5-H coil is provided below. What is the sinusoidal expression for the current?

$$v = 100 \sin 20t$$

Solution:

$$X_L = \omega L = (20\text{ rad/s})(0.5\text{ H}) = 10\ \Omega$$

$$I_m = \frac{V_m}{X_L} = \frac{100\text{ V}}{10\ \Omega} = 10\text{ A}$$

and we know that i lags v by 90°. Therefore,

$$i = \mathbf{10 \sin(20}\boldsymbol{t}\mathbf{ - 90°)}$$

EXAMPLE 14.5 The voltage across a 1-μF capacitor is provided below. What is the sinusoidal expression for the current? Sketch the v and i curves.

$$v = 30 \sin 400t$$

Solution:

$$\text{Eq. (14.6):}\quad X_C = \frac{1}{\omega C} = \frac{1}{(400\text{ rad/s})(1 \times 10^{-6}\text{ F})} = \frac{10^6\ \Omega}{400} = 2500\ \Omega$$

$$\text{Eq. (14.7):}\quad I_m = \frac{V_m}{X_C} = \frac{30\text{ V}}{2500\ \Omega} = 0.0120\text{ A} = 12\text{ mA}$$

and we know that for a capacitor i leads v by 90°. Therefore,

$$i = \mathbf{12 \times 10^{-3} \sin(400}\boldsymbol{t}\mathbf{ + 90°)}$$

The curves are sketched in Fig. 14.17.

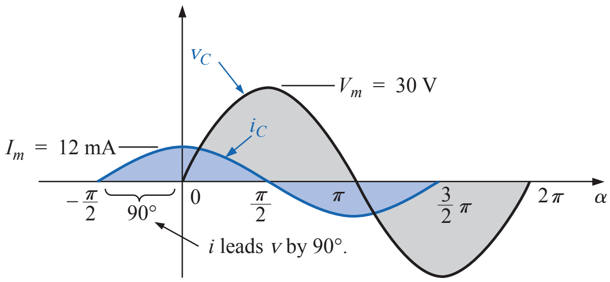


FIG. 14.17
Example 14.5.

EXAMPLE 14.6 The current through a 100-μF capacitor is given. Find the sinusoidal expression for the voltage across the capacitor.

$$i = 40 \sin(500t + 60°)$$

Solution:

$$X_C = \frac{1}{\omega C} = \frac{1}{(500 \text{ rad/s})(100 \times 10^{-6} \text{ F})} = \frac{10^6 \, \Omega}{5 \times 10^4} = \frac{10^2 \, \Omega}{5} = 20 \, \Omega$$

$$V_m = I_m X_C = (40 \text{ A})(20 \, \Omega) = 800 \text{ V}$$

and we know that for a capacitor, v lags i by 90°. Therefore,

$$v = 800 \sin(500t + 60° - 90°)$$

and $$v = \mathbf{800 \sin(500}\boldsymbol{t}\mathbf{ - 30°)}$$

EXAMPLE 14.7 For the following pairs of voltages and currents, determine whether the element involved is a capacitor, an inductor, or a resistor, and determine the value of *C, L,* or *R* if sufficient data are provided (Fig. 14.18):

a. $v = 100 \sin(\omega t + 40°)$
   $i = 20 \sin(\omega t + 40°)$
b. $v = 1000 \sin(377t + 10°)$
   $i = 5 \sin(377t - 80°)$
c. $v = 500 \sin(157t + 30°)$
   $i = 1 \sin(157t + 120°)$
d. $v = 50 \cos(\omega t + 20°)$
   $i = 5 \sin(\omega t + 110°)$

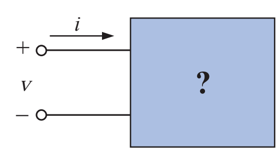


FIG. 14.18
Example 14.7.

Solutions:

a. Since v and i are *in phase,* the element is a *resistor,* and

$$R = \frac{V_m}{I_m} = \frac{100 \text{ V}}{20 \text{ A}} = \mathbf{5 \, \Omega}$$

b. Since v *leads* i by 90°, the element is an *inductor,* and

$$X_L = \frac{V_m}{I_m} = \frac{1000 \text{ V}}{5 \text{ A}} = 200 \, \Omega$$

so that $X_L = \omega L = 200 \, \Omega$ or

$$L = \frac{200\ \Omega}{\omega} = \frac{200\ \Omega}{377\ \text{rad/s}} = \mathbf{0.531\ H}$$

c. Since *i leads v* by 90°, the element is a *capacitor,* and

$$X_C = \frac{V_m}{I_m} = \frac{500\ \text{V}}{1\ \text{A}} = 500\ \Omega$$

so that $X_C = \dfrac{1}{\omega C} = 500\ \Omega$ or

$$C = \frac{1}{\omega 500\ \Omega} = \frac{1}{(157\ \text{rad/s})(500\ \Omega)} = \mathbf{12.74\ \mu F}$$

d. $v = 50\cos(\omega t + 20°) = 50\sin(\omega t + 20° + 90°)$
$= 50\sin(\omega t + 110°)$

Since v and i are *in phase,* the element is a *resistor,* and

$$R = \frac{V_m}{I_m} = \frac{50\ \text{V}}{5\ \text{A}} = \mathbf{10\ \Omega}$$

dc, High-, and Low-Frequency Effects on *L* and *C*

For dc circuits, the frequency is zero, and the reactance of a coil is

$$X_L = 2\pi f L = 2\pi(0)L = 0\ \Omega$$

The use of the short-circuit equivalence for the inductor in dc circuits (Chapter 12) is now validated. At very high frequencies, $X_L\!\uparrow\ = 2\pi f\!\uparrow\! L$ is very large, and for some practical applications the inductor can be replaced by an open circuit. In equation form,

$$\boxed{X_L = 0\ \Omega} \quad \text{dc}, f = 0\ \text{Hz} \tag{14.10}$$

and

$$\boxed{X_L \Rightarrow \infty\ \Omega \qquad \text{as } f \Rightarrow \infty\ \text{Hz}} \tag{14.11}$$

The capacitor can be replaced by an open-circuit equivalence in dc circuits since $f = 0$, and

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(0)C} \Rightarrow \infty\ \Omega$$

once again substantiating our previous action (Chapter 10). At very high frequencies, for finite capacitances,

$$X_C\!\downarrow\ = \frac{1}{2\pi f\!\uparrow\! C}$$

is very small, and for some practical applications the capacitor can be replaced by a short circuit. In equation form

$$\boxed{X_C \Rightarrow \infty\ \Omega \qquad \text{as } f \Rightarrow 0\ \text{Hz}} \tag{14.12}$$

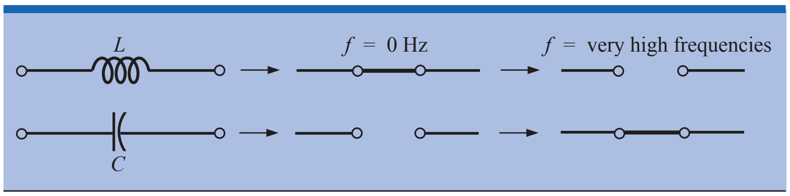
and

$$\boxed{X_C \cong 0\ \Omega} \quad f = \text{very high frequencies} \tag{14.13}$$

Table 14.1 reviews the preceding conclusions.

TABLE 14.1

Effect of high and low frequencies on the circuit model of an inductor and a capacitor.



Phase Angle Measurements between the Applied Voltage and Source Current

Now that we are familiar with phase relationships and understand how the elements affect the phase relationship between the applied voltage and resulting current, the use of the oscilloscope to measure the phase angle can be introduced. Recall from past discussions that the oscilloscope can be used only to display voltage levels versus time. However, now that we realize that the voltage across a resistor is in phase with the current through a resistor, we can consider the phase angle associated with the voltage across any resistor actually to be the phase angle of the current. For example, suppose that we want to find the phase angle introduced by the unknown system of Fig. 14.19(a). In Fig. 14.19(b), a resistor was added to the input leads, and the two channels of a dual trace (most modern-day oscilloscopes can display two signals at the same time) were connected as shown. One channel will display the input voltage v_i, whereas the other will display v_R, as shown in Fig. 14.19(c). However, as noted before, since v_R and i_R are in phase, the phase angle appearing in Fig. 14.19(c) is also the phase angle between v_i and i_i. The addition of a "sensing" resistor (a resistor of a magnitude that will not adversely affect the input characteristics of the system), therefore, can be used to determine the phase angle introduced by the system and can be used to determine the magnitude of the resulting current. The details of the connections that must be made and how the actual phase angle is determined will be left for the laboratory experience.

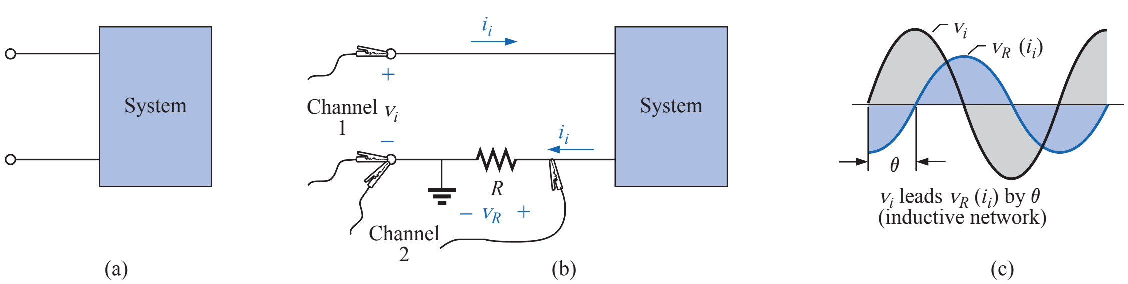


FIG. 14.19

Using an oscilloscope to determine the phase angle between the applied voltage and the source current.

14.4 FREQUENCY RESPONSE OF THE BASIC ELEMENTS

The analysis of Section 14.3 was limited to a particular applied frequency. What is the effect of varying the frequency on the level of opposition offered by a resistive, inductive, or capacitive element? We are aware from the last section that the inductive reactance increases with frequency while the capacitive reactance decreases. However, what is the pattern to this increase or decrease in opposition? Does it continue indefinitely on the same path? Since applied signals may have frequencies extending from a few hertz to megahertz, it is important to be aware of the effect of frequency on the opposition level.

R

Thus far we have assumed that the resistance of a resistor is independent of the applied frequency. However, in the real world each resistive element has stray capacitance levels and lead inductance that are sensitive to the applied frequency. However, the capacitive and inductive levels involved are usually so small that their real effect is not noticed until the megahertz range. The resistance-versus-frequency curves for a number of carbon composition resistors are provided in Fig. 14.20. Note that the lower resistance levels seem to be less affected by the frequency level. The 100-Ω resistor is essentially stable up to about 300 MHz, whereas the 100-kΩ resistor starts its radical decline at about 15 MHz.

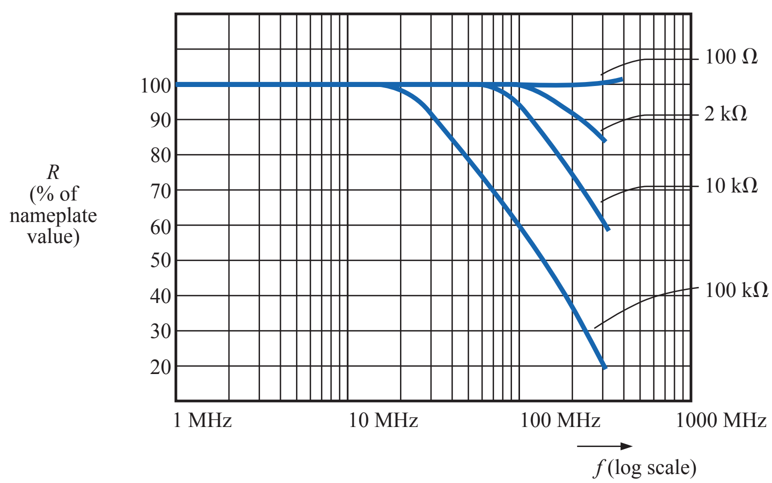


FIG. 14.20
Typical resistance-versus-frequency curves for carbon compound resistors.

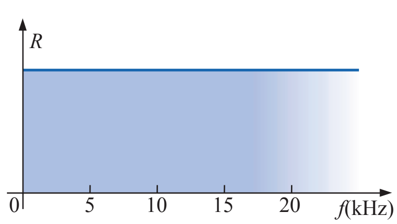


FIG. 14.21
R versus f for the range of interest.

Frequency, therefore, does have impact on the resistance of an element, but for our current frequency range of interest, we will assume the resistance-versus-frequency plot of Fig. 14.21 (like Fig. 14.20 up to 15 MHz), which essentially specifies that the resistance level of a resistor is independent of frequency.

L

For inductors, the equation

$$X_L = \omega L = 2\pi f L = 2\pi L f$$

is directly related to the straight-line equation

$$y = mx + b = (2\pi L)f + 0$$

with a slope (m) of $2\pi L$ and a y-intercept (b) of zero. X_L is the y variable and f is the x variable, as shown in Fig. 14.22.

The larger the inductance, the greater the slope ($m = 2\pi L$) for the same frequency range, as shown in Fig. 14.22. Keep in mind, as reemphasized by Fig. 14.22, that the opposition of an inductor at very low frequencies approaches that of a short circuit, while at high frequencies the reactance approaches that of an open circuit.

For the capacitor, the reactance equation

$$X_C = \frac{1}{2\pi f C}$$

can be written

$$X_C f = \frac{1}{2\pi C}$$

which matches the basic format of a hyperbola,

$$yx = k$$

with $y = X_C$, $x = f$, and the constant $k = 1/(2\pi C)$.

At $f = 0$ Hz, the reactance of the capacitor is so large, as shown in Fig. 14.23, that it can be replaced by an open-circuit equivalent. As the frequency increases, the reactance decreases, until eventually a short-circuit equivalent would be appropriate. Note that an increase in capacitance causes the reactance to drop off more rapidly with frequency.

In summary, therefore, as the applied frequency increases, the resistance of a resistor remains constant, the reactance of an inductor increases linearly, and the reactance of a capacitor decreases nonlinearly.

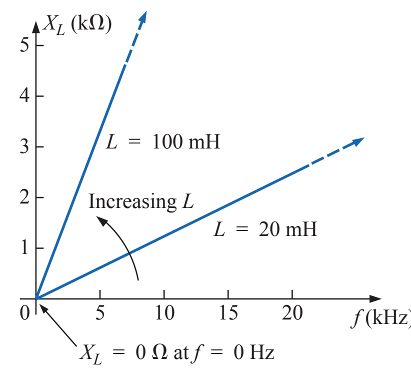


FIG. 14.22
X_L versus frequency.

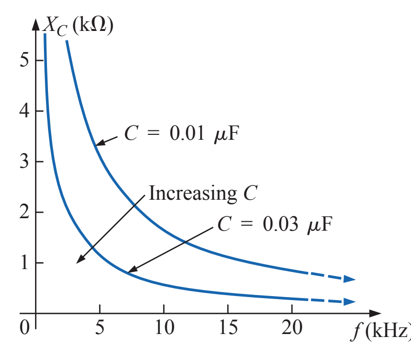


FIG. 14.23
X_C versus frequency.

EXAMPLE 14.8 At what frequency will the reactance of a 200-mH inductor match the resistance level of a 5-kΩ resistor?

Solution: The resistance remains constant at 5 kΩ for the frequency range of the inductor. Therefore,

$$\begin{aligned} R = 5000\ \Omega = X_L &= 2\pi f L = 2\pi L f \\ &= 2\pi(200 \times 10^{-3}\ \text{H})f = 1.257f \end{aligned}$$

and

$$f = \frac{5000\ \text{Hz}}{1.257} \cong \mathbf{3.98\ kHz}$$

EXAMPLE 14.9 At what frequency will an inductor of 5 mH have the same reactance as a capacitor of 0.1 μF?

Solution:

$$X_L = X_C$$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{4\pi^2 LC}$$

and

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(5 \times 10^{-3}\ \text{H})(0.1 \times 10^{-6}\ \text{F})}}$$

$$= \frac{1}{2\pi\sqrt{5 \times 10^{-10}}} = \frac{1}{(2\pi)(2.236 \times 10^{-5})}$$

$$f = \frac{10^5 \text{ Hz}}{14.05} \cong \mathbf{7.12\ kHz}$$

One must also be aware that commercial inductors are not ideal elements. In other words, the terminal characteristics of an inductance will vary with several factors, such as frequency, temperature, and current. A true equivalent for an inductor appears in Fig. 14.24. The series resistance R_s represents the copper losses (resistance of the many turns of thin copper wire); the eddy current losses (which will be described in Chapter 19 and which are losses due to small circular currents in the core when an ac voltage is applied); and the hysteresis losses (which will also be described in Chapter 19 and which are losses due to core losses created by the rapidly reversing field in the core). The capacitance C_p is the stray capacitance that exists between the windings of the inductor. For most inductors, the construction is usually such that the larger the inductance, the lower the frequency at which the parasitic elements become important. That is, for inductors in the millihenry range (which is very typical), frequencies approaching 100 kHz can have an effect on the ideal characteristics of the element. For inductors in the microhenry range, a frequency of 1 MHz may introduce negative effects. This is not to suggest that the inductors lose their effect at these frequencies but more that they can no longer be considered ideal (purely inductive elements).

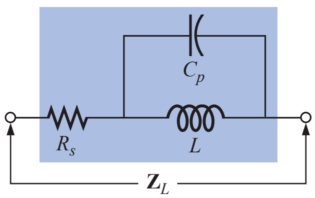


FIG. 14.24

Practical equivalent for an inductor.

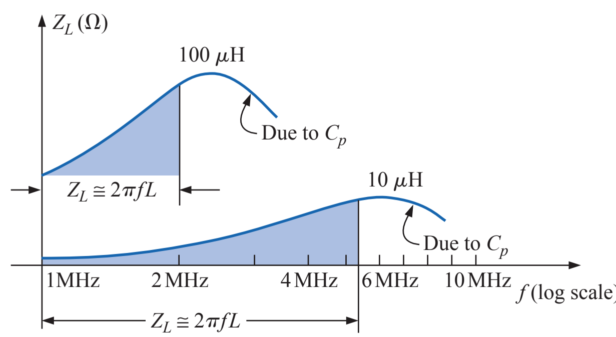


FIG. 14.25

Z_L versus frequency for the practical inductor equivalent of Fig. 14.24.

Figure 14.25 is a plot of the magnitude of the impedance Z_L of Fig. 14.24 versus frequency. Note that up to about 2 MHz the impedance increases almost linearly with frequency, clearly suggesting that the 100-μH inductor is essentially ideal. However, above 2 MHz all the factors contributing to R_s will start to increase, while the reactance due to the capacitive element C_p will be more pronounced. The dropping level of capacitive reactance will begin to have a shorting effect across the windings of the inductor and will reduce the overall inductive effect. Eventually, if the frequency continues to increase, the capacitive effects will overcome the inductive effects, and the element will actually begin to behave in a capacitive fashion. Note the similarities of this region with the curves of Fig. 14.23. Also note that decreasing levels of inductance (available with fewer turns and therefore lower levels of C_p) will not demonstrate the degrading effect until higher frequencies are applied. In general, therefore, the frequency of application for a coil becomes important at increasing frequencies. Inductors lose their ideal characteristics and in fact begin to act as capacitive elements with increasing losses at very high frequencies.

The capacitor, like the inductor, is not ideal at higher frequencies. In fact, a transition point can be defined where the characteristics of the capacitor will actually be inductive. The complete equivalent model for a capacitor is provided in Fig. 14.26. The resistance R_s, defined by the resistivity of the dielectric (typically 10^{12} $\Omega \cdot m$ or better) and the case resistance, will determine the level of leakage current to expect during the discharge cycle. In other words, a charged capacitor can discharge both through the case and through the dielectric at a rate determined by the resistance of each path. Depending on the capacitor, the discharge time can extend from a few seconds for some electrolytic capacitors to hours (paper) or perhaps days (polystyrene). Inversely, therefore, electrolytics obviously have much lower levels of R_s than paper or polystyrene. The resistance R_p reflects the energy lost as the atoms continually realign themselves in the dielectric due to the applied alternating ac voltage. Molecular friction is present due to the motion of the atoms as they respond to the alternating applied electric field. Interestingly enough, however, the relative permittivity will decrease with increasing frequencies but will eventually take a complete turnaround and begin to increase at very high frequencies. The inductance L_s includes the inductance of the capacitor leads and any inductive effects introduced by the design of the capacitor. Be aware that the inductance of the leads is about 0.05 μH per centimeter or 0.2 μH for a capacitor with two 2-cm leads—a level that can be important at high frequencies. As for the inductor, the capacitor will behave quite ideally for the low- and mid-frequency range, as shown by the plot of Fig. 14.27 for a 0.01-μF

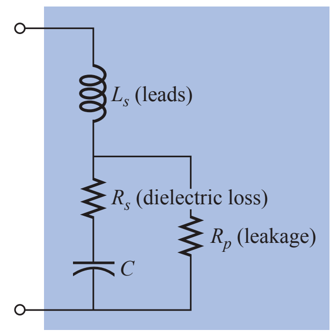


FIG. 14.26
Practical equivalent for a capacitor.

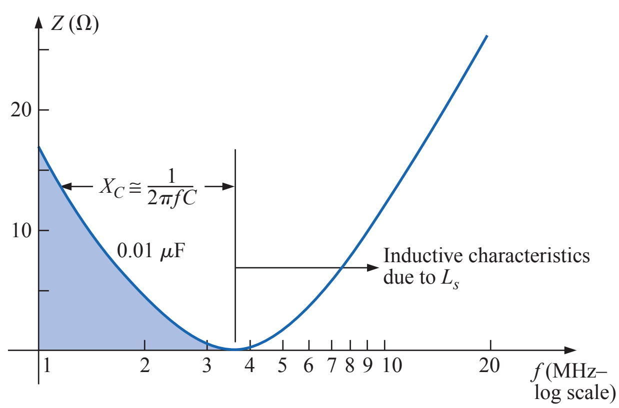


FIG. 14.27
Impedance characteristics of a 0.01-μF metalized film capacitor versus frequency.

metalized film capacitor with 2-cm leads. As the frequency increases, however, and the reactance X_s becomes larger, a frequency will eventually be reached where the reactance of the coil equals that of the capacitor (a resonant condition to be described in Chapter 20). Any additional increase in frequency will simply result in X_s being greater than X_C, and the element will behave like an inductor. In general, therefore, the frequency of application is important for capacitive elements because there comes a point with increasing frequency when the element will take on inductive characteristics. It also points out that the frequency of application defines the type of capacitor (or inductor) that would be applied: Electrolytics are limited to frequencies up to perhaps 10 kHz, while ceramic or mica can handle frequencies beyond 10 MHz.

The expected temperature range of operation can have an important impact on the type of capacitor chosen for a particular application. Electrolytics, tantalum, and some high-k ceramic capacitors are very sensitive to colder temperatures. In fact, most electrolytics lose 20% of their room-temperature capacitance at 0°C (freezing). Higher temperatures (up to 100°C or 212°F) seem to have less of an impact in general than colder temperatures, but high-k ceramics can lose up to 30% of their capacitance level at 100°C compared to room temperature. With exposure and experience, you will learn the type of capacitor employed for each application, and concern will arise only when very high frequencies, extreme temperatures, or very high currents or voltages are encountered.

14.5 AVERAGE POWER AND POWER FACTOR

For any load in a sinusoidal ac network, the voltage across the load and the current through the load will vary in a sinusoidal nature. The questions then arise, How does the power to the load determined by the product $v \cdot i$ vary, and what fixed value can be assigned to the power since it will vary with time?

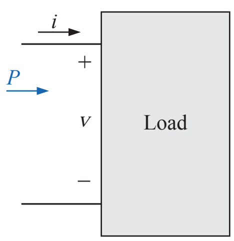


FIG. 14.28
Determining the power delivered in a sinusoidal ac network.

If we take the general case depicted in Fig. 14.28 and use the following for v and i:

$$v = V_m \sin(\omega t + \theta_v)$$
$$i = I_m \sin(\omega t + \theta_i)$$

then the power is defined by

$$\begin{aligned} p = vi &= V_m \sin(\omega t + \theta_v) I_m \sin(\omega t + \theta_i) \\ &= V_m I_m \sin(\omega t + \theta_v) \sin(\omega t + \theta_i) \end{aligned}$$

Using the trigonometric identity

$$\sin A \sin B = \frac{\cos(A - B) - \cos(A + B)}{2}$$

the function $\sin(\omega t + \theta_v) \sin(\omega t + \theta_i)$ becomes

$$\begin{aligned} &\sin(\omega t + \theta_v) \sin(\omega t + \theta_i) \\ &\quad = \frac{\cos[(\omega t + \theta_v) - (\omega t + \theta_i)] - \cos[(\omega t + \theta_v) + (\omega t + \theta_i)]}{2} \\ &\quad = \frac{\cos(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i)}{2} \end{aligned}$$

so that

$$p = \overbrace{\left[\frac{V_m I_m}{2} \cos(\theta_v - \theta_i)\right]}^{\text{Fixed value}} - \overbrace{\left[\frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i)\right]}^{\text{Time-varying (function of } t\text{)}}$$

A plot of v, i, and p on the same set of axes is shown in Fig. 14.29.

Note that the second factor in the preceding equation is a cosine wave with an amplitude of $V_m I_m / 2$ and with a frequency twice that of

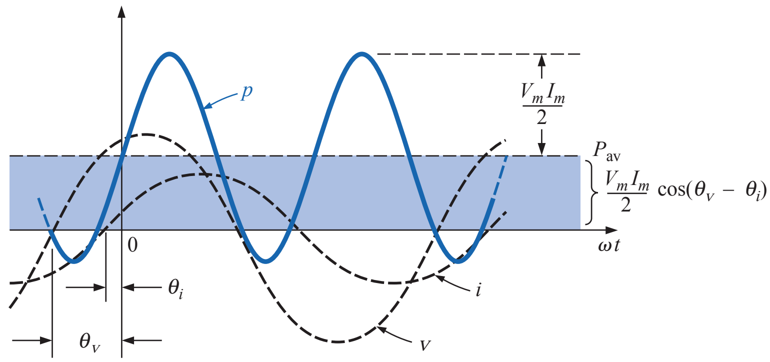


FIG. 14.29
Defining the average power for a sinusoidal ac network.

the voltage or current. The average value of this term is zero over one cycle, producing no net transfer of energy in any one direction.

The first term in the preceding equation, however, has a constant magnitude (no time dependence) and therefore provides some net transfer of energy. This term is referred to as the **average power,** the reason for which is obvious from Fig. 14.29. The average power, or **real power** as it is sometimes called, is the power delivered to and dissipated by the load. It corresponds to the power calculations performed for dc networks. The angle $(\theta_v - \theta_i)$ is the phase angle between v and i. Since $\cos(-\alpha) = \cos \alpha$,

the magnitude of average power delivered is independent of whether v leads i or i leads v.

Defining θ as equal to $|\theta_v - \theta_i|$, where $|\ \ |$ indicates that only the magnitude is important and the sign is immaterial, we have

$$\boxed{P = \frac{V_m I_m}{2}\cos\theta} \qquad \text{(watts, W)} \qquad \textbf{(14.14)}$$

where P is the average power in watts. This equation can also be written

$$P = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right)\cos\theta$$

or, since $\qquad V_{\text{eff}} = \dfrac{V_m}{\sqrt{2}} \quad \text{and} \quad I_{\text{eff}} = \dfrac{I_m}{\sqrt{2}}$

Equation (14.14) becomes

$$\boxed{P = V_{\text{eff}} I_{\text{eff}} \cos\theta} \qquad \textbf{(14.15)}$$

Let us now apply Eqs. (14.14) and (14.15) to the basic R, L, and C elements.

Resistor

In a purely resistive circuit, since v and i are in phase, $|\theta_v - \theta_i| = \theta = 0°$, and $\cos\theta = \cos 0° = 1$, so that

$$\boxed{P = \frac{V_m I_m}{2} = V_{\text{eff}} I_{\text{eff}}} \qquad \text{(W)} \tag{14.16}$$

Or, since
$$I_{\text{eff}} = \frac{V_{\text{eff}}}{R}$$

then
$$\boxed{P = \frac{V_{\text{eff}}^2}{R} = I_{\text{eff}}^2 R} \qquad \text{(W)} \tag{14.17}$$

Inductor

In a purely inductive circuit, since v leads i by 90°, $|\theta_v - \theta_i| = \theta = |-90°| = 90°$. Therefore,

$$P = \frac{V_m I_m}{2} \cos 90° = \frac{V_m I_m}{2}(0) = \mathbf{0\ W}$$

The average power or power dissipated by the ideal inductor (no associated resistance) is zero watts.

Capacitor

In a purely capacitive circuit, since i leads v by 90°, $|\theta_v - \theta_i| = \theta = |-90°| = 90°$. Therefore,

$$P = \frac{V_m I_m}{2} \cos(90°) = \frac{V_m I_m}{2}(0) = \mathbf{0\ W}$$

The average power or power dissipated by the ideal capacitor (no associated resistance) is zero watts.

EXAMPLE 14.10 Find the average power dissipated in a network whose input current and voltage are the following:

$$i = 5 \sin(\omega t + 40°)$$
$$v = 10 \sin(\omega t + 40°)$$

Solution: Since v and i are in phase, the circuit appears to be purely resistive at the input terminals. Therefore,

$$P = \frac{V_m I_m}{2} = \frac{(10\text{ V})(5\text{ A})}{2} = \mathbf{25\ W}$$

or
$$R = \frac{V_m}{I_m} = \frac{10\text{ V}}{5\text{ A}} = 2\ \Omega$$

and
$$P = \frac{V_{\text{eff}}^2}{R} = \frac{[(0.707)(10\text{ V})]^2}{2} = \mathbf{25\ W}$$

or
$$P = I_{\text{eff}}^2 R = [(0.707)(5\text{ A})]^2(2) = \mathbf{25\ W}$$

For the following example, the circuit consists of a combination of resistances and reactances producing phase angles between the input current and voltage different from 0° or 90°.

EXAMPLE 14.11 Determine the average power delivered to networks having the following input voltage and current:

a. $v = 100 \sin(\omega t + 40°)$
   $i = 20 \sin(\omega t + 70°)$
b. $v = 150 \sin(\omega t - 70°)$
   $i = 3 \sin(\omega t - 50°)$

Solutions:

a. $V_m = 100, \quad \theta_v = 40°$
   $I_m = 20, \quad \theta_i = 70°$
   $\theta = |\theta_v - \theta_i| = |40° - 70°| = |-30°| = 30°$
   and

$$P = \frac{V_m I_m}{2}\cos\theta = \frac{(100\text{ V})(20\text{ A})}{2}\cos(30°) = (1000\text{ W})(0.866)$$

$$= \mathbf{866\ W}$$

b. $V_m = 150\text{ V}, \quad \theta_v = -70°$
   $I_m = 3\text{ A}, \quad \theta_i = -50°$
   $\theta = |\theta_v - \theta_i| = |-70° - (-50°)|$
   $= |-70° + 50°| = |-20°| = 20°$
   and

$$P = \frac{V_m I_m}{2}\cos\theta = \frac{(150\text{ V})(3\text{ A})}{2}\cos(20°) = (225\text{ W})(0.9397)$$

$$= \mathbf{211.43\ W}$$

Power Factor

In the equation $P = (V_m I_m/2)\cos\theta$, the factor that has significant control over the delivered power level is the $\cos\theta$. No matter how large the voltage or current, if $\cos\theta = 0$, the power is zero; if $\cos\theta = 1$, the power delivered is a maximum. Since it has such control, the expression was given the name **power factor** and is defined by

$$\boxed{\text{Power factor} = F_p = \cos\theta} \qquad \textbf{(14.18)}$$

For a purely resistive load such as the one shown in Fig. 14.30, the phase angle between v and i is 0° and $F_p = \cos\theta = \cos 0° = 1$. The power delivered is a maximum of $(V_m I_m/2)\cos\theta = ((100\text{ V})(5\text{ A})/2)\cdot(1) = 250\text{ W}$.

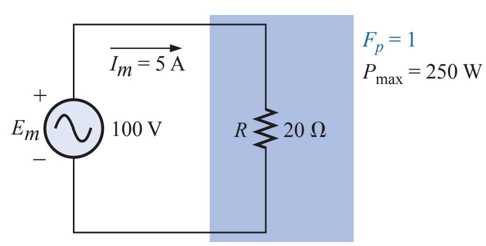


FIG. 14.30
Purely resistive load with $F_p = 1$.

For a purely reactive load (inductive or capacitive) such as the one shown in Fig. 14.31, the phase angle between v and i is 90° and $F_p = \cos\theta = \cos 90° = 0$. The power delivered is then the minimum value of zero watts, **even though the current has the same peak value** as that encountered in Fig. 14.30.

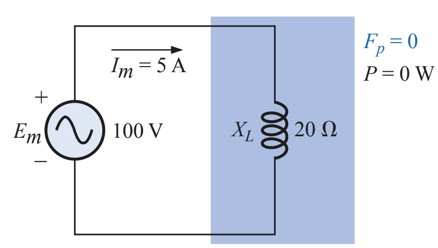


FIG. 14.31
Purely inductive load with $F_p = 0$.

For situations where the load is a combination of resistive and reactive elements, the power factor will vary between 0 and 1. The more resistive the total impedance, the closer the power factor is to 1; the more reactive the total impedance, the closer the power factor is to 0.

In terms of the average power and the terminal voltage and current,

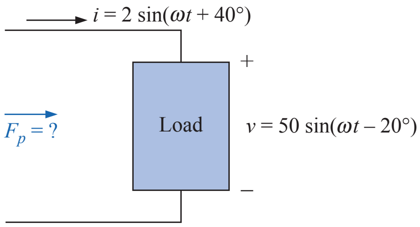


FIG. 14.32
Example 14.12(a).

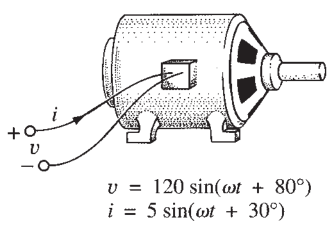


FIG. 14.33
Example 14.12(b).

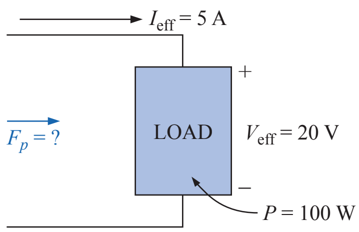


FIG. 14.34
Example 14.12(c).

$$\boxed{F_p = \cos\theta = \frac{P}{V_{\text{eff}}I_{\text{eff}}}} \qquad \textbf{(14.19)}$$

The terms *leading* and *lagging* are often written in conjunction with the power factor. *They are defined by the current through the load.* If the current leads the voltage across a load, the load has a **leading power factor.** If the current lags the voltage across the load, the load has a **lagging power factor.** In other words,

capacitive networks have leading power factors, and inductive networks have lagging power factors.

The importance of the power factor to power distribution systems is examined in Chapter 19. In fact, one section is devoted to power-factor correction.

EXAMPLE 14.12 Determine the power factors of the following loads, and indicate whether they are leading or lagging:

a. Fig. 14.32
b. Fig. 14.33
c. Fig. 14.34

Solutions:

a. $F_p = \cos\theta = \cos|40° - (-20°)| = \cos 60° = \mathbf{0.5\ leading}$
b. $F_p = \cos\theta\,|80° - 30°| = \cos 50° = \mathbf{0.6428\ lagging}$
c. $F_p = \cos\theta = \dfrac{P}{V_{\text{eff}}I_{\text{eff}}} = \dfrac{100\text{ W}}{(20\text{ V})(5\text{ A})} = \dfrac{100\text{ W}}{100\text{ W}} = \mathbf{1}$

The load is resistive, and F_p is neither leading nor lagging.

14.6 COMPLEX NUMBERS

In our analysis of dc networks, we found it necessary to determine the algebraic sum of voltages and currents. Since the same will also be true for ac networks, the question arises, How do we determine the algebraic sum of two or more voltages (or currents) that are varying sinusoidally? Although one solution would be to find the algebraic sum on a point-to-point basis (as shown in Section 14.12), this would be a long and tedious process in which accuracy would be directly related to the scale employed.

It is the purpose of this chapter to introduce a system of **complex numbers** that, when related to the sinusoidal ac waveform, will result in a technique for finding the algebraic sum of sinusoidal waveforms that is quick, direct, and accurate. In the following chapters, the technique will be extended to permit the analysis of sinusoidal ac networks in a manner very similar to that applied to dc networks. The methods and theorems as described for dc networks can then be applied to sinusoidal ac networks with little difficulty.

A **complex number** represents a point in a two-dimensional plane located with reference to two distinct axes. This point can also determine a radius vector drawn from the origin to the point. The horizontal axis is called the *real* axis, while the vertical axis is called the *imaginary* axis. Both are labeled in Fig. 14.35. Every number from zero to ±∞ can be represented by some point along the real axis. Prior to the development of this system of complex numbers, it was believed that

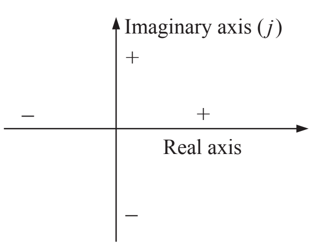


FIG. 14.35
Defining the real and imaginary axes of a complex plane.

any number not on the real axis would not exist—hence the term *imaginary* for the vertical axis.

In the complex plane, the horizontal or real axis represents all positive numbers to the right of the imaginary axis and all negative numbers to the left of the imaginary axis. All positive imaginary numbers are represented above the real axis, and all negative imaginary numbers, below the real axis. The symbol j (or sometimes i) is used to denote the imaginary component.

Two forms are used to represent a complex number: **rectangular** and **polar.** Each can represent a point in the plane or a radius vector drawn from the origin to that point.

14.7 RECTANGULAR FORM

The format for the **rectangular form** is

$$\boxed{\mathbf{C} = X + jY} \tag{14.20}$$

as shown in Fig. 14.36. The letter **C** was chosen from the word "complex." The **boldface** notation is for any number with magnitude and direction. The *italic* is for magnitude only.

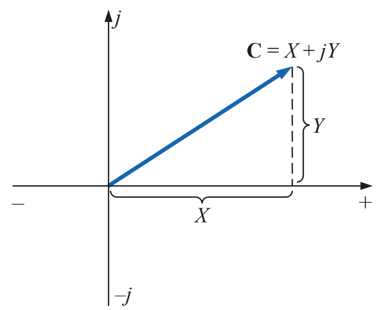


FIG. 14.36
Defining the rectangular form.

EXAMPLE 14.13 Sketch the following complex numbers in the complex plane:

a. $\mathbf{C} = 3 + j4$
b. $\mathbf{C} = 0 - j6$
c. $\mathbf{C} = -10 - j20$

Solutions:

a. See Fig. 14.37.
b. See Fig. 14.38.
c. See Fig. 14.39.

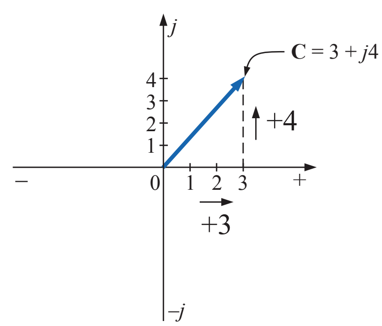


FIG. 14.37
Example 14.13(a).

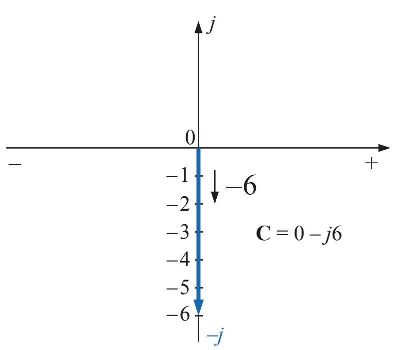


FIG. 14.38
Example 14.13(b).

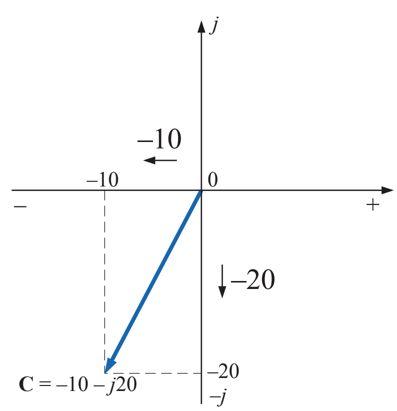


FIG. 14.39
Example 14.13(c).

14.8 POLAR FORM

The format for the **polar form** is

$$\boxed{\mathbf{C} = Z \angle \theta} \tag{14.21}$$

with the letter Z chosen from the sequence X, Y, Z.

where Z indicates magnitude only and θ is **always measured counterclockwise (CCW) from the *positive real axis*,** as shown in Fig. 14.40. Angles measured in the clockwise direction from the positive real axis must have a negative sign associated with them.

A negative sign in front of the polar form has the effect shown in Fig. 14.41. Note that it results in a complex number directly opposite the complex number with a positive sign.

$$\boxed{-\mathbf{C} = -Z\angle\theta = Z\angle\theta \pm 180°} \qquad \textbf{(14.22)}$$

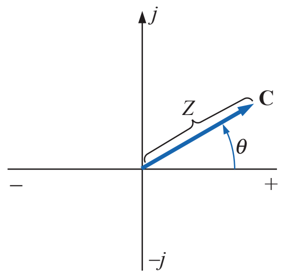


FIG. 14.40
Defining the polar form.

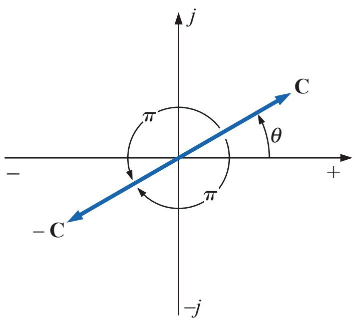


FIG. 14.41
Demonstrating the effect of a negative sign on the polar form.

EXAMPLE 14.14 Sketch the following complex numbers in the complex plane:

a. $\mathbf{C} = 5\angle 30°$
b. $\mathbf{C} = 7\angle -120°$
c. $\mathbf{C} = -4.2\angle 60°$

Solutions:

a. See Fig. 14.42.
b. See Fig. 14.43.
c. See Fig. 14.44.

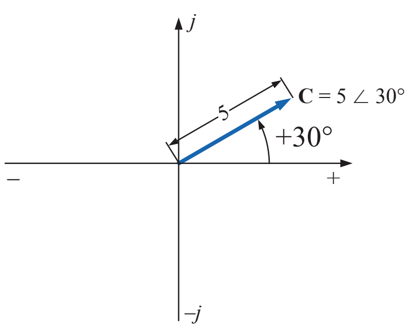


FIG. 14.42
Example 14.14(a).

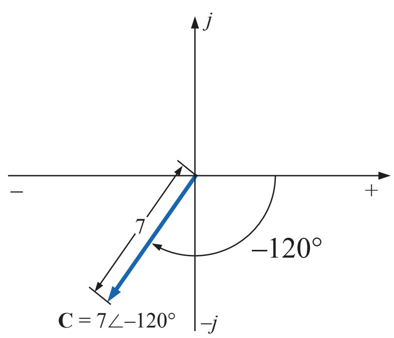


FIG. 14.43
Example 14.14(b).

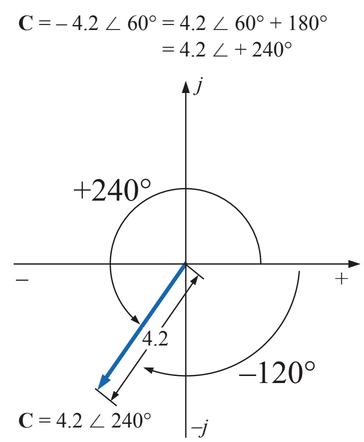


FIG. 14.44
Example 14.14(c).

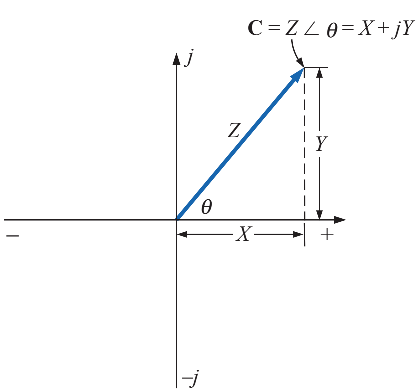


FIG. 14.45
Conversion between forms.

14.9 CONVERSION BETWEEN FORMS

The two forms are related by the following equations, as illustrated in Fig. 14.45.

Rectangular to Polar

$$\boxed{Z = \sqrt{X^2 + Y^2}} \qquad \textbf{(14.23)}$$

$$\boxed{\theta = \tan^{-1}\frac{Y}{X}} \qquad \textbf{(14.24)}$$

Polar to Rectangular

$$\boxed{X = Z\cos\theta} \tag{14.25}$$

$$\boxed{Y = Z\sin\theta} \tag{14.26}$$

EXAMPLE 14.15 Convert the following from rectangular to polar form:

$$\mathbf{C} = 3 + j\,4 \qquad \text{(Fig. 14.46)}$$

Solution:

$$Z = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$$

and $$\mathbf{C = 5\,\angle 53.13°}$$

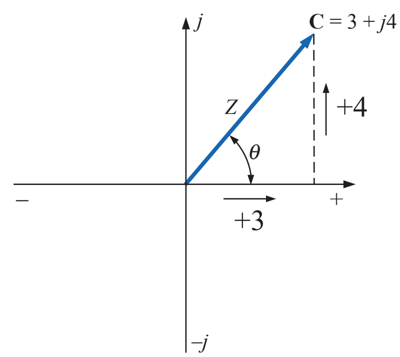


FIG. 14.46
Example 14.15.

EXAMPLE 14.16 Convert the following from polar to rectangular form:

$$\mathbf{C} = 10\,\angle 45° \qquad \text{(Fig. 14.47)}$$

Solution:

$$X = 10\cos 45° = (10)(0.707) = 7.07$$
$$Y = 10\sin 45° = (10)(0.707) = 7.07$$

and $$\mathbf{C = 7.07 + \mathit{j}7.07}$$

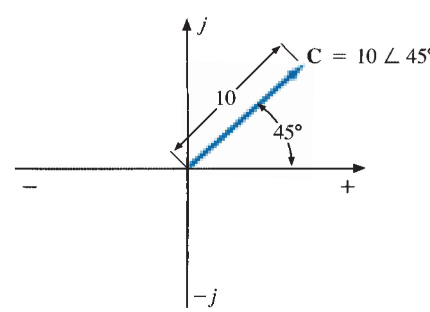


FIG. 14.47
Example 14.16.

If the complex number should appear in the second, third, or fourth quadrant, simply convert it in that quadrant, and carefully determine the proper angle to be associated with the magnitude of the vector.

EXAMPLE 14.17 Convert the following from rectangular to polar form:

$$\mathbf{C} = -6 + j\,3 \qquad \text{(Fig. 14.48)}$$

Solution:

$$Z = \sqrt{(6)^2 + (3)^2} = \sqrt{45} = 6.71$$

$$\beta = \tan^{-1}\left(\frac{3}{6}\right) = 26.57°$$

$$\theta = 180° - 26.57° = 153.43°$$

and $$\mathbf{C = 6.71\,\angle 153.43°}$$

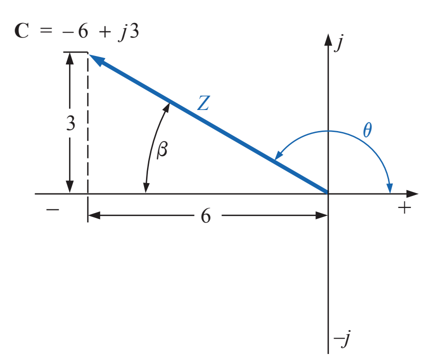


FIG. 14.48
Example 14.17.

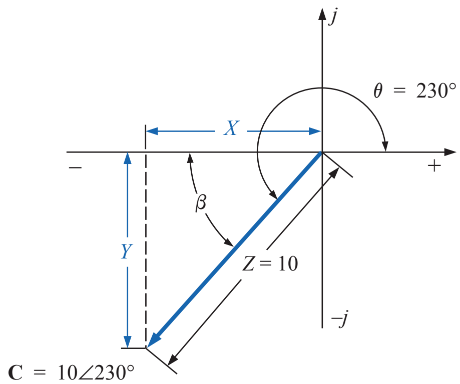


FIG. 14.49
Example 14.18.

EXAMPLE 14.18 Convert the following from polar to rectangular form:

$$\mathbf{C} = 10 \angle 230° \qquad \text{(Fig. 14.49)}$$

Solution:

$$X = Z\cos\beta = 10\cos(230° - 180°) = 10\cos 50°$$
$$= (10)(0.6428) = 6.428$$
$$Y = Z\sin\beta = 10\sin 50° = (10)(0.7660) = 7.660$$

and $$\mathbf{C = -6.428 - \mathit{j}7.660}$$

14.10 MATHEMATICAL OPERATIONS WITH COMPLEX NUMBERS

Complex numbers lend themselves readily to the basic mathematical operations of addition, subtraction, multiplication, and division. A few basic rules and definitions must be understood before considering these operations.

Let us first examine the symbol j associated with imaginary numbers. By definition,

$$\boxed{j = \sqrt{-1}} \qquad \textbf{(14.27)}$$

Thus, $$\boxed{j^2 = -1} \qquad \textbf{(14.28)}$$

and $$j^3 = j^2 j = -1j = -j$$

with $$j^4 = j^2 j^2 = (-1)(-1) = +1$$
$$j^5 = j$$

and so on. Further,

$$\frac{1}{j} = (1)\left(\frac{1}{j}\right) = \left(\frac{j}{j}\right)\left(\frac{1}{j}\right) = \frac{j}{j^2} = \frac{j}{-1}$$

and $$\boxed{\frac{1}{j} = -j} \qquad \textbf{(14.29)}$$

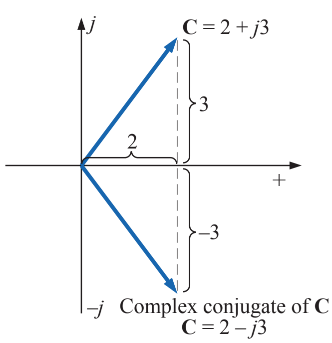


FIG. 14.50
Defining the complex conjugate of a complex number in rectangular form.

Complex Conjugate

The **conjugate** or **complex conjugate** of a complex number can be found by simply changing the sign of the imaginary part in the rectangular form or by using the negative of the angle of the polar form. For example, the conjugate of

$$\mathbf{C} = 2 + j3$$

is $$2 - j3$$

as shown in Fig. 14.50. The conjugate of

$$\mathbf{C} = 2 \angle 30°$$

is $$2 \angle -30°$$

as shown in Fig. 14.51.

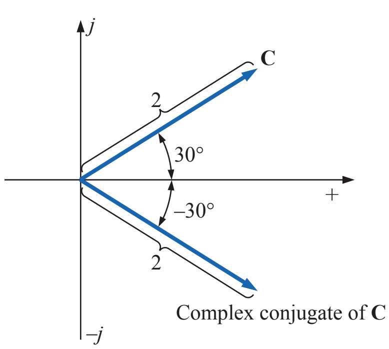


FIG. 14.51
Defining the complex conjugate of a complex number in polar form.

Reciprocal

The **reciprocal** of a complex number is 1 divided by the complex number. For example, the reciprocal of

$$\mathbf{C} = X + jY$$

is

$$\frac{1}{X + jY}$$

and of $Z \angle \theta$,

$$\frac{1}{Z \angle \theta}$$

We are now prepared to consider the four basic operations of *addition, subtraction, multiplication,* and *division* with complex numbers.

Addition

To add two or more complex numbers, simply add the real and imaginary parts separately. For example, if

$$\mathbf{C}_1 = \pm X_1 \pm jY_1 \quad \text{and} \quad \mathbf{C}_2 = \pm X_2 \pm jY_2$$

then

$$\boxed{\mathbf{C}_1 + \mathbf{C}_2 = (\pm X_1 \pm X_2) + j(\pm Y_1 \pm Y_2)} \qquad \textbf{(14.30)}$$

There is really no need to memorize the equation. Simply set one above the other and consider the real and imaginary parts separately, as shown in Example 14.19.

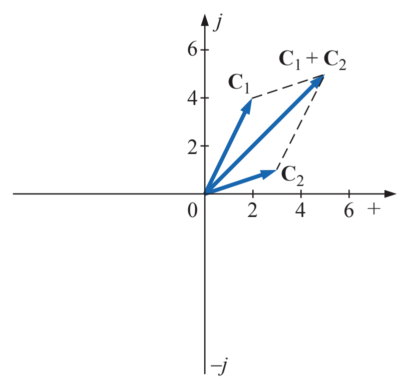


FIG. 14.52
Example 14.19(a).

EXAMPLE 14.19

a. Add $\mathbf{C}_1 = 2 + j4$ and $\mathbf{C}_2 = 3 + j1$.
b. Add $\mathbf{C}_1 = 3 + j6$ and $\mathbf{C}_2 = -6 + j3$.

Solutions:

a. By Eq. (14.30),

$$\mathbf{C}_1 + \mathbf{C}_2 = (2 + 3) + j(4 + 1) = \mathbf{5 + j5}$$

Note Fig. 14.52. An alternative method is

$$\begin{array}{c} 2 + j4 \\ \underline{3 + j1} \\ \downarrow \quad \downarrow \\ \mathbf{5 + j5} \end{array}$$

b. By Eq. (14.30),

$$\mathbf{C}_1 + \mathbf{C}_2 = (3 - 6) + j(6 + 3) = \mathbf{-3 + j9}$$

Note Fig. 14.53. An alternative method is

$$\begin{array}{c} 3 + j6 \\ \underline{-6 + j3} \\ \downarrow \quad \downarrow \\ \mathbf{-3 + j9} \end{array}$$

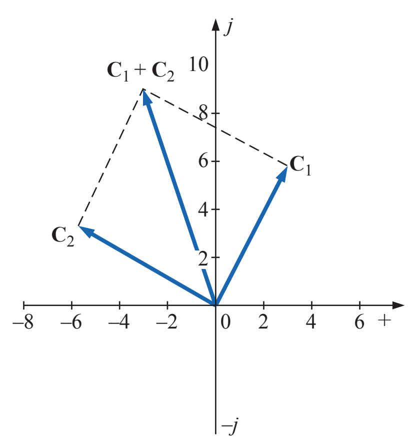


FIG. 14.53
Example 14.19(b).

Subtraction

In subtraction, the real and imaginary parts are again considered separately. For example, if

$$\mathbf{C}_1 = \pm X_1 \pm jY_1 \quad \text{and} \quad \mathbf{C}_2 = \pm X_2 \pm jY_2$$

then

$$\boxed{\mathbf{C}_1 - \mathbf{C}_2 = [\pm X_2 - (\pm X_2)] + j[\pm Y_1 - (\pm Y_2)]} \quad \textbf{(14.31)}$$

Again, there is no need to memorize the equation if the alternative method of Example 14.20 is employed.

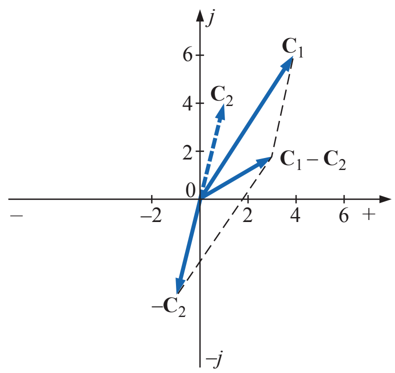


FIG. 14.54
Example 14.20(a).

EXAMPLE 14.20

a. Subtract $\mathbf{C}_2 = 1 + j4$ from $\mathbf{C}_1 = 4 + j6$.
b. Subtract $\mathbf{C}_2 = -2 + j5$ from $\mathbf{C}_1 = +3 + j3$.

Solutions:

a. By Eq. (14.31),

$$\mathbf{C}_1 - \mathbf{C}_2 = (4 - 1) + j(6 - 4) = \mathbf{3 + j2}$$

Note Fig. 14.54. An alternative method is

$$\begin{array}{c} 4 + j6 \\ \underline{-(1 + j4)} \\ \downarrow \quad \downarrow \\ \mathbf{3 + j2} \end{array}$$

b. By Eq. (14.31),

$$\mathbf{C}_1 - \mathbf{C}_2 = [3 - (-2)] + j(3 - 5) = \mathbf{5 - j2}$$

Note Fig. 14.55. An alternative method is

$$\begin{array}{c} 3 + j3 \\ \underline{-(-2 + j5)} \\ \downarrow \quad \downarrow \\ \mathbf{5 - j2} \end{array}$$

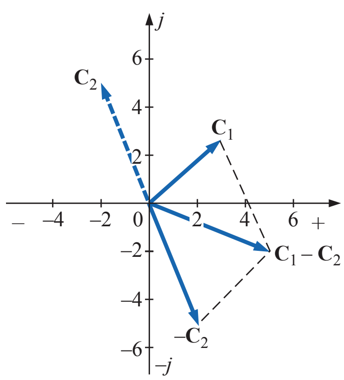


FIG. 14.55
Example 14.20(b).

Addition or subtraction cannot be performed in polar form unless the complex numbers have the same angle θ or unless they differ only by multiples of 180°.

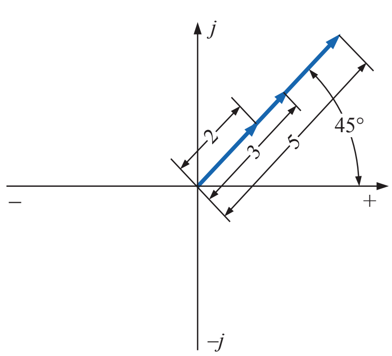


FIG. 14.56
Example 14.21(a).

EXAMPLE 14.21

a. $2\angle 45° + 3\angle 45° = \mathbf{5\angle 45°}$

   Note Fig. 14.56. Or

b. $2\angle 0° - 4\angle 180° = \mathbf{6\angle 0°}$

   Note Fig. 14.57.

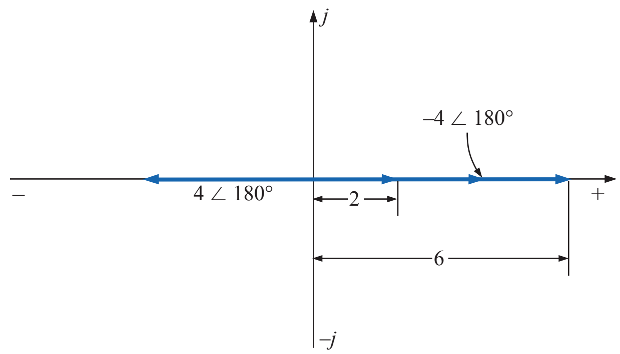


FIG. 14.57
Example 14.21(b).

Multiplication

To multiply two complex numbers in *rectangular* form, multiply the real and imaginary parts of one in turn by the real and imaginary parts of the other. For example, if

$$\mathbf{C}_1 = X_1 + jY_1 \quad \text{and} \quad \mathbf{C}_2 = X_2 + jY_2$$

then $\mathbf{C}_1 \cdot \mathbf{C}_2$:

$$\begin{array}{r} X_1 + jY_1 \\ \underline{X_2 + jY_2} \\ X_1X_2 + jY_1X_2 \\ \underline{+ jX_1Y_2 + j^2Y_1Y_2} \\ X_1X_2 + j(X_1Y_1X_2 + X_1Y_2) + Y_1Y_2(-1) \end{array}$$

and

$$\boxed{\mathbf{C}_1 \cdot \mathbf{C}_2 = (X_1X_2 - Y_1Y_2) + j(Y_1X_2 + X_1Y_2)} \qquad \textbf{(14.32)}$$

In Example 14.22(b), we obtain a solution without resorting to memorizing Eq. (14.32). Simply carry along the j factor when multiplying each part of one vector with the real and imaginary parts of the other.

EXAMPLE 14.22

a. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if

$$\mathbf{C}_1 = 2 + j3 \quad \text{and} \quad \mathbf{C}_2 = 5 + j10$$

b. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if

$$\mathbf{C}_1 = -2 - j3 \quad \text{and} \quad \mathbf{C}_2 = +4 - j6$$

Solutions:

a. Using the format above, we have

$$\begin{aligned} \mathbf{C}_1 \cdot \mathbf{C}_2 &= [(2)(5) - (3)(10)] + j[(3)(5) + (2)(10)] \\ &= \mathbf{-20 + j35} \end{aligned}$$

b. Without using the format, we obtain

$$\begin{array}{l} -2 - j3 \\ \underline{+4 - j6} \\ -8 - j12 \\ \underline{\quad + j12 + j^2 18} \\ -8 + j(-12 + 12) - 18 \end{array}$$

and

$$\mathbf{C}_1 \cdot \mathbf{C}_2 = \mathbf{-26 = 26 \angle 180°}$$

In *polar* form, the magnitudes are multiplied and the angles added algebraically. For example, for

$$\mathbf{C}_1 = Z_1 \angle\theta_1 \quad \text{and} \quad \mathbf{C}_2 = Z_2 \angle\theta_2$$

we write

$$\boxed{\mathbf{C}_1 \cdot \mathbf{C}_2 = Z_1 Z_2 \; \angle\theta_1 + \theta_2} \qquad (14.33)$$

EXAMPLE 14.23

a. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if

$$\mathbf{C}_1 = 5 \angle 20° \quad \text{and} \quad \mathbf{C}_2 = 10 \angle 30°$$

b. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if

$$\mathbf{C}_1 = 2 \angle -40° \quad \text{and} \quad \mathbf{C}_2 = 7 \angle +120°$$

Solutions:

a. $\mathbf{C}_1 \cdot \mathbf{C}_2 = (5 \angle 20°)(10 \angle 30°) = (5)(10) \; \angle 20° + 30° = \mathbf{50 \angle 50°}$

b. $\mathbf{C}_1 \cdot \mathbf{C}_2 = (2 \angle -40°)(7 \angle +120°) = (2)(7) \; \angle -40° + 120°$
$= \mathbf{14 \angle +80°}$

To multiply a complex number in rectangular form by a real number requires that both the real part and the imaginary part be multiplied by the real number. For example,

$$(10)(2 + j3) = 20 + j30$$

and $\quad 50 \angle 0°(0 + j6) = j300 = 300 \angle 90°$

Division

To divide two complex numbers in *rectangular* form, multiply the numerator and denominator by the conjugate of the denominator and the resulting real and imaginary parts collected. That is, if

$$\mathbf{C}_1 = X_1 + jY_1 \quad \text{and} \quad \mathbf{C}_2 = X_2 + jY_2$$

then

$$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{(X_1 + jY_1)(X_2 - jY_2)}{(X_2 + jY_2)(X_2 - jY_2)} = \frac{(X_1X_2 + Y_1Y_2) + j(X_2Y_1 - X_1Y_2)}{X_2^2 + Y_2^2}$$

and

$$\boxed{\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{X_1X_2 + Y_1Y_2}{X_2^2 + Y_2^2} + j\frac{X_2Y_1 - X_1Y_2}{X_2^2 + Y_2^2}} \qquad (14.34)$$

The equation does not have to be memorized if the steps above used to obtain it are employed. That is, first multiply the numerator by the complex conjugate of the denominator and separate the real and imaginary terms. Then divide each term by the sum of each term of the denominator squared.

EXAMPLE 14.24

a. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 1 + j4$ and $\mathbf{C}_2 = 4 + j5$.

b. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = -4 - j8$ and $\mathbf{C}_2 = +6 - j1$.

Solutions:

a. By Eq. (14.34),

$$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{(1)(4) + (4)(5)}{4^2 + 5^2} + j\frac{(4)(4) - (1)(5)}{4^2 + 5^2}$$

$$= \frac{24}{41} + \frac{j11}{41} \cong \mathbf{0.585 + \mathit{j}\,0.268}$$

b. Using an alternative method, we obtain

$$\begin{array}{l} \;\;-4 - j8 \\ \;\;\underline{+6 + j1} \\ -24 - j48 \\ \underline{\qquad -j4 - j^2 8} \\ -24 - j52 + 8 = -16 - j52 \end{array}$$

$$\begin{array}{l} +6 - j1 \\ \underline{+6 + j1} \\ 36 + j6 \\ \underline{\quad -j6 - j^2 1} \\ 36 + 0 + 1 \quad = 37 \end{array}$$

and $$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{-16}{37} - \frac{j52}{37} = \mathbf{-0.432 - \mathit{j}1.405}$$

To divide a complex number in rectangular form by a real number, both the real part and the imaginary part must be divided by the real number. For example,

$$\frac{8 + j10}{2} = 4 + j5$$

and $$\frac{6.8 - j0}{2} = 3.4 - j0 = 3.4 \angle 0°$$

In *polar* form, division is accomplished by simply dividing the magnitude of the numerator by the magnitude of the denominator and subtracting the angle of the denominator from that of the numerator. That is, for

$$\mathbf{C}_1 = Z_1 \angle \theta_1 \quad \text{and} \quad \mathbf{C}_2 = Z_2 \angle \theta_2$$

we write

$$\boxed{\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{Z_1}{Z_2} \angle \theta_1 - \theta_2} \qquad \textbf{(14.35)}$$

EXAMPLE 14.25

a. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 15 \angle 10°$ and $\mathbf{C}_2 = 2 \angle 7°$.
b. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 8 \angle 120°$ and $\mathbf{C}_2 = 16 \angle -50°$.

Solutions:

a. $$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{15 \angle 10°}{2 \angle 7°} = \frac{15}{2} \angle 10° - 7° = \mathbf{7.5 \angle 3°}$$

b. $\dfrac{\mathbf{C}_1}{\mathbf{C}_2} = \dfrac{8\angle 120^\circ}{16\angle -50^\circ} = \dfrac{8}{16}\underline{/120^\circ - (-50^\circ)} = \mathbf{0.5\angle 170^\circ}$

We obtain the *reciprocal* in the rectangular form by multiplying the numerator and denominator by the complex conjugate of the denominator:

$$\frac{1}{X + jY} = \left(\frac{1}{X + jY}\right)\left(\frac{X - jY}{X - jY}\right) = \frac{X - jY}{X^2 + Y^2}$$

and

$$\boxed{\frac{1}{X + jY} = \frac{X}{X^2 + Y^2} - j\frac{Y}{X^2 + Y^2}} \tag{14.36}$$

In polar form, the reciprocal is

$$\boxed{\frac{1}{Z\angle\theta} = \frac{1}{Z}\angle -\theta} \tag{14.37}$$

A concluding example using the four basic operations follows.

EXAMPLE 14.26 Perform the following operations, leaving the answer in polar or rectangular form:

a.
$$\begin{aligned}
\frac{(2 + j3) + (4 + j6)}{(7 + j7) - (3 - j3)} &= \frac{(2 + 4) + j(3 + 6)}{(7 - 3) + j(7 + 3)} \\
&= \frac{(6 + j9)(4 - j10)}{(4 + j10)(4 - j10)} \\
&= \frac{[(6)(4) + (9)(10)] + j[(4)(9) - (6)(10)]}{4^2 + 10^2} \\
&= \frac{114 - j24}{116} = \mathbf{0.983 - j0.207}
\end{aligned}$$

b.
$$\begin{aligned}
\frac{(50\angle 30^\circ)(5 + j5)}{10\angle -20^\circ} &= \frac{(50\angle 30^\circ)(7.07\angle 45^\circ)}{10\angle -20^\circ} = \frac{353.5\angle 75^\circ}{10\angle -20^\circ} \\
&= 35.35\,\underline{/75^\circ - (-20^\circ)} = \mathbf{35.35\angle 95^\circ}
\end{aligned}$$

c.
$$\begin{aligned}
\frac{(2\angle 20^\circ)^2(3 + j4)}{8 - j6} &= \frac{(2\angle 20^\circ)(2\angle 20^\circ)(5\angle 53.13^\circ)}{10\angle -36.87^\circ} \\
&= \frac{(4\angle 40^\circ)(5\angle 53.13^\circ)}{10\angle -36.87^\circ} = \frac{20\angle 93.13^\circ}{10\angle -36.87^\circ} \\
&= 2\,\underline{/93.13^\circ - (-36.87^\circ)} = \mathbf{2.0\angle 130^\circ}
\end{aligned}$$

d.
$$\begin{aligned}
3\angle 27^\circ - 6\angle -40^\circ &= (2.673 + j1.362) - (4.596 - j3.857) \\
&= (2.673 - 4.596) + j(1.362 + 3.857) \\
&= \mathbf{-1.923 + j5.219}
\end{aligned}$$

14.11 CALCULATOR AND COMPUTER METHODS WITH COMPLEX NUMBERS

The process of converting from one form to another or working through lengthy operations with complex numbers can be time-consuming and often frustrating if one lost minus sign or decimal point invalidates the solution. Fortunately, technologists of today have calculators and computer methods that make the process measurably easier with higher degrees of reliability and accuracy.

Calculators

The TI-86 calculator of Fig. 14.58 is only one of numerous calculators that can convert from one form to another and perform lengthy calculations with complex numbers in a concise, neat form. Not all of the details of using a specific calculator will be included here because each has its own format and sequence of steps. However, the basic operations with the TI-86 will be included primarily to demonstrate the ease with which the conversions can be made and the format for more complex operations.

For the TI-86 calculator, one must first call up the 2nd function CPLX from the keyboard, which results in a menu at the bottom of the display including conj, real, imag, abs, and angle. If we choose the key MORE, ▶ Rec and ▶ Pol will appear as options (for the conversion process). To convert from one form to another, simply enter the current form in brackets with a comma between components for the rectangular form and an angle symbol for the polar form. Follow this form with the operation to be performed, and press the ENTER key—the result will appear on the screen in the desired format.

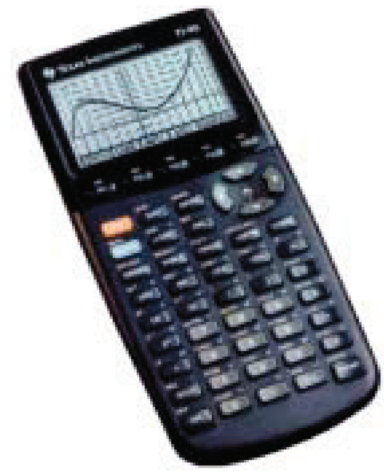


FIG. 14.58
TI-86 scientific calculator. (Courtesy of Texas Instruments, Inc.)

EXAMPLE 14.27 This example is for demonstration purposes only. It is not expected that all readers will have a TI-86 calculator. The sole purpose of the example is to demonstrate the power of today's calculators.

Using the TI-86 calculator, perform the following conversions:

a. $3 - j4$ to polar form.
b. $0.006 \angle 20.6°$ to rectangular form.

Solutions:

a. The TI-86 display for part (a) is the following:

(3, −4) ▶ Pol ENTER
(5.000E0∠−53.130E0)

CALC. 14.1

b. The TI-86 display for part (b) is the following:

(0.006∠20.6) ▶ Rec ENTER
(5.616E−3, 2.111E−3)

CALC. 14.2

EXAMPLE 14.28 Using the TI-86 calculator, perform the desired operations required in part (c) of Example 14.26, and compare solutions.

Solution: One must now be aware of the hierarchy of mathematical operations. In other words, in which sequence will the calculator perform the desired operations? In most cases, the sequence is the same as that used in longhand calculations, although one must become adept at setting up the parentheses to ensure the correct order of operations. For this example, the TI-86 display is the following:

((2∠20)^2∗(3,4))/(8,−6)► Pol (ENTER)
(2.000E0∠130.000E0)

CALC. 14.3

which is a perfect match with the earlier solution.

Mathcad

The Mathcad format for complex numbers will now be introduced in preparation for the chapters to follow. We will continue to use j when we define a complex number in rectangular form even though the Mathcad result will always appear with the letter i. You can change this by going to the **Format** menu, but for this presentation we decided to use the default operators as much as possible.

When entering j to define the imaginary component of a complex number, be sure to enter it as 1j; but do not put a multiplication operator between the 1 and the j. Just type 1 and then j. In addition, place the j after the constant rather than before as in the text material.

When Mathcad operates on an angle, it will assume that the angle is in radians and not degrees. Further, all results will appear in radians rather than degrees.

The first operation to be developed is the conversion from rectangular to polar form. In Fig. 14.59 the rectangular number $4 + j\,3$ is being converted to polar form using Mathcad. First X and Y are defined using the colon operator. Next the equation for the magnitude of the polar form is written in terms of the two variables just defined. The magnitude of the polar form is then revealed by writing the variable again and using the equal sign. It will take some practice, but be careful when writing the equation for Z in the sense that you pay particular attention to the location of the bracket before performing the next operation. The resulting magnitude of 5 is as expected.

For the angle, the sequence **View-Toolbars-Greek** is first applied to obtain the **Greek** toolbar appearing in Fig. 14.59. It can be moved to any location by simply clicking on the blue at the top of the toolbar and dragging it to the preferred location. Then 0 is selected from the toolbar as the variable to be defined. The $\tan^{-1}\theta$ is obtained through the sequence **Insert-*f(x)*-Insert Function** dialog box-**trigonometric-atan-OK** in which Y/X is inserted. Then bring the controlling bracket to the outside of the entire expression, and multiply by the ratio of $180/\pi$ with π selected from the **Calculator** toolbar (available from the same sequence used to obtain the **Greek** toolbar). The multiplication by the last factor of the equation will ensure that the angle is in degrees. Selecting θ again followed by an equal sign will result in the correct angle of 36.87° as shown in Fig. 14.59.

We will now look at two forms for the polar form of a complex number. The first is defined by the basic equations introduced in this chapter, while the second uses a special format. For all the Mathcad analyses to be provided in this text, the latter format will be employed. First

German-American
(Breslau, Germany; Yonkers and Schenectady, NY, USA)
(1865–1923)
Mathematician, Scientist, Engineer, Inventor, Professor of Electrical Engineering and Electrophysics, Union College
Department Head, General Electric Co.

Courtesy of the Hall of History Foundation, Schenectady, New York

Although the holder of some 200 patents and recognized worldwide for his contributions to the study of hysteresis losses and electrical transients, Charles Proteus Steinmetz is best recognized for his contribution to the study of ac networks. His "Symbolic Method of Alternating-current Calculations" provided an approach to the analysis of ac networks that removed a great deal of the confusion and frustration experienced by engineers of that day as they made the transition from dc to ac systems. His approach (from which the phasor notation of this text is premised) permitted a direct analysis of ac systems using many of the theorems and methods of analysis developed for dc systems. In 1897 he authored the epic work *Theory and Calculation of Alternating Current Phenomena*, which became the "bible" for practicing engineers. Dr. Steinmetz was fondly referred to as "The Doctor" at General Electric Company where he worked for some 30 years in a number of important capacities. His recognition as a "multigifted genius" is supported by the fact that he maintained active friendships with such individuals as Albert Einstein, Guglielmo Marconi (radio), and Thomas A. Edison, to name just a few. He was President of the American Institute of Electrical Engineers (AIEE) and the National Association of Corporation Schools and actively supported his local community (Schenectady) as president of the Board of Education and the Commission on Parks and City Planning.

CHARLES PROTEUS STEINMETZ

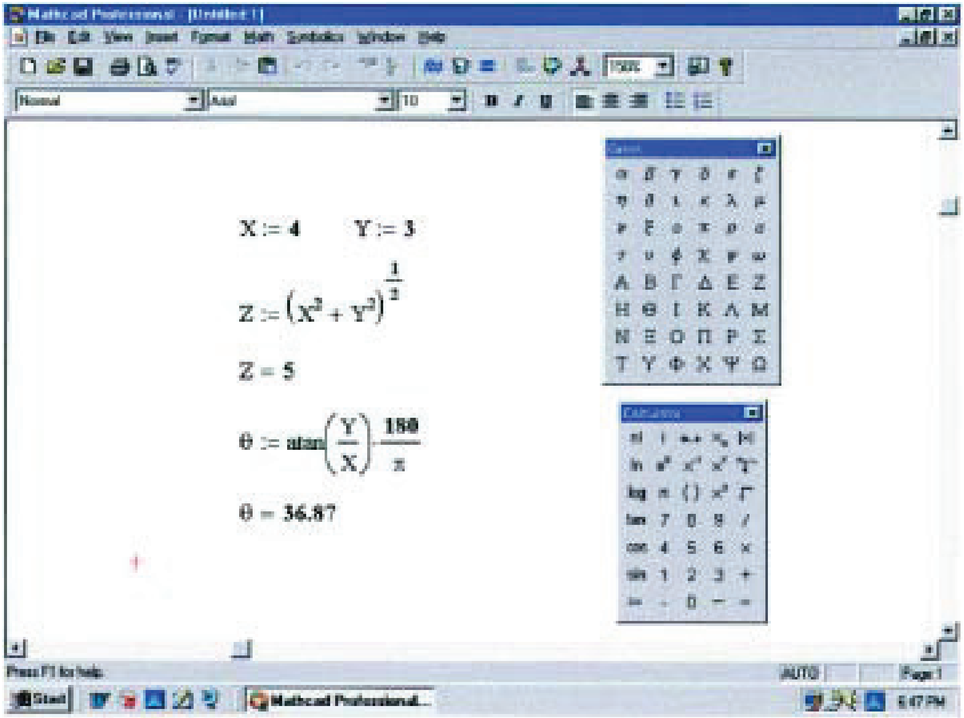


FIG. 14.59
Using Mathcad to convert from rectangular to polar form.

the magnitude of the polar form is defined followed by the conversion of the angle of 60° to radians by multiplying by the factor $\pi/180$ as shown in Fig. 14.60. In this example the resulting angular measure is $\pi/3$ radians. Next the rectangular format is defined by a real part $X = Z \cos \theta$ and by an imaginary part $Y = Z \sin \theta$. Both the cos and the sin are obtained by the sequence **Insert-*f(x)*-trigonometric-cos(**or **sin)-OK.** Note the multiplication by j which was actually entered as $1j$. Entering C again followed by an equal sign will result in the correct conversion shown in Fig. 14.60.

The next format is based on the mathematical relationship that $e^{j\theta} = \cos \theta + j \sin \theta$. Both Z and θ are as defined above, but now the complex number is written as shown in Fig. 14.60 using the notation just introduced. Note that both Z and θ are part of this defining form. The e^x is obtained directly from the **Calculator** toolbar. Remember to enter the j as $1j$ without a multiplication sign between the 1 and the j. However, there is a multiplication operator placed between the j and θ. When entered again followed by an equal sign, the rectangular form appears to match the above results. As mentioned above, it is this latter format that will be used throughout the text due to its cleaner form and more direct entering path.

The last example using Mathcad will be a confirmation of the results of Example 14.26(b) as shown in Fig. 14.61. The three complex numbers are first defined as shown. Then the equation for the desired result

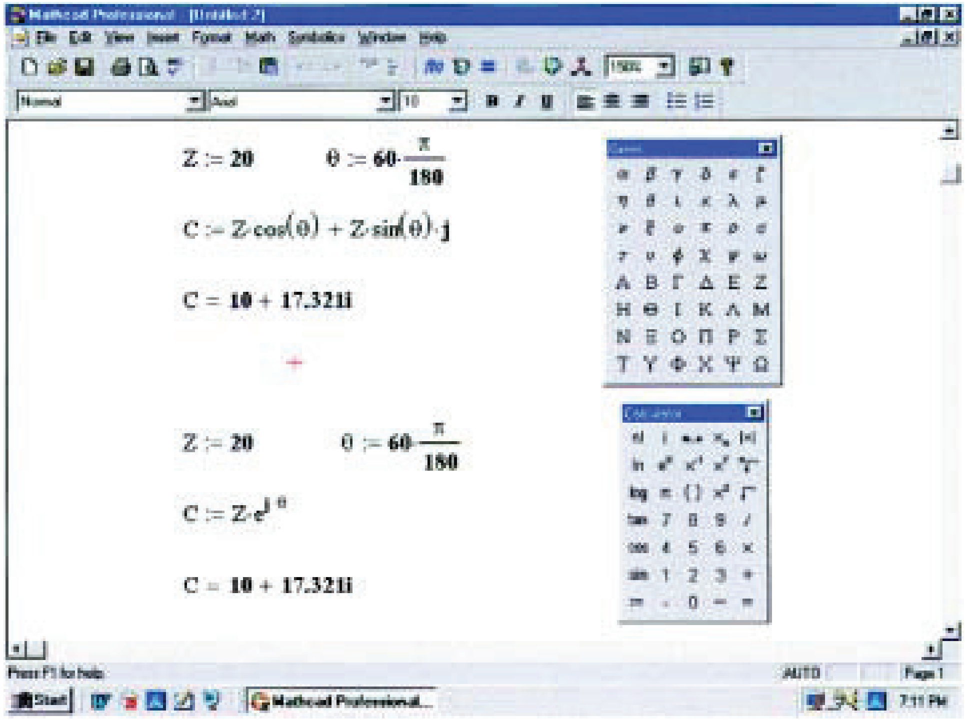


FIG. 14.60
Using Mathcad to convert from polar to rectangular form.

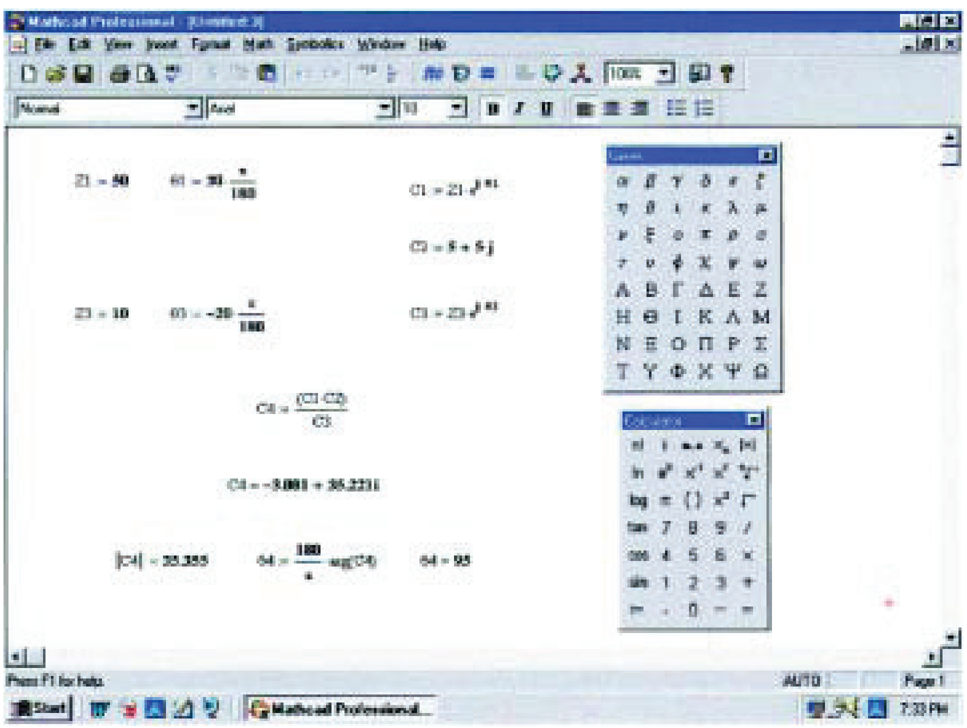


FIG. 14.61
Using Mathcad to confirm the results of Example 14.26(b).

is entered using C_4, and finally the results are called for. Note the relative simplicity of the equation for C_4 now that all the other variables have been defined. As shown, however, the immediate result is in the rectangular form using the magnitude feature from the calculator and the **arg** function from **Insert-*f(x)*-Complex Numbers-arg.** There will be a number of other examples in the chapters to follow on the use of Mathcad with complex numbers.

14.12 PHASORS

As noted earlier in this chapter, the addition of sinusoidal voltages and currents will frequently be required in the analysis of ac circuits. One lengthy but valid method of performing this operation is to place both sinusoidal waveforms on the same set of axes and add algebraically the magnitudes of each at every point along the abscissa, as shown for $c = a + b$ in Fig. 14.62. This, however, can be a long and tedious process with limited accuracy. A shorter method uses the rotating radius vector first appearing in Fig. 13.16. This *radius vector,* having a *constant magnitude* (length) with *one end fixed at the origin,* is called a **phasor** when applied to electric circuits. During its rotational development of the sine wave, the phasor will, at the instant $t = 0$, have the positions shown in Fig. 14.63(a) for each waveform in Fig. 14.63(b).

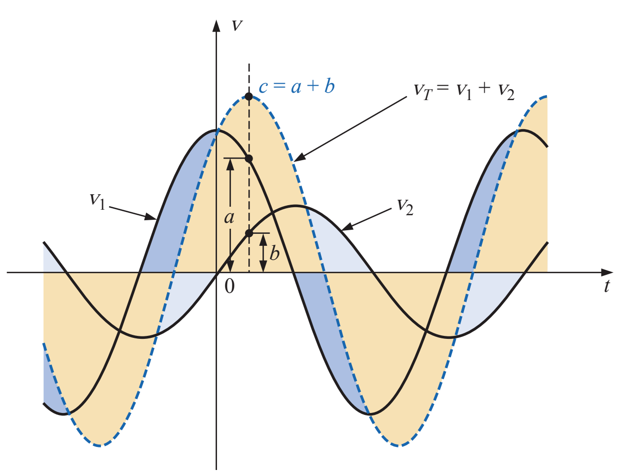


FIG. 14.62
Adding two sinusoidal waveforms on a point-by-point basis.

Note in Fig. 14.63(b) that v_2 passes through the horizontal axis at $t = 0$ s, requiring that the radius vector in Fig. 14.63(a) be on the horizontal axis to ensure a vertical projection of zero volts at $t = 0$ s. Its length in Fig. 14.63(a) is equal to the peak value of the sinusoid as required by the radius vector of Fig. 13.16. The other sinusoid has passed through 90° of its rotation by the time $t = 0$ s is reached and

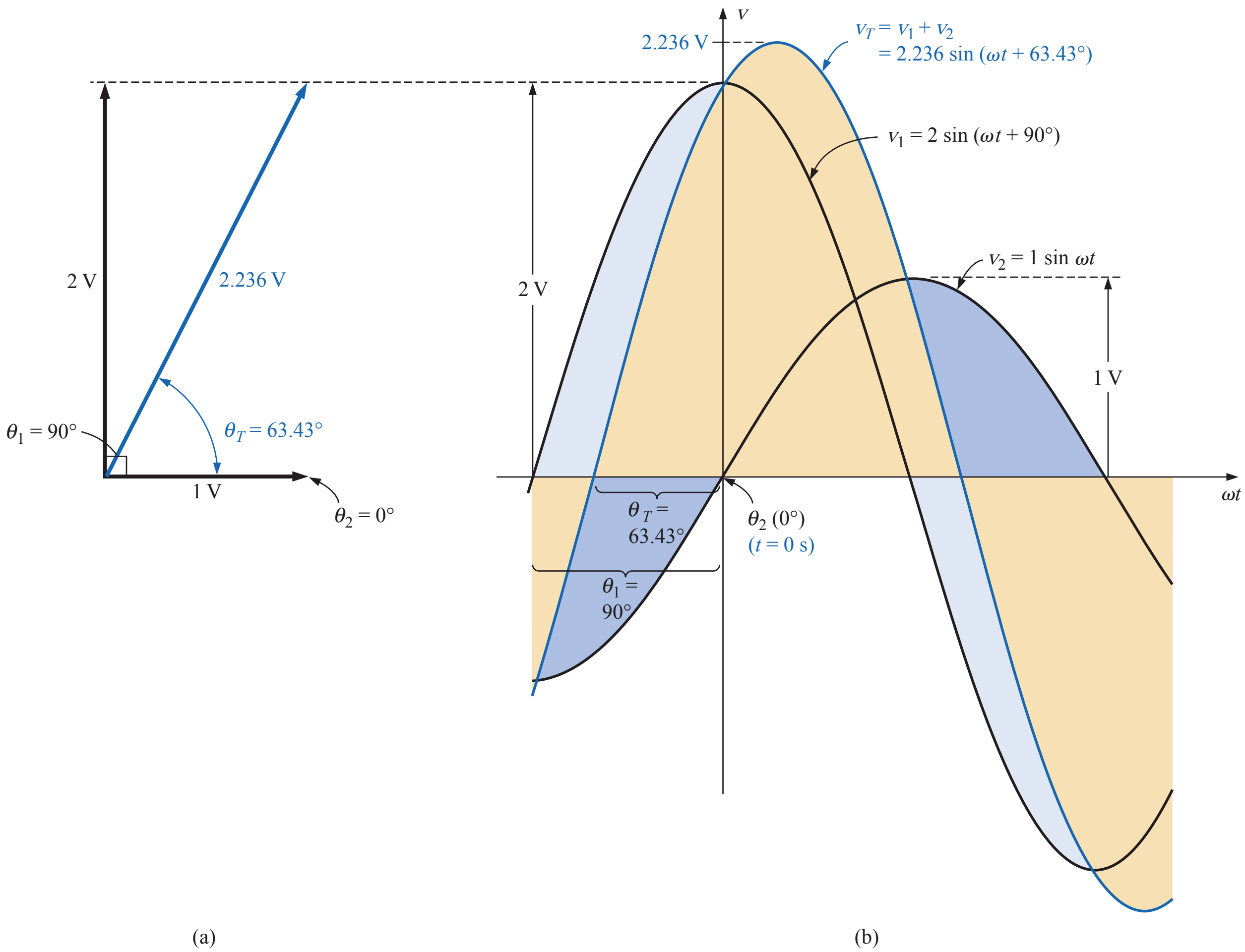


FIG. 14.63
(a) The phasor representation of the sinusoidal waveforms of Fig. 14.63(b); (b) finding the sum of two sinusoidal waveforms of v_1 and v_2.

therefore has its maximum vertical projection as shown in Fig. 14.63(a). Since the vertical projection is a maximum, the peak value of the sinusoid that it will generate is also attained at $t = 0$ s, as shown in Fig. 14.63(b). Note also that $v_T = v_1$ at $t = 0$ s since $v_2 = 0$ V at this instant.

It can be shown [see Fig. 14.63(a)] using the vector algebra described in Section 14.10 that

$$1 \text{ V} \angle 0° + 2 \text{ V} \angle 90° = 2.236 \text{ V} \angle 63.43°$$

In other words, if we convert v_1 and v_2 to the phasor form using

$$v = V_m \sin(\omega t \pm \theta) \Rightarrow V_m \angle \pm\theta$$

and add them using complex number algebra, we can find the phasor form for v_T with very little difficulty. It can then be converted to the time domain and plotted on the same set of axes, as shown in Fig. 14.63(b). Figure 14.63(a), showing the magnitudes and relative positions of the various phasors, is called a **phasor diagram.** It is actually a “snapshot” of the rotating radius vectors at $t = 0$ s.

In the future, therefore, if the addition of two sinusoids is required, they should first be converted to the phasor domain and the sum found using complex algebra. The result can then be converted to the time domain.

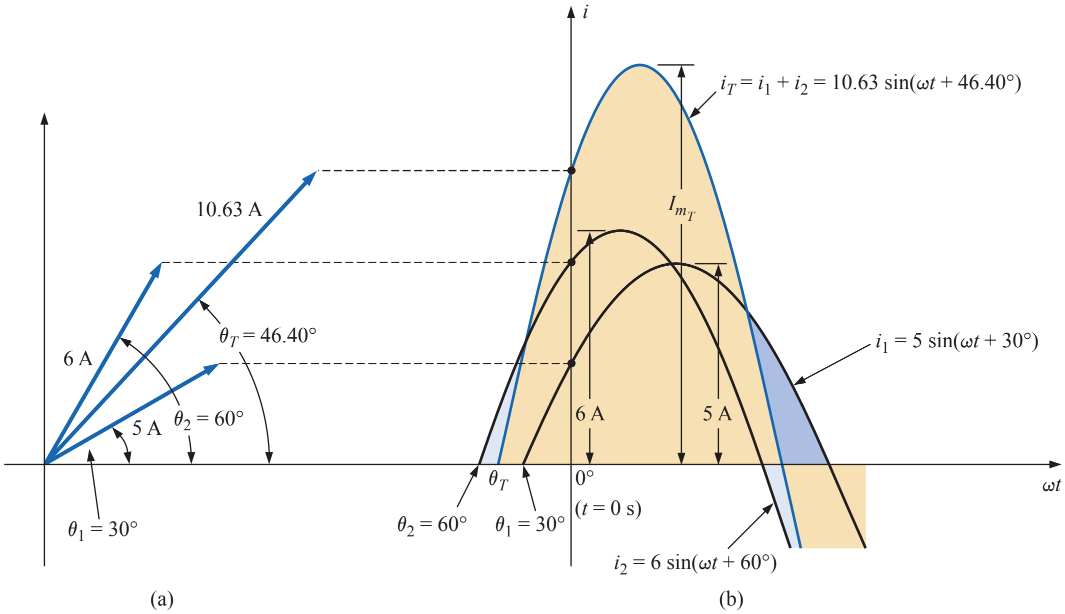


FIG. 14.64
Adding two sinusoidal currents with phase angles other than 90°.

The case of two sinusoidal functions having phase angles different from 0° and 90° appears in Fig. 14.64. Note again that the vertical height of the functions in Fig. 14.64(b) at $t = 0$ s is determined by the rotational positions of the radius vectors in Fig. 14.64(a).

Since the rms, rather than the peak, values are used almost exclusively in the analysis of ac circuits, the phasor will now be redefined for the purposes of practicality and uniformity as having a magnitude equal to the *rms value* of the sine wave it represents. The angle associated with the phasor will remain as previously described—the phase angle.

In general, for all of the analyses to follow, the phasor form of a sinusoidal voltage or current will be

$$\mathbf{V} = V \angle \theta \quad \text{and} \quad \mathbf{I} = I \angle \theta$$

where V and I are rms values and θ is the phase angle. It should be pointed out that in phasor notation, the sine wave is always the reference, and the frequency is not represented.

Phasor algebra for sinusoidal quantities is applicable only for waveforms having the same frequency.

EXAMPLE 14.29 Convert the following from the time to the phasor domain:

Time Domain	Phasor Domain
a. $\sqrt{2}(50) \sin \omega t$	$\mathbf{50 \angle 0°}$
b. $69.6 \sin(\omega t + 72°)$	$(0.707)(69.6) \angle 72° = \mathbf{49.21 \angle 72°}$
c. $45 \cos \omega t$	$(0.707)(45) \angle 90° = \mathbf{31.82 \angle 90°}$

EXAMPLE 14.30 Write the sinusoidal expression for the following phasors if the frequency is 60 Hz:

Phasor Domain	Time Domain
a. $\mathbf{I} = 10 \angle 30°$	$i = \sqrt{2}(10)\sin(2\pi 60t + 30°)$ and $i = \mathbf{14.14\ sin(377}\boldsymbol{t}\mathbf{ + 30°)}$
b. $\mathbf{V} = 115 \angle -70°$	$v = \sqrt{2}(115)\sin(377t - 70°)$ and $v = \mathbf{162.6\ sin(377}\boldsymbol{t}\mathbf{ - 70°)}$

EXAMPLE 14.31 Find the input voltage of the circuit of Fig. 14.65 if

$$\left.\begin{aligned} v_a &= 50\sin(377t + 30°) \\ v_b &= 30\sin(377t + 60°) \end{aligned}\right\} f = 60\text{ Hz}$$

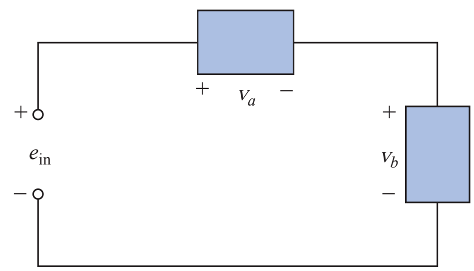


FIG. 14.65
Example 14.31.

Solution: Applying Kirchhoff's voltage law, we have

$$e_{in} = v_a + v_b$$

Converting from the time to the phasor domain yields

$$v_a = 50\sin(377t + 30°) \Rightarrow \mathbf{V}_a = 35.35\text{ V} \angle 30°$$
$$v_b = 30\sin(377t + 60°) \Rightarrow \mathbf{V}_b = 21.21\text{ V} \angle 60°$$

Converting from polar to rectangular form for addition yields

$$\mathbf{V}_a = 35.35\text{ V} \angle 30° = 30.61\text{ V} + j\,17.68\text{ V}$$
$$\mathbf{V}_b = 21.21\text{ V} \angle 60° = 10.61\text{ V} + j\,18.37\text{ V}$$

Then

$$\begin{aligned} \mathbf{E}_{in} = \mathbf{V}_a + \mathbf{V}_b &= (30.61\text{ V} + j\,17.68\text{ V}) + (10.61\text{ V} + j\,18.37\text{ V}) \\ &= 41.22\text{ V} + j\,36.05\text{ V} \end{aligned}$$

Converting from rectangular to polar form, we have

$$\mathbf{E}_{in} = 41.22\text{ V} + j\,36.05\text{ V} = 54.76\text{ V} \angle 41.17°$$

Converting from the phasor to the time domain, we obtain

$$\mathbf{E}_{in} = 54.76\text{ V} \angle 41.17° \Rightarrow e_{in} = \sqrt{2}(54.76)\sin(377t + 41.17°)$$

and $$e_{in} = \mathbf{77.43\ sin(377}\boldsymbol{t}\mathbf{ + 41.17°)}$$

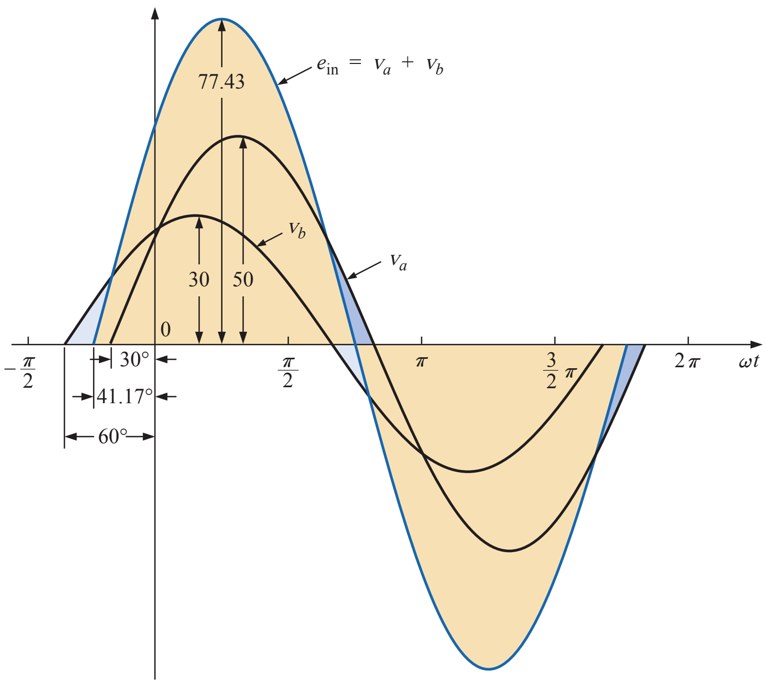


FIG. 14.66
Solution to Example 14.31.

A plot of the three waveforms is shown in Fig. 14.66. Note that at each instant of time, the sum of the two waveforms does in fact add up to e_{in}. At $t = 0$ ($\omega t = 0$), e_{in} is the sum of the two positive values, while at a value of ωt, almost midway between $\pi/2$ and π, the sum of the positive value of v_a and the negative value of v_b results in $e_{in} = 0$.

EXAMPLE 14.32 Determine the current i_2 for the network of Fig. 14.67.

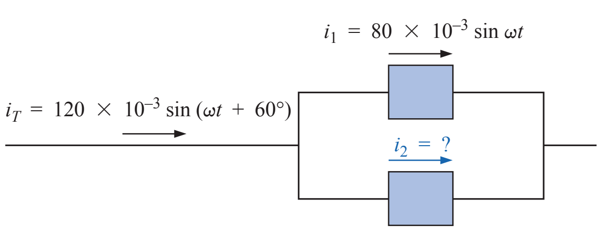


FIG. 14.67
Example 14.32.

Solution: Applying Kirchhoff's current law, we obtain

$$i_T = i_1 + i_2 \quad \text{or} \quad i_2 = i_T - i_1$$

Converting from the time to the phasor domain yields

$$i_T = 120 \times 10^{-3} \sin(\omega t + 60°) \Rightarrow 84.84 \text{ mA } \angle 60°$$
$$i_1 = 80 \times 10^{-3} \sin \omega t \Rightarrow 56.56 \text{ mA } \angle 0°$$

Converting from polar to rectangular form for subtraction yields

$$\mathbf{I}_T = 84.84 \text{ mA} \angle 60° = 42.42 \text{ mA} + j\,73.47 \text{ mA}$$
$$\mathbf{I}_1 = 56.56 \text{ mA} \angle 0° = 56.56 \text{ mA} + j\,0$$

Then

$$\begin{aligned}\mathbf{I}_2 &= \mathbf{I}_T - \mathbf{I}_1\\ &= (42.42 \text{ mA} + j\,73.47 \text{ mA}) - (56.56 \text{ mA} + j\,0)\end{aligned}$$

and $$\mathbf{I}_2 = -14.14 \text{ mA} + j\,73.47 \text{ mA}$$

Converting from rectangular to polar form, we have

$$\mathbf{I}_2 = 74.82 \text{ mA} \angle 100.89°$$

Converting from the phasor to the time domain, we have

$$\begin{aligned}\mathbf{I}_2 &= 74.82 \text{ mA} \angle 100.89° \Rightarrow\\ i_2 &= \sqrt{2}(74.82 \times 10^{-3}) \sin(\omega t + 100.89°)\end{aligned}$$

and $$\boldsymbol{i_2 = 105.8 \times 10^{-3} \sin(\omega t + 100.89°)}$$

A plot of the three waveforms appears in Fig. 14.68. The waveforms clearly indicate that $i_T = i_1 + i_2$.

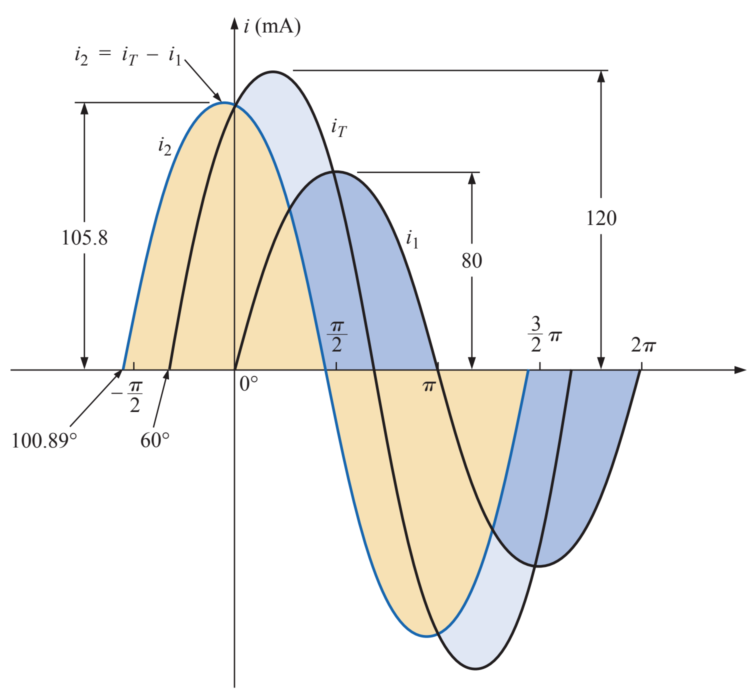


FIG. 14.68
Solution to Example 14.32.

14.13 COMPUTER ANALYSIS

PSpice

Capacitors and the ac Response The simplest of ac capacitive circuits will now be analyzed to introduce the process of setting up an ac source and running an ac transient simulation. The ac source of Fig.

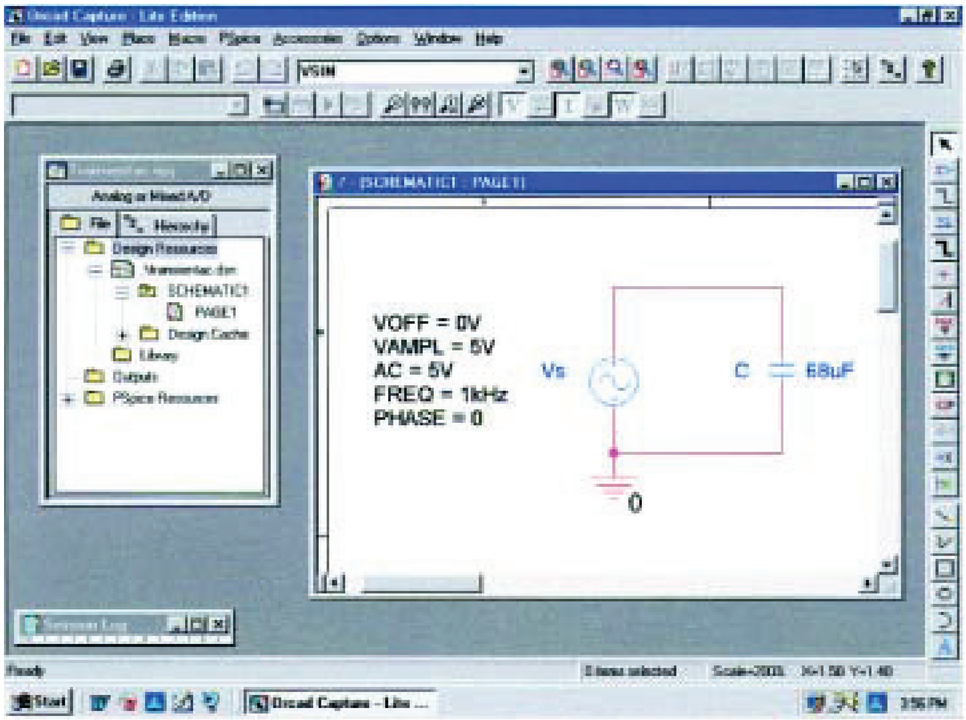


FIG. 14.69
Using PSpice to analyze the response of a capacitor to a sinusoidal ac signal.

14.69 is obtained through **Place part** key-**SOURCE-VSIN-OK.** The name or value of any parameter can be changed by simply double-clicking on the parameter on the display or by double-clicking on the source symbol to get the **Property Editor** dialog box. Within the dialog box the values appearing in Fig. 14.69 were set, and under **Display, Name and Value** were selected. After you have selected **Apply** and exited the dialog box, the parameters will appear as shown in the figure.

The simulation process is initiated by selecting the **New Simulation Profile** and under **New Simulation** entering **Transientac** for the **Name** followed by **Create.** In the **Simulation Settings** dialog box, **Analysis** is selected and **Time Domain(Transient)** is chosen under **Analysis type.** The **Run to time** will be set at 3 ms to permit a display of three cycles of the sinusoidal waveforms ($T = 1/f = 1/1000$ Hz $= 1$ ms). The **Start saving data after** will be left at 0 s, and the **Maximum step size** will be 3 ms/1000 $= 3\ \mu$s. Clicking **OK** and then selecting the **Run PSpice** icon will result in a plot having a horizontal axis that extends from 0 to 3 ms.

Now we have to tell the computer which waveforms we are interested in. First, we should take a look at the applied ac source by selecting **Trace-Add Trace-V(Vs:+)** followed by **OK.** The result is the sweeping ac voltage in the botttom region of the screen of Fig. 14.70. Note that it has a peak value of 5 V, and three cycles appear in the 3-ms time frame. The current for the capacitor can be added by selecting **Trace-Add Trace** and choosing **I(C)** followed by **OK.** The resulting waveform for **I(C)** appears at a 90° phase shift from the applied voltage, with the current leading the voltage (the current has already peaked

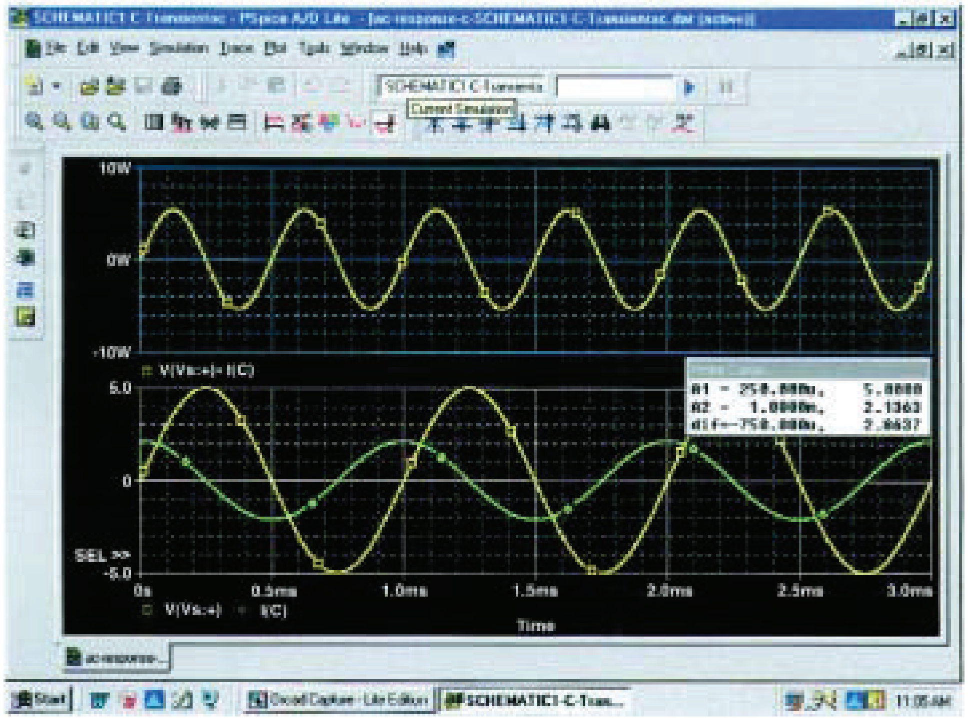


FIG. 14.70
A plot of the voltage, current, and power for the capacitor of Fig. 14.69.

as the voltage crosses the 0-V axis). Since the peak value of each plot is in the same magnitude range, the 5 appearing on the vertical scale can be used for both. A theoretical analysis would result in $X_C = 2.34\ \Omega$, and the peak value of $I_C = E/X_C = 5\ \text{V}/2.34\ \Omega = 2.136\ \text{A}$, as shown in Fig. 14.70.

For interest sake, and a little bit of practice, let us obtain the curve for the power delivered to the capacitor over the same time period. First select **Plot-Add Plot to Window-Trace-Add Trace** to obtain the **Add Traces** dialog box. Then chose **V(Vs:+)**, follow it with a * for multiplication, and finish by selecting **I(C).** The result is the expression **V(Vs:+)*I(C)** of the power format: $p = vi$. Click **OK,** and the power plot at the top of Fig 14.70 will appear. Note that over the full three cycles, the area above the axis equals the area below—there is no net transfer of power over the 3-ms period. Note also that the power curve is sinusoidal (which is quite interesting) with a frequency twice that of the applied signal. Using the cursor control, we can determine that the maximum power (peak value of the sinusoidal waveform) is 5.34 W. The cursors, in fact, have been added to the lower curves to show the peak value of the applied sinusoid and the resulting current.

After selecting the **Toggle cursor** icon, left-click the mouse to surround the **V(Vs:+)** at the bottom of the plot with a dashed line to show that the cursor is providing the levels of that quantity. When placed at $\frac{1}{4}$ of the total period of 250 μs **(A1),** the peak value is exactly 5 V as shown in the **Probe Cursor** dialog box. Placing the cursor over the symbol next to **I(C)** at the bottom of the plot and right-clicking the mouse will assign the right cursor to the current. Placing it at exactly 1 ms **(A2)** will result in a peak value of 2.136 A to match the solution above. To further distinguish between the voltage and current waveforms, the color and the width of the lines of the traces were changed. Place the cursor right on the plot line and perform a right click. Then the **Properties** option appears. When **Properties** is selected, a **Trace Properties** dialog box will appear in which the yellow color can be selected and the width widened to improve the visibility on the black background. Note that yellow was chosen for **Vs** and green for **I(C)**. Note also that the axis and the grid have been changed to a more visible color using the same procedure.

Electronics Workbench

Since PSpice reviewed the response of a capacitive element to an ac voltage, Electronics Workbench will repeat the analysis for an inductive element. The ac voltage source was derived from the **Sources** parts bin as described in Chapter 13 with the values appearing in Fig. 14.71 set in the **AC Voltage** dialog box. Since the transient response of Electronics Workbench is limited to a plot of voltage versus time, a plot of the current of the circuit will require the addition of a resistor of 1 Ω in series

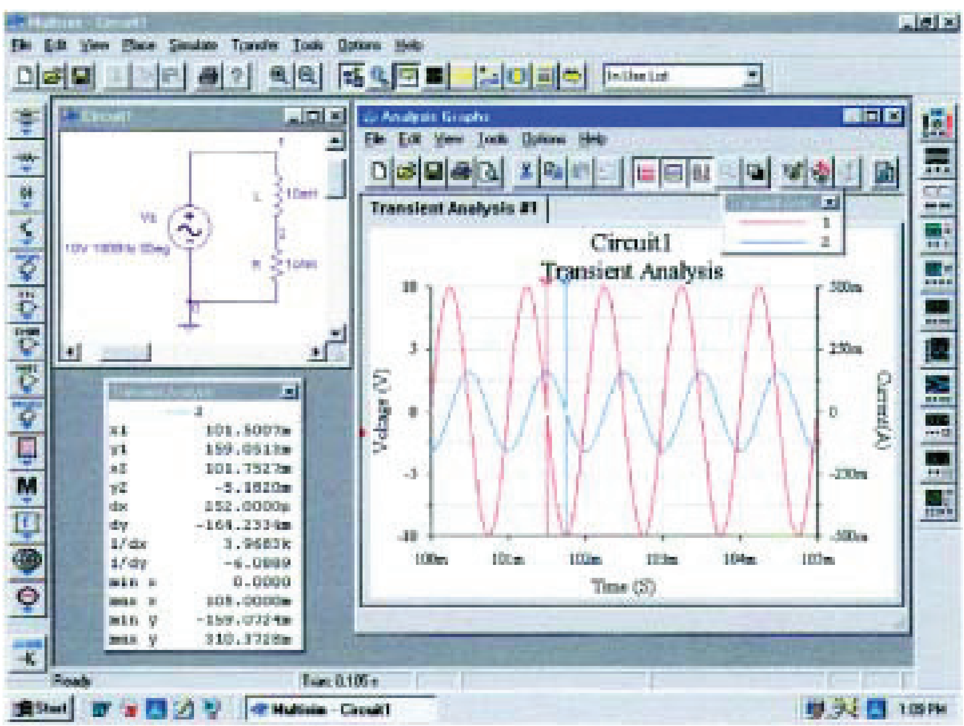


FIG. 14.71
Using Electronics Workbench to review the response of an inductive element to a sinusoidal ac signal.

with the inductive element. The magnitude of the current through the resistor and, of course, the series inductor will then be determined by

$$|i_R| = \left|\frac{v_R}{R}\right| = \left|\frac{v_R}{1\Omega}\right| = |v_R| = |i_L|$$

revealing that the current will have the same peak value as the voltage across the resistor due to the division by 1. When viewed on the graph, it can simply be considered a plot of the current. In actuality, all inductors require a series resistance, so the 1-Ω resistor serves an important dual purpose. The 1-Ω resistance is also so small compared to the reactance of the coil at the 1-kHz frequency that its effect on the total impedance or voltage across the coil can be ignored.

Once the circuit has been constructed, the sequence **Simulate-Analyses-Transient Analysis** will result in a **Transient Analysis** dialog box in which the **Start time** is set at 0 s and the **End time** at 105 ms. The 105 ms was set as the **End time** to give the network 100 ms to settle down in its steady-state mode and 5 ms for five cycles in the output display. The **Minimum number of time points** was set at 10,000 to ensure a good display for the rapidly changing waveforms.

Next the **Output variables** heading was chosen within the dialog box, and nodes **1** and **2** were moved from the **Variables in Circuit** to **Selected variables** for analysis using the **Plot during simulation** key pad. Choosing **Simulate** will then result in a waveform that extends from 0 s to 105 ms. Even though we plan to save only the response that occurs after 100 ms, the computer is unaware of our interest, and it plots the response for the entire period, This is corrected by selecting the **Properties** key pad in the toolbar at the top of the graph (it looks like a tag and pencil) to obtain the **Graph Properties** dialog box. Selecting **Bottom Axis** will permit setting the **Range** from a **Minimum of 0.100s=100ms** to a **Maximum of 0.105s=105ms.** Click **OK**, and the time period of Fig. 14.71 will be displayed. The grid structure is added by selecting the **Show/Hide Grid** key pad, and the color associated with each nodal voltage will be displayed if we choose the **Show/Hide Legend** key next to it.

The scale for the plot of i_L can be improved by first going to **Traces** and setting the **Trace** to the number **2** representing the voltage across the 1-Ω resistor. When **2** is selected, the **Color** displayed will automaticaly change to blue. In the **Y Range**, select **Right Axis** followed by **OK.** Then select the **Right Axis** heading, and enter **Current(A)** for the **Label**, enable **Axis**, change the **Pen Size** to 1, and change the **Range** from −500 mA to +500 mA. Finally, set the **Total Ticks** at 8 with **Minor Ticks** at 2 to match the **Left Axis**, and leave the box with an **OK.** The plot of Fig. 14.71 will result. Take immediate note of the new axis on the right and the **Current(A)** label. We can now see that the current has a peak of about 160 mA. For more detail on the peak values, simply click on the **Show/Hide Cursors** key pad on the top toolbar. A **Transient Analysis** dialog box will appear with a **1** and a red line to indicate that it is working on the full source voltage at node **1.** To switch to the current curve (the blue curve), simply bring the cursor to any point on the blue curve and perform a left click. A blue line and the number **2** will appear at the heading of the **Transient Analysis** dialog box. Clicking on the **1** in the small inverted arrow at the top will allow you to drag the vertical red line to any horizontal point on the graph. As shown in Fig. 14.71, when the cursor is set on 101.5 ms **(x1),** the peak value of the current curve is 159.05 mA **(y1)**. A second cursor appears in blue with a number **2** in the inverted arrowhead that can also be moved with a left click on the number **2** at the top of the line. If set at 101.75 ms **(x2),** it has a minimum value of -5.18 mA **(y2)**, the smallest value available for the calculated data points. Note that the difference between horizontal time values **dx** $= 252\ \mu s = 0.25$ ms which is ¼ of the period of the wave (at 1 ms).

C++

The versatility of the C++ programming language is clearly demonstrated by the following program designed to perform conversions between the polar and rectangular forms. Comments are provided on the right side of the program to help identify the function of specific lines or sections of the program. Recall that any comments to the right of the parallel slash bars // are ignored by the compiler. In this case the file *math.h* must be added to the preprocessor directive list, as shown in Fig. 14.72, to provide the mathematical functions to be employed in the program. A complete list of operations can be found in the compiler reference manual. The *#define* directive defines the level of *PI* to be employed when called for in the program and specifies the operations to be performed when *SQR*(*N*) and *SGN*(*N*) appear. The *?* associated with the *SGN*(*N*) directive is a *conditional operator* that specifies $+1$ if *N* is greater than or equal to 0 and -1 if not.

Next the variables are introduced and defined as floating points. The next entry includes the term *void* to indicate that the variable *to_polar* will not return a specific numerical value when part of an execution but rather may identify a subroutine or string of words or characters. The *void* within the parentheses reveals that the variable does not have a list of parameters associated with it for possible use in an application.

As described in earlier programs the *main* ( ) defines the point at which execution will begin, with the body of *main* defined by the opening and closing braces { }. Within *main,* an integer variable *choice* is introduced to handle the integer number (1 or 2) which the user will choose in response to the question posed under *cout.* Through *cin* the user will respond with a 1 or 2, which will define the variable *choice.* The *switch* is a conditional response that will follow a path defined by the variable *choice.* The possible paths for the program to follow under *switch* are enclosed in the braces { }. Since a numerical value will determine the path, the options must begin with the word *case.* In this case, a 1 will follow the *to_polar* structured variable, and a 2 will follow the *to_rectangular* structured variable. The *break* simply marks the end of the selection process.

On a *to_polar* choice the program will move to the subroutine *void to_polar* and will convert the number to the polar form. The first six lines simply create line shifts and ask for the values of *X* and *Y*. The next line calculates the magnitude of the polar form (*Z*) using *SQR*(*N*)*,* defined above, and the *sqrt* from the *math.h* header file. An *if* statement sensitive to the value of *X* and *Y* will then delineate which line will determine the phase angle of the polar form. The *SGN*(*N*)*,* as introduced in the preprocessor listing, will determine the sign to be employed in the equation. The *a* preceding the tan function indicates arc tan or $\tan^{-1}$*,* while *PI* is as defined above in the preprocessor section. Note also that the angles must

```
//C++ Rectangular/Polar Conversion

#include <iostream.h>            //needed for input/output
#include <math.h>                //needed for sqrt() and fabs()

#define PI 3.14159265
#define SQR(N) N * N              //calculates N squared
#define SGN(N) ((N >= 0) ? 1 : -1) //calculates sign of number

        float X, Y, Z, TH;  //define system variables

void to_polar(void);        //convert into polar function
void to_rectangular(void);  //convert into rectangular function

main()
{
     int choice;    //needed for user choice input

     cout << "Enter (1) for rectangular to polar conversion\n";
     cout << "      (2) for polar to rectangular conversion\n";
     cout << "                   Choice=? ";
     cin >> choice;            //get choice from user
     switch(choice)            //match choice with case value
     {
             case 1 : to_polar(); break;
             case 2 : to_rectangular(); break;
     }
}

void to_polar()            //convert from rectangular to polar
{
     cout << "\n";
     cout << "Enter rectangular data:\n\n";
     cout << "X=? ";
     cin >> X;             //get X value from user
     cout << "Y=? ";
     cin >> Y;             //get Y value from user
     Z = sqrt(SQR(X) + SQR(Y));    //calculate magnitude
     //now do all the angle tests
     if (X > 0) TH = atan(Y / X) * 180.0 / PI;
     if (X < 0) TH = 180.0 * SGN(Y) + atan(Y / X) * 180.0 / PI;
     if (X == 0) TH = 90.0 * SGN(Y);
     if ((Y == 0) && (X < 0)) TH = 180.0;
     cout << "\n";
     cout << "Polar form is " << Z;
     cout << " at an angle of " << TH << " degrees\n";
}

void to_rectangular()   //convert from polar into rectangular
{
     cout << "\n";
     cout << "Enter polar data:\n\n";
     cout << "Z=? ";
     cin >> Z;            //get magnitude from user
     cout << "Angle(degrees). TH=? ";
     cin >> TH;           //get angle from user
     X = Z * cos(TH * PI / 180.0);    //calculate X value
     Y = Z * sin(TH * PI / 180.0);    //calculate Y value
     cout << "\n";
     cout << "Rectangular form is " << X;

       if (Y >= 0)       //test Y value
              cout << " +j ";
       else
              cout << " -j ";
       cout << fabs(Y) << "\n";    //display absolute value of Y value
}
```

Heading

Preprocessor directives

Defines π, SQR(N), and SGN(N)

Define variables and data type

Identify subroutines

Body of program

Choose type of conversion

Define choice variable as integer

Question to user

Choice from user

Subroutine to_polar

Rectangular-to-polar

Request and enter X and Y

Calculate magnitude of Z

Determine angle

Display results of conversion

Subroutine to_rectangular

Polar-to-rectangular

Request Z and θ

Calculate X and Y

Display X

Determine sign for Y component

Display Y

FIG. 14.72

C++ program for complex number conversions.

first be converted to radians by multiplying by the ratio $180°/\pi$. Once determined, the polar form is printed out using the *cout* statements.

Choosing the *to_rectangular* structured variable will cause the program to bypass the above subroutine and move directly to the polar-to-rectangular-conversion sequence. Again, the first six lines simply ask for the components of the polar form. The real and imaginary parts are then calculated and the results printed out. Note the *if-else* statement required to associate the properly signed *j* with the imaginary part.

In an effort to clearly identify the major components of the program, brackets have been added at the edge of the program with a short description of the function performed. As mentioned earlier, do not be concerned if a number of questions arise about the program structure or specific commands or statements. The purpose here is simply to introduce the basic format of the C++ programming language and not to provide all the details required to write your own programs.

Two runs of the program have been provided in Figs. 14.73 and 14.74, one for a polar-to-rectangular conversion and the other for a rectangular-to-polar conversion. Note in each case the result of the *cout* and *cin* statements and in general the clean, clear, and direct format of the resulting output.

```
Enter (1) for rectangular to polar conversion
      (2) for polar to rectangular conversion
                  Choice=? 2

Enter polar data:

Z=? 12
Angle(degrees). TH=? 35

Rectangular form is 9.829824 +j 6.882917
```

FIG. 14.73
Polar-to-rectangular conversion using the C++ program of Fig. 14.72.

```
Enter (1) for rectangular to polar conversion
      (2) for polar to rectangular conversion
                  Choice=? 1

Enter rectangular data:
X=? -10
Y=? 20

Polar form is 22.36068 at an angle of 116.565048 degrees
```

FIG. 14.74
Rectangular-to-polar conversion using the C++ program of Fig. 14.72.

PROBLEMS

SECTION 14.2 The Derivative

1. Plot the following waveform versus time showing one clear, complete cycle. Then determine the derivative of the waveform using Eq. (14.1), and sketch one complete cycle of the derivative directly under the original waveform. Compare the magnitude of the derivative at various points versus the slope of the original sinusoidal function.

$$v = 1 \sin 3.14t$$

2. Repeat Problem 1 for the following sinusoidal function, and compare results. In particular, determine the frequency of the waveforms of Problems 1 and 2, and compare the magnitude of the derivative.

$$v = 1 \sin 15.71t$$

3. What is the derivative of each of the following sinusoidal expressions?
   a. $10 \sin 377t$ **b.** $0.6 \sin(754t + 20°)$
   c. $\sqrt{2}\, 20 \sin(157t - 20°)$ **d.** $-200 \sin(t + 180°)$

SECTION 14.3 Response of Basic *R, L,* and *C* Elements to a Sinusoidal Voltage or Current

4. The voltage across a 5-Ω resistor is as indicated. Find the sinusoidal expression for the current. In addition, sketch the v and i sinusoidal waveforms on the same axis.
   a. $150 \sin 377t$ **b.** $30 \sin(377t + 20°)$
   c. $40 \cos(\omega t + 10°)$ **d.** $-80 \sin(\omega t + 40°)$
5. The current through a 7-kΩ resistor is as indicated. Find the sinusoidal expression for the voltage. In addition, sketch the v and i sinusoidal waveforms on the same axis.
   a. $0.03 \sin 754t$
   b. $2 \times 10^{-3} \sin(400t - 120°)$
   c. $6 \times 10^{-6} \cos(\omega t - 2°)$
   d. $-0.004 \cos(\omega t - 90°)$
6. Determine the inductive reactance (in ohms) of a 2-H coil for
   a. dc
   and for the following frequencies:
   b. 25 Hz **c.** 60 Hz
   d. 2000 Hz **e.** 100,000 Hz
7. Determine the inductance of a coil that has a reactance of
   a. 20 Ω at $f = 2$ Hz.
   b. 1000 Ω at $f = 60$ Hz.
   c. 5280 Ω at $f = 1000$ Hz.
8. Determine the frequency at which a 10-H inductance has the following inductive reactances:
   a. 50 Ω **b.** 3770 Ω
   c. 15.7 kΩ **d.** 243 Ω
9. The current through a 20-Ω inductive reactance is given. What is the sinusoidal expression for the voltage? Sketch the v and i sinusoidal waveforms on the same axis.
   a. $i = 5 \sin \omega t$ **b.** $i = 0.4 \sin(\omega t + 60°)$
   c. $i = -6 \sin(\omega t - 30°)$ **d.** $i = 3 \cos(\omega t + 10°)$
10. The current through a 0.1-H coil is given. What is the sinusoidal expression for the voltage?
    a. $30 \sin 30t$
    b. $0.006 \sin 377t$
    c. $5 \times 10^{-6} \sin(400t + 20°)$
    d. $-4 \cos(20t - 70°)$
11. The voltage across a 50-Ω inductive reactance is given. What is the sinusoidal expression for the current? Sketch the v and i sinusoidal waveforms on the same set of axes.
    a. $50 \sin \omega t$ **b.** $30 \sin(\omega t + 20°)$
    c. $40 \cos(\omega t + 10°)$ **d.** $-80 \sin(377t + 40°)$
12. The voltage across a 0.2-H coil is given. What is the sinusoidal expression for the current?
    a. $1.5 \sin 60t$
    b. $0.016 \sin(t + 4°)$
    c. $-4.8 \sin(0.05t + 50°)$
    d. $9 \times 10^{-3} \cos(377t + 360°)$
13. Determine the capacitive reactance (in ohms) of a 5-μF capacitor for
    a. dc
    and for the following frequencies:
    b. 60 Hz **c.** 120 Hz
    d. 1800 Hz **e.** 24,000 Hz
14. Determine the capacitance in microfarads if a capacitor has a reactance of
    a. 250 Ω at $f = 60$ Hz.
    b. 55 Ω at $f = 312$ Hz.
    c. 10 Ω at $f = 25$ Hz.
15. Determine the frequency at which a 50-μF capacitor has the following capacitive reactances:
    a. 342 Ω **b.** 684 Ω
    c. 171 Ω **d.** 2000 Ω
16. The voltage across a 2.5-Ω capacitive reactance is given. What is the sinusoidal expression for the current? Sketch the v and i sinusoidal waveforms on the same set of axes.
    a. $100 \sin \omega t$ **b.** $0.4 \sin(\omega t + 20°)$
    c. $8 \cos(\omega t + 10°)$ **d.** $-70 \sin(\omega t + 40°)$
17. The voltage across a 1-μF capacitor is given. What is the sinusoidal expression for the current?
    a. $30 \sin 200t$ **b.** $90 \sin 377t$
    c. $-120 \sin(374t + 30°)$ **d.** $70 \cos(800t - 20°)$
18. The current through a 10-Ω capacitive reactance is given. Write the sinusoidal expression for the voltage. Sketch the v and i sinusoidal waveforms on the same set of axes.
    a. $i = 50 \sin \omega t$ **b.** $i = 40 \sin(\omega t + 60°)$
    c. $i = -6 \sin(\omega t - 30°)$ **d.** $i = 3 \cos(\omega t + 10°)$
19. The current through a 0.5-μF capacitor is given. What is the sinusoidal expression for the voltage?
    a. $0.20 \sin 300t$ **b.** $0.007 \sin 377t$
    c. $0.048 \cos 754t$ **d.** $0.08 \sin(1600t - 80°)$

***20.** For the following pairs of voltages and currents, indicate whether the element involved is a capacitor, an inductor, or a resistor, and the value of C, L, or R if sufficient data are given:

a. $v = 550 \sin(377t + 40°)$
$i = 11 \sin(377t - 50°)$

b. $v = 36 \sin(754t + 80°)$
$i = 4 \sin(754t + 170°)$

c. $v = 10.5 \sin(\omega t + 13°)$
$i = 1.5 \sin(\omega t + 13°)$

***21.** Repeat Problem 20 for the following pairs of voltages and currents:

a. $v = 2000 \sin \omega t$
$i = 5 \cos \omega t$

b. $v = 80 \sin(157t + 150°)$
$i = 2 \sin(157t + 60°)$

c. $v = 35 \sin(\omega t - 20°)$
$i = 7 \cos(\omega t - 110°)$

SECTION 14.4 Frequency Response of the Basic Elements

22. Plot X_L versus frequency for a 5-mH coil using a frequency range of zero to 100 kHz on a linear scale.

23. Plot X_C versus frequency for a 1-μF capacitor using a frequency range of zero to 10 kHz on a linear scale.

24. At what frequency will the reactance of a 1-μF capacitor equal the resistance of a 2-kΩ resistor?

25. The reactance of a coil equals the resistance of a 10-kΩ resistor at a frequency of 5 kHz. Determine the inductance of the coil.

26. Determine the frequency at which a 1-μF capacitor and a 10-mH inductor will have the same reactance.

27. Determine the capacitance required to establish a capacitive reactance that will match that of a 2-mH coil at a frequency of 50 kHz.

SECTION 14.5 Average Power and Power Factor

28. Find the average power loss in watts for each set in Problem 20.

29. Find the average power loss in watts for each set in Problem 21.

***30.** Find the average power loss and power factor for each of the circuits whose input current and voltage are as follows:

a. $v = 60 \sin(\omega t + 30°)$
$i = 15 \sin(\omega t + 60°)$

b. $v = -50 \sin(\omega t - 20°)$
$i = -2 \sin(\omega t + 40°)$

c. $v = 50 \sin(\omega t + 80°)$
$i = 3 \cos(\omega t + 20°)$

d. $v = 75 \sin(\omega t - 5°)$
$i = 0.08 \sin(\omega t - 35°)$

31. If the current through and voltage across an element are $i = 8 \sin(\omega t + 40°)$ and $v = 48 \sin(\omega t + 40°)$, respectively, compute the power by I^2R, $(V_mI_m/2) \cos \theta$, and $VI \cos \theta$, and compare answers.

32. A circuit dissipates 100 W (average power) at 150 V (effective input voltage) and 2 A (effective input current). What is the power factor? Repeat if the power is 0 W; 300 W.

***33.** The power factor of a circuit is 0.5 lagging. The power delivered in watts is 500. If the input voltage is $50 \sin(\omega t + 10°)$, find the sinusoidal expression for the input current.

34. In Fig. 14.75, $e = 30 \sin(377t + 20°)$.

a. What is the sinusoidal expression for the current?

b. Find the power loss in the circuit.

c. How long (in seconds) does it take the current to complete six cycles?

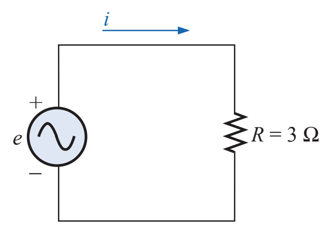


FIG. 14.75
Problem 34.

35. In Fig. 14.76, $e = 100 \sin(157t + 30°)$.

a. Find the sinusoidal expression for i.

b. Find the value of the inductance L.

c. Find the average power loss by the inductor.

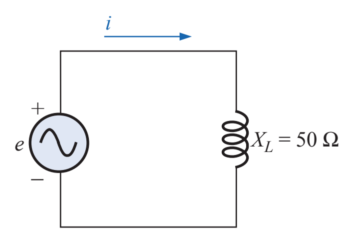


FIG. 14.76
Problem 35.

36. In Fig. 14.77, $i = 3 \sin(377t - 20°)$.

a. Find the sinusoidal expression for e.

b. Find the value of the capacitance C in microfarads.

c. Find the average power loss in the capacitor.

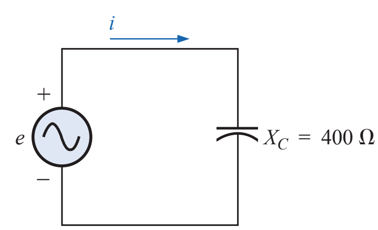
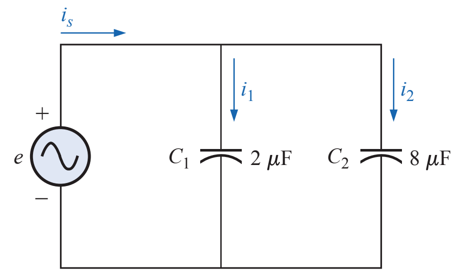


FIG. 14.77
Problem 36.

***37.** For the network of Fig. 14.78 and the applied signal:
a. Determine i_1 and i_2.
b. Find i_s.



$e = \sqrt{2}\,100 \sin(10^4 t + 60°)$

FIG. 14.78
Problem 37.

***38.** For the network of Fig. 14.79 and the applied source:
a. Determine the source voltage v_s.
b. Find the currents i_1 and i_2.

$i_s = \sqrt{2}\,6 \sin(10^3 t + 30°)$

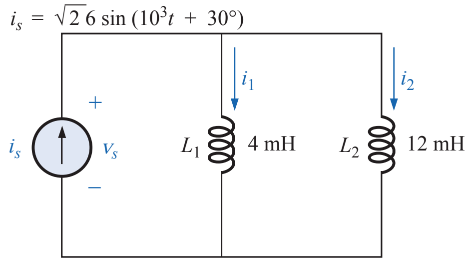


FIG. 14.79
Problem 38.

SECTION 14.9 Conversion between Forms

39. Convert the following from rectangular to polar form:
a. $4 + j3$ **b.** $2 + j2$
c. $3.5 + j16$ **d.** $100 + j800$
e. $1000 + j400$ **f.** $0.001 + j0.0065$
g. $7.6 - j9$ **h.** $-8 + j4$
i. $-15 - j60$ **j.** $+78 - j65$
k. $-2400 + j3600$
l. $5 \times 10^{-3} - j25 \times 10^{-3}$

40. Convert the following from polar to rectangular form:
a. $6 \angle 30°$ **b.** $40 \angle 80°$
c. $7400 \angle 70°$ **d.** $4 \times 10^{-4} \angle 8°$
e. $0.04 \angle 80°$ **f.** $0.0093 \angle 23°$
g. $65 \angle 150°$ **h.** $1.2 \angle 135°$
i. $500 \angle 200°$ **j.** $6320 \angle -35°$
k. $7.52 \angle -125°$ **l.** $0.008 \angle 310°$

41. Convert the following from rectangular to polar form:
a. $1 + j15$ **b.** $60 + j5$
c. $0.01 + j0.3$ **d.** $100 - j2000$
e. $-5.6 + j86$ **f.** $-2.7 - j38.6$

42. Convert the following from polar to rectangular form:
a. $13 \angle 5°$ **b.** $160 \angle 87°$
c. $7 \times 10^{-6} \angle 2°$ **d.** $8.7 \angle 177°$
e. $76 \angle -4°$ **f.** $396 \angle +265°$

SECTION 14.10 Mathematical Operations with Complex Numbers

Perform the following operations.

43. Addition and subtraction (express your answers in rectangular form):
a. $(4.2 + j6.8) + (7.6 + j0.2)$
b. $(142 + j7) + (9.8 + j42) + (0.1 + j0.9)$
c. $(4 \times 10^{-6} + j76) + (7.2 \times 10^{-7} - j5)$
d. $(9.8 + j6.2) - (4.6 + j4.6)$
e. $(167 + j243) - (-42.3 - j68)$
f. $(-36.0 + j78) - (-4 - j6) + (10.8 - j72)$
g. $6 \angle 20° + 8 \angle 80°$
h. $42 \angle 45° + 62 \angle 60° - 70 \angle 120°$

44. Multiplication [express your answers in rectangular form for parts (a) through (d), and in polar form for parts (e) through (h)]:
a. $(2 + j3)(6 + j8)$
b. $(7.8 + j1)(4 + j2)(7 + j6)$
c. $(0.002 + j0.006)(-2 + j2)$
d. $(400 - j200)(-0.01 - j0.5)(-1 + j3)$
e. $(2 \angle 60°)(4 \angle 22°)$
f. $(6.9 \angle 8°)(7.2 \angle -72°)$
g. $0.002 \angle 120°)(0.5 \angle 200°)(40 \angle -60°)$
h. $(540 \angle -20°)(-5 \angle 180°)(6.2 \angle 0°)$

45. Division (express your answers in polar form):
a. $(42 \angle 10°)/(7 \angle 60°)$
b. $(0.006 \angle 120°)/(30 \angle -20°)$
c. $(4360 \angle -20°)/(40 \angle 210°)$
d. $(650 \angle -80°)/(8.5 \angle 360°)$
e. $(8 + j8)/(2 + j2)$
f. $(8 + j42)/(-6 + j60)$
g. $(0.05 + j0.25)/(8 - j60)$
h. $(-4.5 - j6)/(0.1 - j0.4)$

***46.** Perform the following operations (express your answers in rectangular form):

a. $\dfrac{(4 + j3) + (6 - j8)}{(3 + j3) - (2 + j3)}$

b. $\dfrac{8 \angle 60°}{(2 \angle 0°) + (100 + j100)}$

c. $\dfrac{(6 \angle 20°)(120 \angle -40°)(3 + j4)}{2 \angle -30°}$

d. $\dfrac{(0.4 \angle 60°)^2(300 \angle 40°)}{3 + j9}$

e. $\left(\dfrac{1}{(0.02 \angle 10°)^2}\right)\left(\dfrac{2}{j}\right)^3\left(\dfrac{1}{6^2 - j\sqrt{900}}\right)$

***47. a.** Determine a solution for x and y if

$$(x + j4) + (3x + jy) - j7 = 16 \angle 0^\circ$$

b. Determine x if

$$(10 \angle 20^\circ)(x \angle -60^\circ) = 30.64 - j25.72$$

c. Determine a solution for x and y if

$$(5x + j10)(2 - jy) = 90 - j70$$

d. Determine θ if

$$\frac{80 \angle 0^\circ}{20 \angle \theta} = 3.464 - j2$$

SECTION 14.12 Phasors

48. Express the following in phasor form:
a. $\sqrt{2}(100)\sin(\omega t + 30^\circ)$
b. $\sqrt{2}(0.25)\sin(157t - 40^\circ)$
c. $100\sin(\omega t - 90^\circ)$
d. $42\sin(377t + 0^\circ)$
e. $6 \times 10^{-6}\cos\omega t$
f. $3.6 \times 10^{-6}\cos(754t - 20^\circ)$

49. Express the following phasor currents and voltages as sine waves if the frequency is 60 Hz:
a. $\mathbf{I} = 40\text{ A} \angle 20^\circ$ **b.** $\mathbf{V} = 120\text{ V} \angle 0^\circ$
c. $\mathbf{I} = 8 \times 10^{-3}\text{ A} \angle 120^\circ$ **d.** $\mathbf{V} = 5\text{ V} \angle 90^\circ$
e. $\mathbf{I} = 1200\text{ A} \angle -120^\circ$ **f.** $\mathbf{V} = \dfrac{6000}{\sqrt{2}}\text{ V} \angle -180^\circ$

50. For the system of Fig. 14.80, find the sinusoidal expression for the unknown voltage v_a if

$$e_{in} = 60\sin(377t + 20^\circ)$$
$$v_b = 20\sin 377t$$

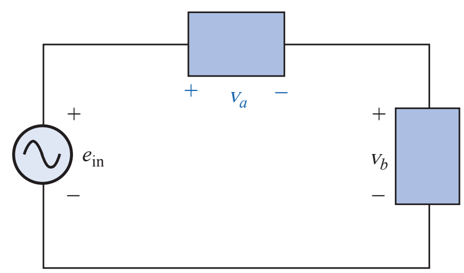


FIG. 14.80
Problem 50.

51. For the system of Fig. 14.81, find the sinusoidal expression for the unknown current i_1 if

$$i_s = 20 \times 10^{-6}\sin(\omega t + 90^\circ)$$
$$i_2 = 6 \times 10^{-6}\sin(\omega t - 60^\circ)$$

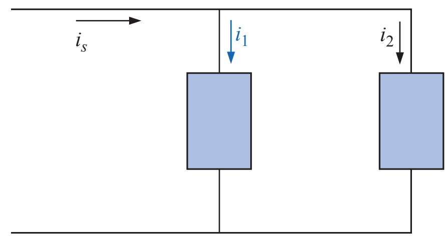


FIG. 14.81
Problem 51.

52. Find the sinusoidal expression for the applied voltage e for the system of Fig. 14.82 if

$$v_a = 60\sin(\omega t + 30^\circ)$$
$$v_b = 30\sin(\omega t - 30^\circ)$$
$$v_c = 40\sin(\omega t + 120^\circ)$$

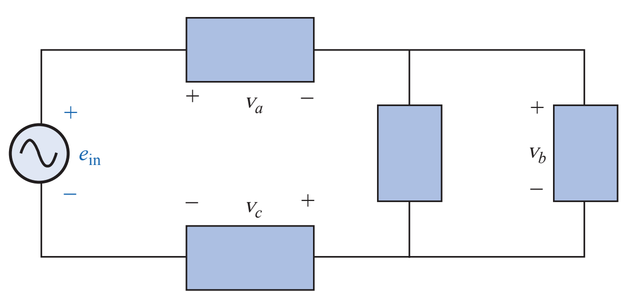


FIG. 14.82
Problem 52.

53. Find the sinusoidal expression for the current i_s for the system of Fig. 14.83 if

$$i_1 = 6 \times 10^{-3} \sin(377t + 180°)$$
$$i_2 = 8 \times 10^{-3} \sin 377t$$
$$i_3 = 2i_2$$

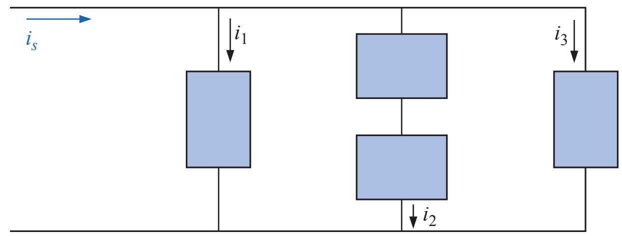


FIG. 14.83
Problem 53.

SECTION 14.13 Computer Analysis

PSpice or Electronics Workbench

54. Plot i_C and v_C versus time for the network of Fig. 14.69 for two cycles if the frequency is 0.2 kHz.

55. Plot the magnitude and phase angle of the current i_C versus frequency (100 Hz to 100 kHz) for the network of Fig. 14.69.

56. Plot the total impedance of the configuration of Fig. 14.26 versus frequency (100 kHz to 100 MHz) for the following parameter values: $C = 0.1\ \mu\text{F}$, $L_s = 0.2\ \mu\text{H}$, $R_s = 2\ \text{M}\Omega$, and $R_p = 100\ \text{M}\Omega$. For what frequency range is the capacitor "capacitive"?

Programming Language (C++, QBASIC, Pascal, etc.)

57. Given a sinusoidal function, write a program to print out the derivative.

58. Given the sinusoidal expression for the current, determine the expression for the voltage across a resistor, a capacitor, or an inductor, depending on the element involved. In other words, the program will ask which element is to be investigated and will then request the pertinent data to obtain the mathematical expression for the sinusoidal voltage.

59. Write a program to tabulate the reactance versus frequency for an inductor or a capacitor for a specified frequency range.

60. Given the sinusoidal expression for the voltage and current of a load, write a program to determine the average power and power factor.

61. Given two sinusoidal functions, write a program to convert each to the phasor domain, add the two, and print out the sum in the phasor and time domains.

GLOSSARY

Average or **real power** The power delivered to and dissipated by the load over a full cycle.

Complex conjugate A complex number defined by simply changing the sign of an imaginary component of a complex number in the rectangular form.

Complex number A number that represents a point in a two-dimensional plane located with reference to two distinct axes. It defines a vector drawn from the origin to that point.

Derivative The instantaneous rate of change of a function with respect to time or another variable.

Leading and **lagging power factors** An indication of whether a network is primarily capacitive or inductive in nature. Leading power factors are associated with capacitive networks, and lagging power factors with inductive networks.

Phasor A radius vector that has a constant magnitude at a fixed angle from the positive real axis and that represents a sinusoidal voltage or current in the vector domain.

Phasor diagram A "snapshot" of the phasors that represent a number of sinusoidal waveforms at $t = 0$.

Polar form A method of defining a point in a complex plane that includes a single magnitude to represent the distance from the origin, and an angle to reflect the counterclockwise distance from the positive real axis.

Power factor (F_p) An indication of how reactive or resistive an electrical system is. The higher the power factor, the greater the resistive component.

Reactance The opposition of an inductor or a capacitor to the flow of charge that results in the continual exchange of energy between the circuit and magnetic field of an inductor or the electric field of a capacitor.

Reciprocal A format defined by 1 divided by the complex number.

Rectangular form A method of defining a point in a complex plane that includes the magnitude of the real component and the magnitude of the imaginary component, the latter component being defined by an associated letter j.

15

Series and Parallel ac Circuits

15.1 INTRODUCTION

In this chapter, phasor algebra will be used to develop a quick, direct method for solving both the series and the parallel ac circuits. The close relationship that exists between this method for solving for unknown quantities and the approach used for dc circuits will become apparent after a few simple examples are considered. Once this association is established, many of the rules (current divider rule, voltage divider rule, and so on) for dc circuits can be readily applied to ac circuits.

SERIES ac CIRCUITS

15.2 IMPEDANCE AND THE PHASOR DIAGRAM

Resistive Elements

In Chapter 14, we found, for the purely resistive circuit of Fig. 15.1, that v and i were in phase, and the magnitude

$$I_m = \frac{V_m}{R} \quad \text{or} \quad V_m = I_m R$$

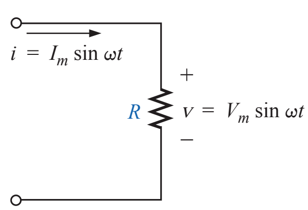


FIG. 15.1
Resistive ac circuit.

In phasor form,

$$v = V_m \sin \omega t \Rightarrow \mathbf{V} = V \angle 0°$$

where $V = 0.707V_m$.

Applying Ohm's law and using phasor algebra, we have

$$\mathbf{I} = \frac{V \angle 0°}{R \angle \theta_R} = \frac{V}{R} \underline{/0° - \theta_R}$$

Since i and v are in phase, the angle associated with i also must be 0°. To satisfy this condition, θ_R must equal 0°. Substituting $\theta_R = 0°$, we find

$$\mathbf{I} = \frac{V \angle 0°}{R \angle 0°} = \frac{V}{R} \underline{/0° - 0°} = \frac{V}{R} \angle 0°$$

so that in the time domain,

$$i = \sqrt{2}\left(\frac{V}{R}\right) \sin \omega t$$

The fact that $\theta_R = 0°$ will now be employed in the following polar format to ensure the proper phase relationship between the voltage and current of a resistor:

$$\boxed{\mathbf{Z}_R = R \angle 0°} \qquad \textbf{(15.1)}$$

The boldface roman quantity $\mathbf{Z}_R$, having both magnitude and an associated angle, is referred to as the *impedance* of a resistive element. It is measured in ohms and is a measure of how much the element will "impede" the flow of charge through the network. The above format will prove to be a useful "tool" when the networks become more complex and phase relationships become less obvious. It is important to realize, however, that $\mathbf{Z}_R$ is *not a phasor,* even though the format $R \angle 0°$ is very similar to the phasor notation for sinusoidal currents and voltages. The term *phasor* is reserved for quantities that vary with time, and R and its associated angle of 0° are fixed, nonvarying quantities.

EXAMPLE 15.1 Using complex algebra, find the current i for the circuit of Fig. 15.2. Sketch the waveforms of v and i.

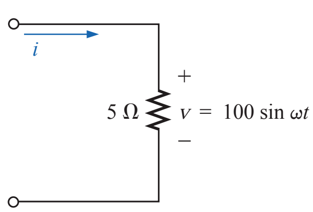


FIG. 15.2
Example 15.1.

Solution: Note Fig. 15.3:

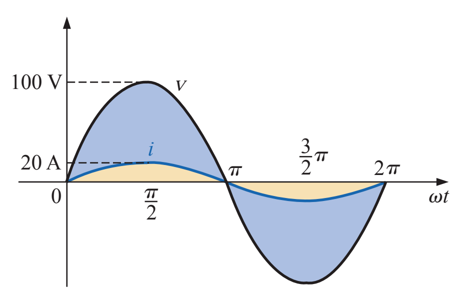


FIG. 15.3
Waveforms for Example 15.1.

$$v = 100 \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = 70.71 \text{ V} \angle 0°$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_R} = \frac{V \angle \theta}{R \angle 0°} = \frac{70.71 \text{ V} \angle 0°}{5\ \Omega \angle 0°} = 14.14 \text{ A} \angle 0°$$

and $$i = \sqrt{2}(14.14) \sin \omega t = \mathbf{20 \sin \omega t}$$

EXAMPLE 15.2 Using complex algebra, find the voltage v for the circuit of Fig. 15.4. Sketch the waveforms of v and i.

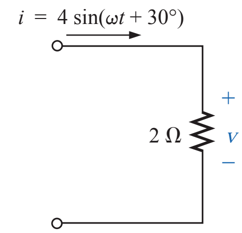


FIG. 15.4
Example 15.2.

Solution: Note Fig. 15.5:

$$i = 4 \sin(\omega t + 30°) \Rightarrow \text{phasor form } \mathbf{I} = 2.828 \text{ A} \angle 30°$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_R = (I \angle \theta)(R \angle 0°) = (2.828 \text{ A} \angle 30°)(2\ \Omega \angle 0°)$$
$$= 5.656 \text{ V} \angle 30°$$

and $$v = \sqrt{2}(5.656) \sin(\omega t + 30°) = \mathbf{8.0 \sin(\omega t + 30°)}$$

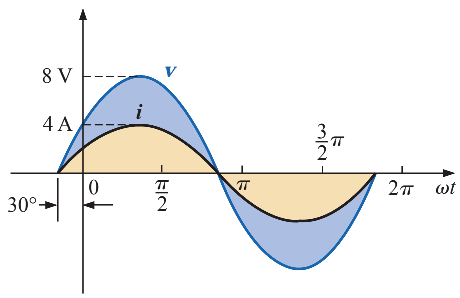


FIG. 15.5
Waveforms for Example 15.2.

It is often helpful in the analysis of networks to have a **phasor diagram**, which shows at a glance the *magnitudes* and *phase relations* among the various quantities within the network. For example, the phasor diagrams of the circuits considered in the two preceding examples would be as shown in Fig. 15.6. In both cases, it is immediately obvious that v and i are in phase since they both have the same phase angle.

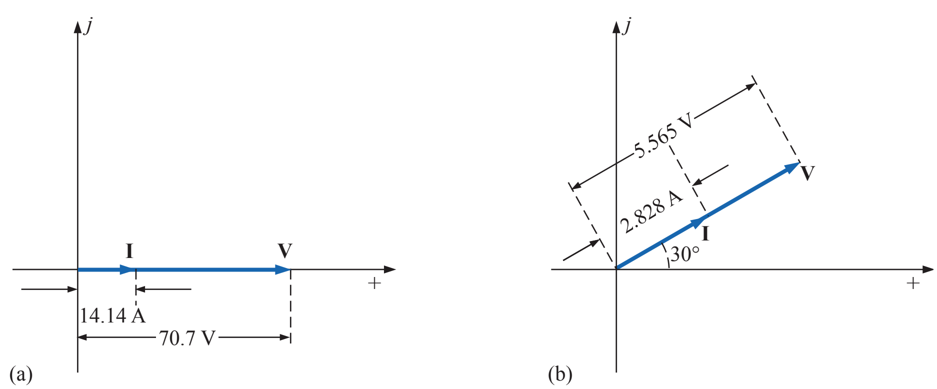


FIG. 15.6
Phasor diagrams for Examples 15.1 and 15.2.

Inductive Reactance

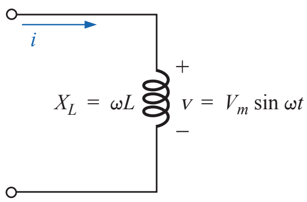


FIG. 15.7
Inductive ac circuit.

It was learned in Chapter 13 that for the pure inductor of Fig. 15.7, the voltage leads the current by 90° and that the reactance of the coil X_L is determined by ωL.

$$v = V_m \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = V \angle 0^\circ$$

By Ohm's law,

$$\mathbf{I} = \frac{V \angle 0^\circ}{X_L \angle \theta_L} = \frac{V}{X_L} \underline{/0^\circ - \theta_L}$$

Since v leads i by 90°, i must have an angle of $-90°$ associated with it. To satisfy this condition, θ_L must equal $+90°$. Substituting $\theta_L = 90°$, we obtain

$$\mathbf{I} = \frac{V \angle 0^\circ}{X_L \angle 90^\circ} = \frac{V}{X_L} \underline{/0^\circ - 90^\circ} = \frac{V}{X_L} \angle -90^\circ$$

so that in the time domain,

$$i = \sqrt{2}\left(\frac{V}{X_L}\right) \sin(\omega t - 90^\circ)$$

The fact that $\theta_L = 90°$ will now be employed in the following polar format for inductive reactance to ensure the proper phase relationship between the voltage and current of an inductor.

$$\boxed{\mathbf{Z}_L = X_L \angle 90^\circ} \qquad \textbf{(15.2)}$$

The boldface roman quantity $\mathbf{Z}_L$, having both magnitude and an associated angle, is referred to as the *impedance* of an inductive element. It is measured in ohms and is a measure of how much the inductive element will "control or impede" the level of current through the network (always keep in mind that inductive elements are storage devices and do not dissipate like resistors). The above format, like that defined for the resistive element, will prove to be a useful "tool" in the analysis of ac networks. Again, be aware that $\mathbf{Z}_L$ is not a phasor quantity, for the same reasons indicated for a resistive element.

EXAMPLE 15.3 Using complex algebra, find the current i for the circuit of Fig. 15.8. Sketch the v and i curves.

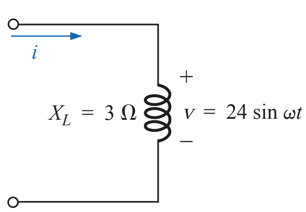


FIG. 15.8
Example 15.3.

Solution: Note Fig. 15.9:

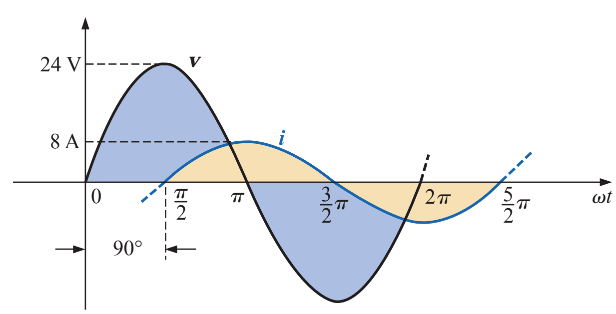


FIG. 15.9
Waveforms for Example 15.3.

$$v = 24 \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = 16.968 \text{ V} \angle 0°$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_L} = \frac{V \angle \theta}{X_L \angle 90°} = \frac{16.968 \text{ V} \angle 0°}{3 \ \Omega \angle 90°} = 5.656 \text{ A} \angle -90°$$

and $$i = \sqrt{2}(5.656) \sin(\omega t - 90°) = \mathbf{8.0 \sin(\omega t - 90°)}$$

EXAMPLE 15.4 Using complex algebra, find the voltage v for the circuit of Fig. 15.10. Sketch the v and i curves.

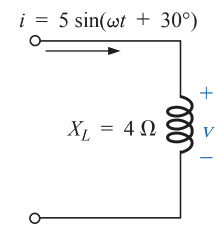


FIG. 15.10
Example 15.4.

Solution: Note Fig. 15.11:

$$i = 5 \sin(\omega t + 30°) \Rightarrow \text{phasor form } \mathbf{I} = 3.535 \text{ A} \angle 30°$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_L = (I \angle \theta)(X_L \angle 90°) = (3.535 \text{ A} \angle 30°)(4 \ \Omega \angle +90°)$$
$$= 14.140 \text{ V} \angle 120°$$

and $$v = \sqrt{2}(14.140) \sin(\omega t + 120°) = \mathbf{20 \sin(\omega t + 120°)}$$

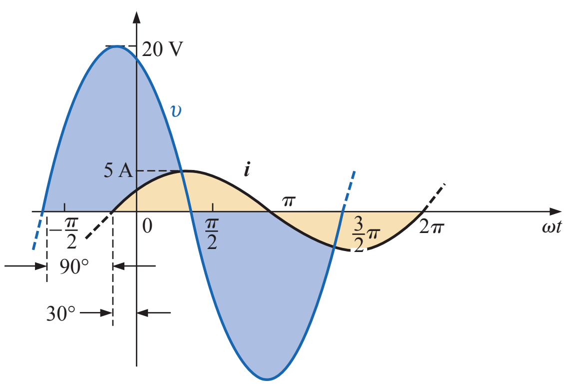


FIG. 15.11
Waveforms for Example 15.4.

The phasor diagrams for the two circuits of the two preceding examples are shown in Fig. 15.12. Both indicate quite clearly that the voltage leads the current by 90°.

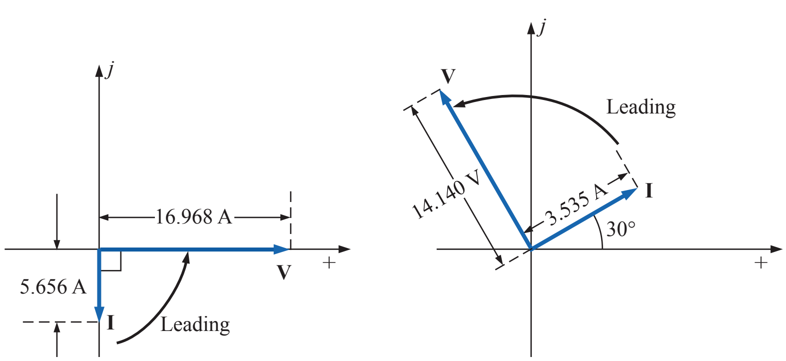


FIG. 15.12
Phasor diagrams for Examples 15.3 and 15.4.

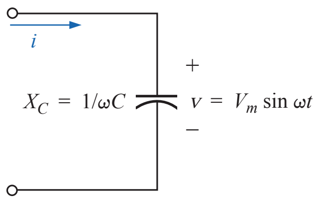


FIG. 15.13
Capacitive ac circuit.

Capacitive Reactance

It was learned in Chapter 13 that for the pure capacitor of Fig. 15.13, the current leads the voltage by 90° and that the reactance of the capacitor X_C is determined by $1/\omega C$.

$$v = V_m \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = V \angle 0^\circ$$

Applying Ohm's law and using phasor algebra, we find

$$\mathbf{I} = \frac{V \angle 0^\circ}{X_C \angle \theta_C} = \frac{V}{X_C} \angle 0^\circ - \theta_C$$

Since i leads v by 90°, i must have an angle of +90° associated with it. To satisfy this condition, θ_C must equal −90°. Substituting $\theta_C = -90^\circ$ yields

$$\mathbf{I} = \frac{V \angle 0^\circ}{X_C \angle -90^\circ} = \frac{V}{X_C} \angle 0^\circ - (-90^\circ) = \frac{V}{X_C} \angle 90^\circ$$

so, in the time domain,

$$i = \sqrt{2}\left(\frac{V}{X_C}\right) \sin(\omega t + 90^\circ)$$

The fact that $\theta_C = -90^\circ$ will now be employed in the following polar format for capacitive reactance to ensure the proper phase relationship between the voltage and current of a capacitor.

$$\boxed{\mathbf{Z}_C = X_C \angle -90^\circ} \tag{15.3}$$

The boldface roman quantity $\mathbf{Z}_C$, having both magnitude and an associated angle, is referred to as the *impedance* of a capacitive element. It is measured in ohms and is a measure of how much the capacitive element will "control or impede" the level of current through the network (always keep in mind that capacitive elements are storage devices and do not dissipate like resistors). The above format, like that defined for the resistive element, will prove a very useful "tool" in the analysis of ac networks. Again, be aware that $\mathbf{Z}_C$ is not a phasor quantity, for the same reasons indicated for a resistive element.

EXAMPLE 15.5 Using complex algebra, find the current i for the circuit of Fig. 15.14. Sketch the v and i curves.

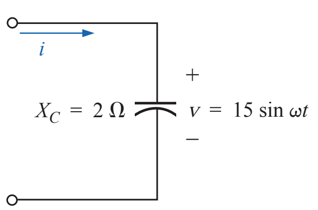


FIG. 15.14
Example 15.5.

Solution: Note Fig. 15.15:

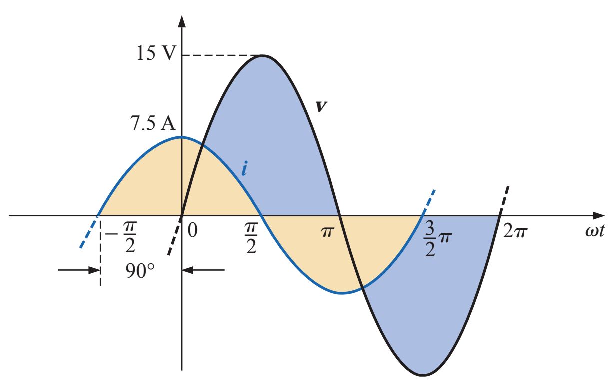


FIG. 15.15
Waveforms for Example 15.5.

$$v = 15 \sin \omega t \Rightarrow \text{phasor notation } \mathbf{V} = 10.605 \text{ V} \angle 0°$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_C} = \frac{V \angle \theta}{X_C \angle -90°} = \frac{10.605 \text{ V} \angle 0°}{2\ \Omega \angle -90°} = 5.303 \text{ A} \angle 90°$$

and $$i = \sqrt{2}(5.303) \sin(\omega t + 90°) = \mathbf{7.5 \sin(\boldsymbol{\omega} t + 90°)}$$

EXAMPLE 15.6 Using complex algebra, find the voltage v for the circuit of Fig. 15.16. Sketch the v and i curves.

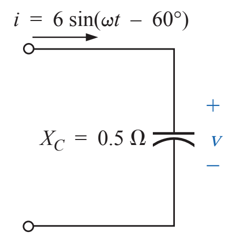


FIG. 15.16
Example 15.6.

Solution: Note Fig. 15.17:

$$i = 6 \sin(\omega t - 60°) \Rightarrow \text{phasor notation } \mathbf{I} = 4.242 \text{ A} \angle -60°$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C = (I \angle \theta)(X_C \angle -90°) = (4.242 \text{ A} \angle -60°)(0.5\ \Omega \angle -90°)$$
$$= 2.121 \text{ V} \angle -150°$$

and $$v = \sqrt{2}(2.121) \sin(\omega t - 150°) = \mathbf{3.0 \sin(\boldsymbol{\omega} t - 150°)}$$

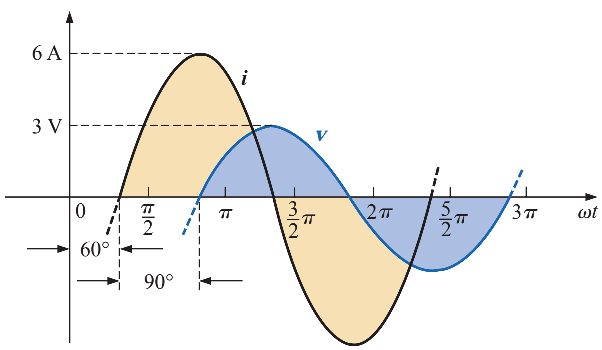
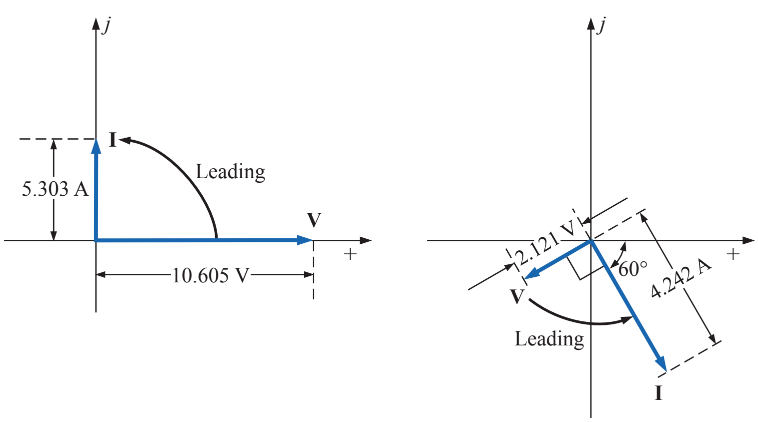


FIG. 15.17
Waveforms for Example 15.6.

The phasor diagrams for the two circuits of the two preceding examples are shown in Fig. 15.18. Both indicate quite clearly that the current i leads the voltage v by 90°.



(a) (b)

FIG. 15.18
Phasor diagrams for Examples 15.5 and 15.6.

Impedance Diagram

Now that an angle is associated with resistance, inductive reactance, and capacitive reactance, each can be placed on a complex plane dia-

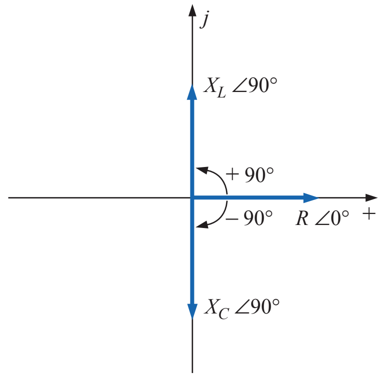


FIG. 15.19
Impedance diagram.

gram, as shown in Fig. 15.19. For any network, the resistance will *always* appear on the positive real axis, the inductive reactance on the positive imaginary axis, and the capacitive reactance on the negative imaginary axis. The result is an **impedance diagram** that can reflect the individual and total impedance levels of an ac network.

We will find in the sections and chapters to follow that networks combining different types of elements will have total impedances that extend from $-90°$ to $+90°$. If the total impedance has an angle of 0°, it is said to be resistive in nature. If it is closer to 90°, it is inductive in nature; and if it is closer to $-90°$, it is capacitive in nature.

Of course, for single-element networks the angle associated with the impedance will be the same as that of the resistive or reactive element, as revealed by Eqs. (15.1) through (15.3). It is important to stay aware that impedance, like resistance or reactance, is not a phasor quantity representing a time-varying function with a particular phase shift. It is simply an operating "tool" that is extremely useful in determining the magnitude and angle of quantities in a sinusoidal ac network.

Once the total impedance of a network is determined, its magnitude will define the resulting current level (through Ohm's law), whereas its angle will reveal whether the network is primarily inductive or capacitive or simply resistive.

For any configuration (series, parallel, series-parallel, etc.), the angle associated with the total impedance is the angle by which the applied voltage leads the source current. For inductive networks, θ_T will be positive, whereas for capacitive networks, θ_T will be negative.

15.3 SERIES CONFIGURATION

The overall properties of series ac circuits (Fig. 15.20) are the same as those for dc circuits. For instance, the total impedance of a system is the sum of the individual impedances:

$$\boxed{\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \cdots + \mathbf{Z}_N} \tag{15.4}$$

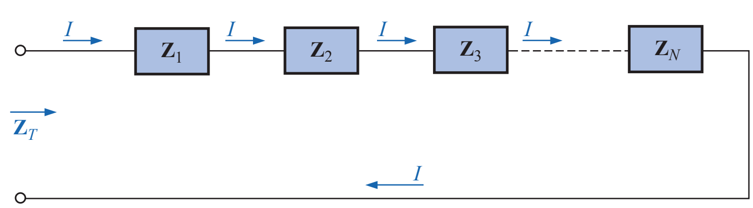


FIG. 15.20
Series impedances.

EXAMPLE 15.7 Draw the impedance diagram for the circuit of Fig. 15.21, and find the total impedance.

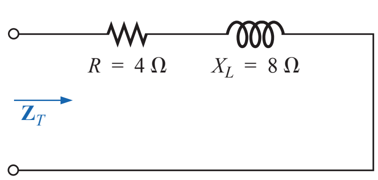


FIG. 15.21
Example 15.7.

Solution: As indicated by Fig. 15.22, the input impedance can be found graphically from the impedance diagram by properly scaling the real and imaginary axes and finding the length of the resultant vector Z_T and angle θ_T. Or, by using vector algebra, we obtain

$$\begin{aligned} \mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 \\ &= R\angle 0° + X_L\angle 90° \\ &= R + jX_L = 4\ \Omega + j8\ \Omega \\ \mathbf{Z}_T &= \mathbf{8.944\ \Omega\ \angle 63.43°} \end{aligned}$$

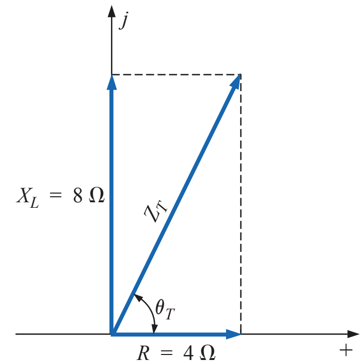


FIG. 15.22
Impedance diagram for Example 15.7.

EXAMPLE 15.8 Determine the input impedance to the series network of Fig. 15.23. Draw the impedance diagram.

Solution:

$$\begin{aligned} \mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 \\ &= R\angle 0° + X_L\angle 90° + X_C\angle -90° \\ &= R + jX_L - jX_C \\ &= R + j(X_L - X_C) = 6\ \Omega + j(10\ \Omega - 12\ \Omega) = 6\ \Omega - j2\ \Omega \\ \mathbf{Z}_T &= \mathbf{6.325\ \Omega\ \angle -18.43°} \end{aligned}$$

The impedance diagram appears in Fig. 15.24. Note that in this example, series inductive and capacitive reactances are in direct opposition. For the circuit of Fig. 15.23, if the inductive reactance were equal to the capacitive reactance, the input impedance would be purely resistive. We will have more to say about this particular condition in a later chapter.

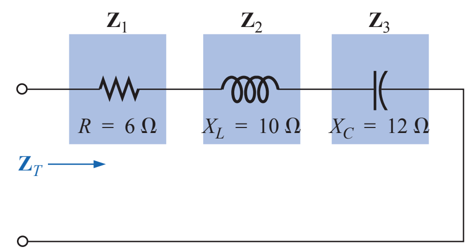


FIG. 15.23
Example 15.8

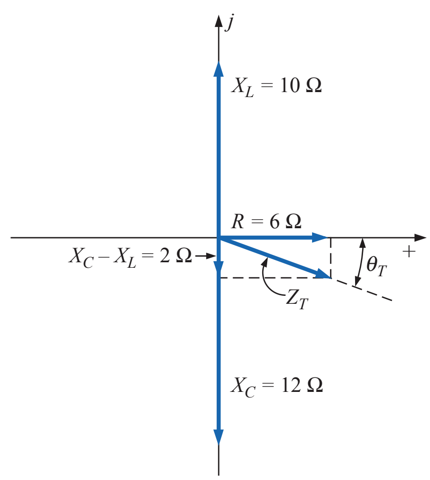


FIG. 15.24
Impedance diagram for Example 15.8.

For the representative **series ac configuration** of Fig. 15.25 having two impedances, *the current is the same through each element* (as it was for the series dc circuits) and is determined by Ohm's law:

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2$$

and

$$\boxed{\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T}} \tag{15.5}$$

The voltage across each element can then be found by another application of Ohm's law:

$$\boxed{\mathbf{V}_1 = \mathbf{IZ}_1} \tag{15.6a}$$

$$\boxed{\mathbf{V}_2 = \mathbf{IZ}_2} \tag{15.6b}$$

Kirchhoff's voltage law can then be applied in the same manner as it is employed for dc circuits. However, keep in mind that we are now dealing with the algebraic manipulation of quantities that have both magnitude and direction.

$$\mathbf{E} - \mathbf{V}_1 - \mathbf{V}_2 = 0$$

or

$$\boxed{\mathbf{E} = \mathbf{V}_1 + \mathbf{V}_2} \tag{15.7}$$

The power to the circuit can be determined by

$$\boxed{P = EI\cos\theta_T} \tag{15.8}$$

where θ_T is the phase angle between **E** and **I**.

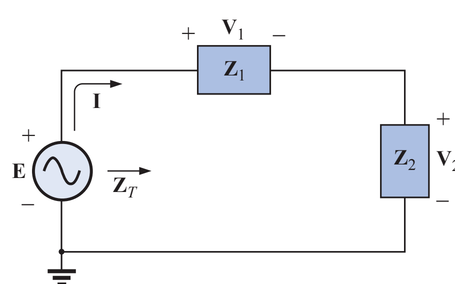


FIG. 15.25
Series ac circuit.

Now that a general approach has been introduced, the simplest of series configurations will be investigated in detail to further emphasize the similarities in the analysis of dc circuits. In many of the circuits to be considered, $3 + j4 = 5\angle 53.13^\circ$ and $4 + j3 = 5\angle 36.87^\circ$ will be used quite frequently to ensure that the approach is as clear as possible and not lost in mathematical complexity. Of course, the problems at the end of the chapter will provide plenty of experience with random values.

R-L

Refer to Fig. 15.26.

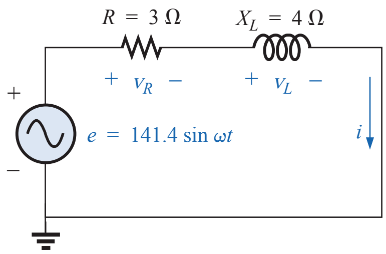


FIG. 15.26
Series R-L circuit.

Phasor Notation

$$e = 141.4 \sin \omega t \Rightarrow \mathbf{E} = 100\text{ V}\angle 0^\circ$$

Note Fig. 15.27.

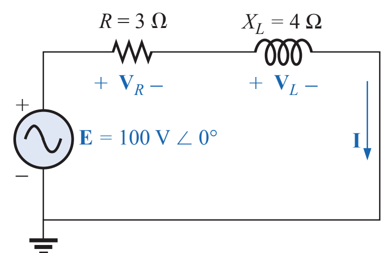


FIG. 15.27
Applying phasor notation to the network of Fig. 15.26.

$\mathbf{Z}_T$

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 3\ \Omega\angle 0^\circ + 4\ \Omega\angle 90^\circ = 3\ \Omega + j4\ \Omega$$

and $\mathbf{Z}_T = \mathbf{5\ \Omega\angle 53.13^\circ}$

Impedance diagram: See Fig. 15.28.

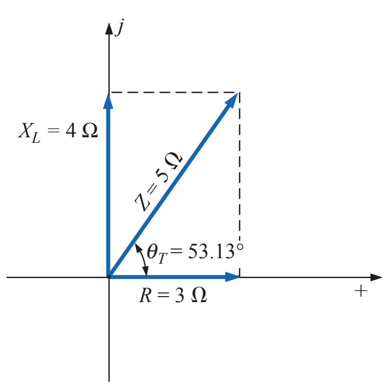


FIG. 15.28
Impedance diagram for the series R-L circuit of Fig. 15.26.

I

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\text{ V}\angle 0^\circ}{5\ \Omega\angle 53.13^\circ} = \mathbf{20\text{ A}\angle -53.13^\circ}$$

$\mathbf{V}_R$ and $\mathbf{V}_L$

Ohm's law:

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (20\text{ A}\angle -53.13^\circ)(3\ \Omega\angle 0^\circ) = \mathbf{60\text{ V}\angle -53.13^\circ}$$

$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (20\text{ A}\angle -53.13^\circ)(4\ \Omega\angle 90^\circ) = \mathbf{80\text{ V}\angle 36.87^\circ}$$

Kirchhoff's voltage law:

$$\Sigma_{\circlearrowright} \mathbf{V} = \mathbf{E} - \mathbf{V}_R - \mathbf{V}_L = 0$$

or

$$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_L$$

In rectangular form,

$$\mathbf{V}_R = 60\text{ V} \angle -53.13° = 36\text{ V} - j\,48\text{ V}$$
$$\mathbf{V}_L = 80\text{ V} \angle +36.87° = 64\text{ V} + j\,48\text{ V}$$

and

$$\begin{aligned}\mathbf{E} = \mathbf{V}_R + \mathbf{V}_L &= (36\text{ V} - j\,48\text{ V}) + (64\text{ V} + j\,48\text{ V}) = 100\text{ V} + j\,0\\ &= 100\text{ V} \angle 0°\end{aligned}$$

as applied.

Phasor diagram: Note that for the phasor diagram of Fig. 15.29, **I** is in phase with the voltage across the resistor and lags the voltage across the inductor by 90°.

Power: The total power in watts delivered to the circuit is

$$\begin{aligned}P_T &= EI\cos\theta_T\\ &= (100\text{ V})(20\text{ A})\cos 53.13° = (2000\text{ W})(0.6)\\ &= \mathbf{1200\text{ W}}\end{aligned}$$

where E and I are effective values and θ_T is the phase angle between E and I, or

$$\begin{aligned}P_T &= I^2R\\ &= (20\text{ A})^2(3\ \Omega) = (400)(3)\\ &= \mathbf{1200\text{ W}}\end{aligned}$$

where I is the effective value, or, finally,

$$\begin{aligned}P_T = P_R + P_L &= V_RI\cos\theta_R + V_LI\cos\theta_L\\ &= (60\text{ V})(20\text{ A})\cos 0° + (80\text{ V})(20\text{ A})\cos 90°\\ &= 1200\text{ W} + 0\\ &= \mathbf{1200\text{ W}}\end{aligned}$$

where θ_R is the phase angle between $\mathbf{V}_R$ and **I**, and θ_L is the phase angle between $\mathbf{V}_L$ and **I**.

Power factor: The power factor F_p of the circuit is cos 53.13° = **0.6 lagging,** where 53.13° is the phase angle between **E** and **I.**

If we write the basic power equation $P = EI\cos\theta$ as follows:

$$\cos\theta = \frac{P}{EI}$$

where E and I are the input quantities and P is the power delivered to the network, and then perform the following substitutions from the basic series ac circuit:

$$\cos\theta = \frac{P}{EI} = \frac{I^2R}{EI} = \frac{IR}{E} = \frac{R}{E/I} = \frac{R}{Z_T}$$

we find

$$\boxed{F_p = \cos\theta_T = \frac{R}{Z_T}} \qquad \mathbf{(15.9)}$$

Reference to Fig. 15.28 also indicates that θ is the impedance angle θ_T as written in Eq. (15.9), further supporting the fact that the impedance angle θ_T is also the phase angle between the input voltage and current for a series ac circuit. To determine the power factor, it is necessary

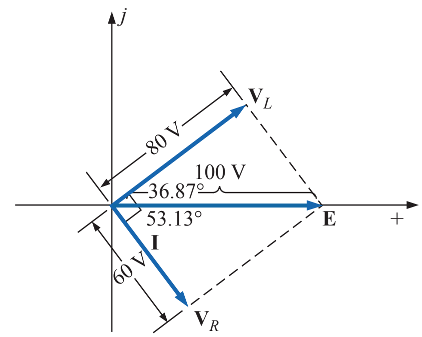


FIG. 15.29
Phasor diagram for the series R-L circuit of Fig. 15.26.

only to form the ratio of the total resistance to the magnitude of the input impedance. For the case at hand,

$$F_p = \cos\theta = \frac{R}{Z_T} = \frac{3\ \Omega}{5\ \Omega} = \mathbf{0.6\ lagging}$$

as found above.

R-C

Refer to Fig. 15.30.

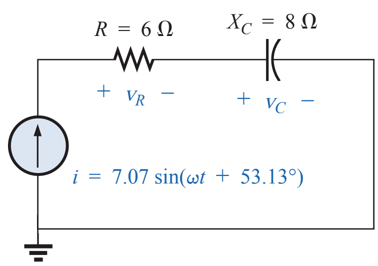


FIG. 15.30
Series R-C ac circuit.

Phasor Notation

$$i = 7.07 \sin(\omega t + 53.13°) \Rightarrow \mathbf{I} = 5\ \text{A}\ \angle 53.13°$$

Note Fig. 15.31.

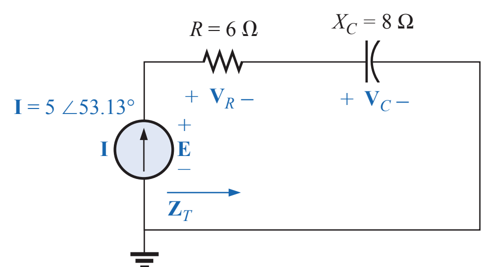


FIG. 15.31
Applying phasor notation to the circuit of Fig. 15.30.

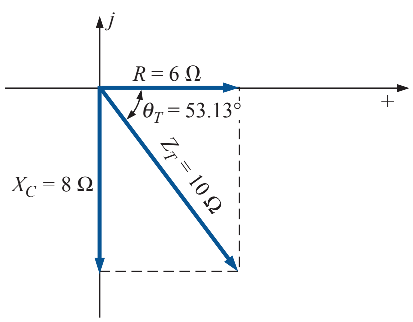


FIG. 15.32
Impedance diagram for the series R-C circuit of Fig. 15.30.

Z_T

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 6\ \Omega\ \angle 0° + 8\ \Omega\ \angle -90° = 6\ \Omega - j8\ \Omega$$

and
$$\mathbf{Z}_T = \mathbf{10\ \Omega\ \angle -53.13°}$$

Impedance diagram: As shown in Fig. 15.32.

E

$$\mathbf{E} = \mathbf{I}\mathbf{Z}_T = (5\ \text{A}\ \angle 53.13°)(10\ \Omega\ \angle -53.13°) = \mathbf{50\ V\ \angle 0°}$$

V_R and V_C

$$\begin{aligned}\mathbf{V}_R &= \mathbf{I}\mathbf{Z}_R = (I\ \angle\theta)(R\ \angle 0°) = (5\ \text{A}\ \angle 53.13°)(6\ \Omega\ \angle 0°)\\ &= \mathbf{30\ V\ \angle 53.13°}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_C &= \mathbf{I}\mathbf{Z}_C = (I\ \angle\theta)(X_C\ \angle -90°) = (5\ \text{A}\ \angle 53.13°)(8\ \Omega\ \angle -90°)\\ &= \mathbf{40\ V\ \angle -36.87°}\end{aligned}$$

Kirchhoff's voltage law:

$$\Sigma_{\circlearrowright} \mathbf{V} = \mathbf{E} - \mathbf{V}_R - \mathbf{V}_C = 0$$

or
$$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_C$$

which can be verified by vector algebra as demonstrated for the *R-L* circuit.

Phasor diagram: Note on the phasor diagram of Fig. 15.33 that the current **I** is in phase with the voltage across the resistor and leads the voltage across the capacitor by 90°.

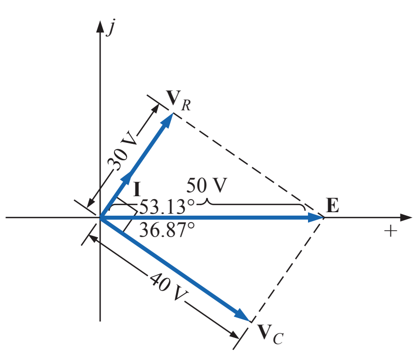


FIG. 15.33
Phasor diagram for the series R-C circuit of Fig. 15.30.

Time domain: In the time domain,

$$e = \sqrt{2}(50) \sin \omega t = \mathbf{70.70 \sin \omega t}$$
$$v_R = \sqrt{2}(30) \sin(\omega t + 53.13°) = \mathbf{42.42 \sin(\omega t + 53.13°)}$$
$$v_C = \sqrt{2}(40) \sin(\omega t - 36.87°) = \mathbf{56.56 \sin(\omega t - 36.87°)}$$

A plot of all of the voltages and the current of the circuit appears in Fig. 15.34. Note again that i and v_R are in phase and that v_C lags i by 90°.

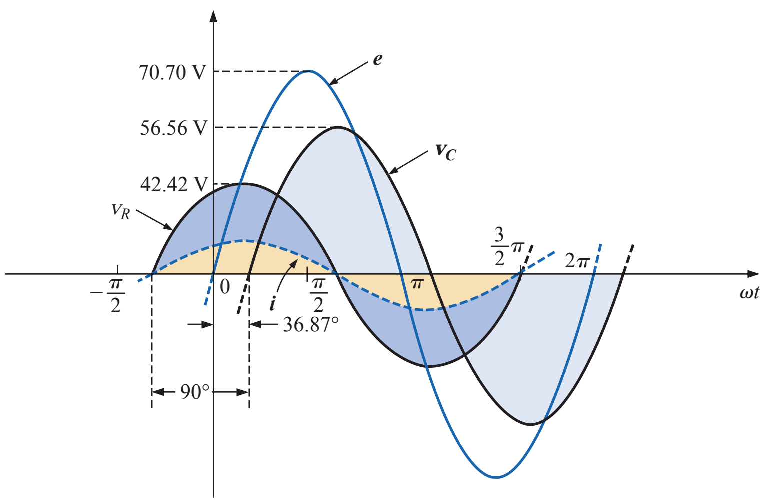


FIG. 15.34
Waveforms for the series R-C circuit of Fig. 15.30.

Power: The total power in watts delivered to the circuit is

$$P_T = EI \cos \theta_T = (50 \text{ V})(5 \text{ A}) \cos 53.13°$$
$$= (250)(0.6) = \mathbf{150 \text{ W}}$$

or

$$P_T = I^2R = (5 \text{ A})^2(6 \ \Omega) = (25)(6)$$
$$= \mathbf{150 \text{ W}}$$

or, finally,

$$P_T = P_R + P_C = V_R I \cos \theta_R + V_C I \cos \theta_C$$
$$= (30 \text{ V})(5 \text{ A}) \cos 0° + (40 \text{ V})(5 \text{ A}) \cos 90°$$
$$= 150 \text{ W} + 0$$
$$= \mathbf{150 \text{ W}}$$

Power factor: The power factor of the circuit is

$$F_p = \cos \theta = \cos 53.13° = \mathbf{0.6 \text{ leading}}$$

Using Eq. (15.9), we obtain

$$F_p = \cos \theta = \frac{R}{Z_T} = \frac{6 \ \Omega}{10 \ \Omega}$$
$$= \mathbf{0.6 \text{ leading}}$$

as determined above.

R-L-C

Refer to Fig. 15.35.

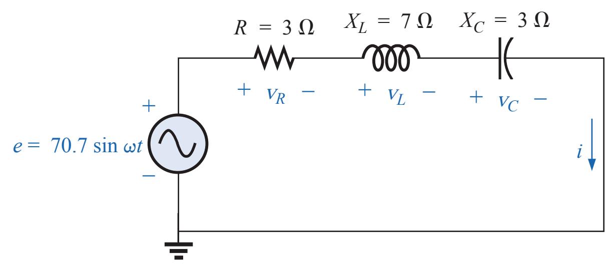


FIG. 15.35
Series R-L-C ac circuit.

Phasor Notation As shown in Fig. 15.36.

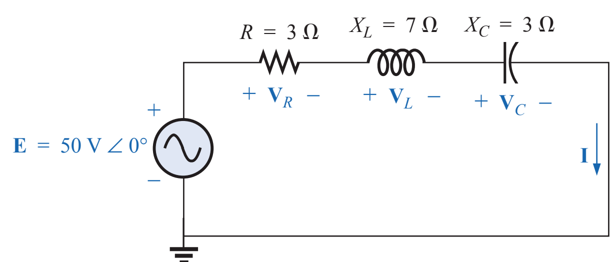


FIG. 15.36
Applying phasor notation to the circuit of Fig. 15.35.

$\mathbf{Z}_T$

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 = R\ \angle 0° + X_L\ \angle 90° + X_C\ \angle -90°$$
$$= 3\ \Omega + j\,7\ \Omega - j\,3\ \Omega = 3\ \Omega + j\,4\ \Omega$$

and $\mathbf{Z}_T = \mathbf{5\ \Omega\ \angle 53.13°}$

Impedance diagram: As shown in Fig. 15.37.

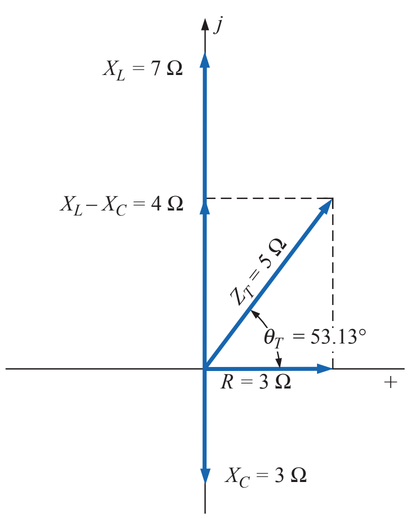


FIG. 15.37
Impedance diagram for the series R-L-C circuit of Fig. 15.35.

I

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{50\ \text{V}\ \angle 0°}{5\ \Omega\ \angle 53.13°} = \mathbf{10\ A\ \angle -53.13°}$$

$\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (I\ \angle\theta)(R\ \angle 0°) = (10\ \text{A}\ \angle -53.13°)(3\ \Omega\ \angle 0°)$$
$$= \mathbf{30\ V\ \angle -53.13°}$$

$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (I\ \angle\theta)(X_L\ \angle 90°) = (10\ \text{A}\ \angle -53.13°)(7\ \Omega\ \angle 90°)$$
$$= \mathbf{70\ V\ \angle 36.87°}$$

$$\mathbf{V}_C = \mathbf{I}\mathbf{Z}_C = (I\ \angle\theta)(X_C\ \angle -90°) = (10\ \text{A}\ \angle -53.13°)(3\ \Omega\ \angle -90°)$$
$$= \mathbf{30\ V\ \angle -143.13°}$$

Kirchhoff's voltage law:

$$\Sigma_{\circlearrowright}\ \mathbf{V} = \mathbf{E} - \mathbf{V}_R - \mathbf{V}_L - \mathbf{V}_C = 0$$

or $$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C$$

which can also be verified through vector algebra.

Phasor diagram: The phasor diagram of Fig. 15.38 indicates that the current **I** is in phase with the voltage across the resistor, lags the voltage across the inductor by 90°, and leads the voltage across the capacitor by 90°.

Time domain:

$$i = \sqrt{2}(10)\sin(\omega t - 53.13°) = \mathbf{14.14\ sin(\boldsymbol{\omega} t - 53.13°)}$$
$$v_R = \sqrt{2}(30)\sin(\omega t - 53.13°) = \mathbf{42.42\ sin(\boldsymbol{\omega} t - 53.13°)}$$
$$v_L = \sqrt{2}(70)\sin(\omega t + 36.87°) = \mathbf{98.98\ sin(\boldsymbol{\omega} t + 36.87°)}$$
$$v_C = \sqrt{2}(30)\sin(\omega t - 143.13°) = \mathbf{42.42\ sin(\boldsymbol{\omega} t - 143.13°)}$$

A plot of all the voltages and the current of the circuit appears in Fig. 15.39.

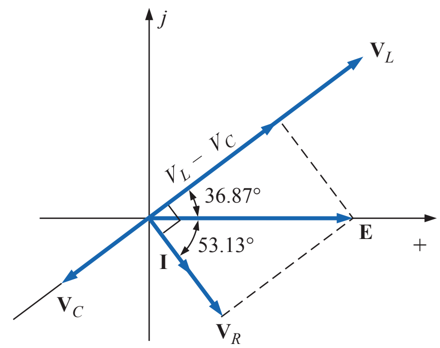


FIG. 15.38
Phasor diagram for the series R-L-C circuit of Fig. 15.35.

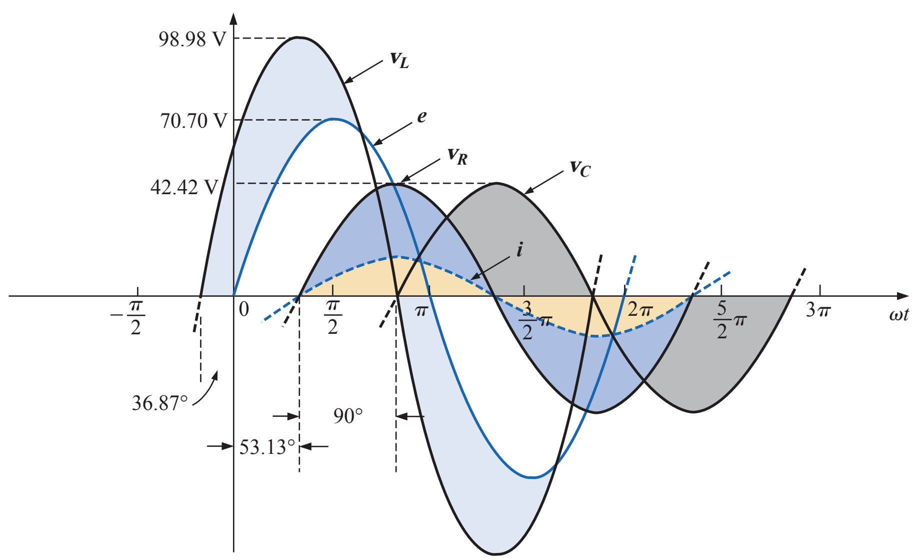


FIG. 15.39
Waveforms for the series R-L circuit of Fig. 15.35.

Power: The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta_T = (50\text{ V})(10\text{ A})\cos 53.13° = (500)(0.6) = \mathbf{300\ W}$$

or $$P_T = I^2R = (10\text{ A})^2(3\ \Omega) = (100)(3) = \mathbf{300\ W}$$

or

$$\begin{aligned}P_T &= P_R + P_L + P_C\\ &= V_RI\cos\theta_R + V_LI\cos\theta_L + V_CI\cos\theta_C\\ &= (30\text{ V})(10\text{ A})\cos 0° + (70\text{ V})(10\text{ A})\cos 90° + (30\text{ V})(10\text{ A})\cos 90°\\ &= (30\text{ V})(10\text{ A}) + 0 + 0 = \mathbf{300\ W}\end{aligned}$$

Power factor: The power factor of the circuit is

$$F_p = \cos\theta_T = \cos 53.13° = \mathbf{0.6\ lagging}$$

Using Eq. (15.9), we obtain

$$F_p = \cos\theta = \frac{R}{Z_T} = \frac{3\ \Omega}{5\ \Omega} = \mathbf{0.6\ lagging}$$

15.4 VOLTAGE DIVIDER RULE

The basic format for the **voltage divider rule** in ac circuits is exactly the same as that for dc circuits:

$$\mathbf{V}_x = \frac{\mathbf{Z}_x\mathbf{E}}{\mathbf{Z}_T} \qquad \textbf{(15.10)}$$

where $\mathbf{V}_x$ is the voltage across one or more elements in series that have total impedance $\mathbf{Z}_x$, $\mathbf{E}$ is the total voltage appearing across the series circuit, and $\mathbf{Z}_T$ is the total impedance of the series circuit.

EXAMPLE 15.9 Using the voltage divider rule, find the voltage across each element of the circuit of Fig. 15.40.

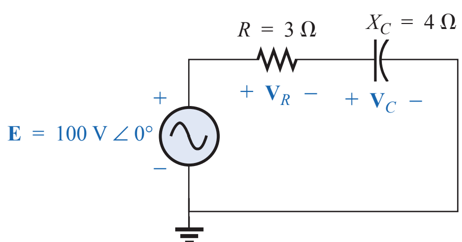


FIG. 15.40
Example 15.9.

Solution:

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(4\ \Omega \angle -90^\circ)(100\ \text{V} \angle 0^\circ)}{4\ \Omega \angle -90^\circ + 3\ \Omega \angle 0^\circ} = \frac{400 \angle -90^\circ}{3 - j\,4}$$

$$= \frac{400 \angle -90^\circ}{5 \angle -53.13^\circ} = \mathbf{80\ V \angle -36.87^\circ}$$

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(3\ \Omega \angle 0^\circ)(100\ \text{V} \angle 0^\circ)}{5\ \Omega \angle -53.13^\circ} = \frac{300 \angle 0^\circ}{5 \angle -53.13^\circ}$$

$$= \mathbf{60\ V \angle +53.13^\circ}$$

EXAMPLE 15.10 Using the voltage divider rule, find the unknown voltages $\mathbf{V}_R$, $\mathbf{V}_L$, $\mathbf{V}_C$, and $\mathbf{V}_1$ for the circuit of Fig. 15.41.

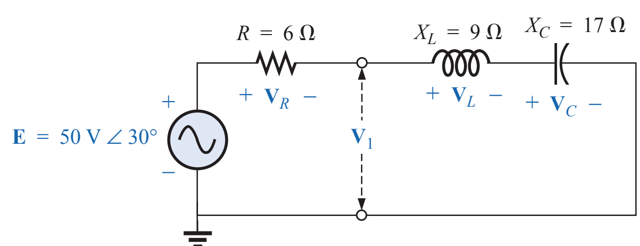


FIG. 15.41
Example 15.10.

Solution:

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_R + \mathbf{Z}_L + \mathbf{Z}_C} = \frac{(6\ \Omega \angle 0^\circ)(50\ \text{V} \angle 30^\circ)}{6\ \Omega \angle 0^\circ + 9\ \Omega \angle 90^\circ + 17\ \Omega \angle -90^\circ}$$

$$= \frac{300 \angle 30^\circ}{6 + j\,9 - j\,17} = \frac{300 \angle 30^\circ}{6 - j\,8}$$

$$= \frac{300 \angle 30^\circ}{10 \angle -53.13^\circ} = \mathbf{30\ V \angle 83.13^\circ}$$

Calculator The above calculation provides an excellent opportunity to demonstrate the power of today's calculators. Using the notation of the TI-86 calculator, the above calculation and the result are as follows:

```
(6∠0)*(50∠30)/((6∠0)+(9∠90)+(17∠−90))
      (3.588E0,29.785E0)
Ans ▶ Pol
      (30.000E0∠83.130E0)
```

CALC. 15.1

$$\mathbf{V}_L = \frac{\mathbf{Z}_L\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega\ \angle 90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ} = \frac{450\ \text{V}\angle 120^\circ}{10\ \angle -53.13^\circ}$$
$$= \mathbf{45\ V\ \angle 173.13^\circ}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(17\ \Omega\ \angle -90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ} = \frac{850\ \text{V}\angle -60^\circ}{10\ \angle -53^\circ}$$
$$= \mathbf{85\ V\ \angle -6.87^\circ}$$

$$\mathbf{V}_1 = \frac{(\mathbf{Z}_L + \mathbf{Z}_C)\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega\ \angle 90^\circ + 17\ \Omega\ \angle -90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ}$$
$$= \frac{(8\ \angle -90^\circ)(50\ \angle 30^\circ)}{10\ \angle -53.13^\circ}$$
$$= \frac{400\ \angle -60^\circ}{10\ \angle -53.13^\circ} = \mathbf{40\ V\ \angle -6.87^\circ}$$

EXAMPLE 15.11 For the circuit of Fig. 15.42:

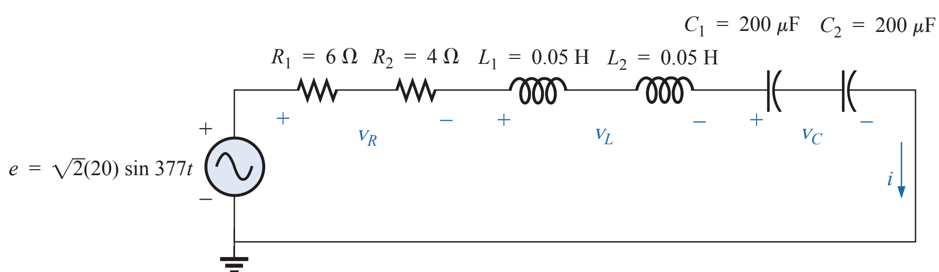


FIG. 15.42
Example 15.11.

a. Calculate $\mathbf{I}$, $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ in phasor form.
b. Calculate the total power factor.
c. Calculate the average power delivered to the circuit.
d. Draw the phasor diagram.
e. Obtain the phasor sum of $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$, and show that it equals the input voltage $\mathbf{E}$.
f. Find $\mathbf{V}_R$ and $\mathbf{V}_C$ using the voltage divider rule.

Solutions:

a. Combining common elements and finding the reactance of the inductor and capacitor, we obtain

$$R_T = 6\ \Omega + 4\ \Omega = 10\ \Omega$$
$$L_T = 0.05\ \text{H} + 0.05\ \text{H} = 0.1\ \text{H}$$
$$C_T = \frac{200\ \mu\text{F}}{2} = 100\ \mu\text{F}$$

$$X_L = \omega L = (377 \text{ rad/s})(0.1 \text{ H}) = 37.70\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(377 \text{ rad/s})(100 \times 10^{-6} \text{ F})} = \frac{10^6\ \Omega}{37{,}700} = 26.53\ \Omega$$

Redrawing the circuit using phasor notation results in Fig. 15.43.

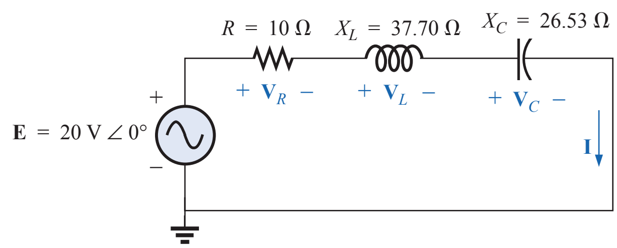


FIG. 15.43
Applying phasor notation to the circuit of Fig. 15.42.

For the circuit of Fig. 15.43,

$$\begin{aligned}\mathbf{Z}_T &= R\angle 0° + X_L\angle 90° + X_C\angle -90° \\ &= 10\ \Omega + j\,37.70\ \Omega - j\,26.53\ \Omega \\ &= 10\ \Omega + j\,11.17\ \Omega = \mathbf{15\ \Omega\ \angle 48.16°}\end{aligned}$$

The current **I** is

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{20 \text{ V} \angle 0°}{15\ \Omega \angle 48.16°} = \mathbf{1.33\ A\ \angle -48.16°}$$

The voltage across the resistor, inductor, and capacitor can be found using Ohm's law:

$$\begin{aligned}\mathbf{V}_R = \mathbf{IZ}_R &= (I\angle\theta)(R\angle 0°) = (1.33 \text{ A} \angle -48.16°)(10\ \Omega \angle 0°) \\ &= \mathbf{13.30\ V\ \angle -48.16°}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_L = \mathbf{IZ}_L &= (I\angle\theta)(X_L\angle 90°) = (1.33 \text{ A} \angle -48.16°)(37.70\ \Omega \angle 90°) \\ &= \mathbf{50.14\ V\ \angle 41.84°}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_C = \mathbf{IZ}_C &= (I\angle\theta)(X_C\angle -90°) = (1.33 \text{ A} \angle -48.16°)(26.53\ \Omega \angle -90°) \\ &= \mathbf{35.28\ V\ \angle -138.16°}\end{aligned}$$

b. The total power factor, determined by the angle between the applied voltage **E** and the resulting current **I**, is 48.16°:

$$F_p = \cos\theta = \cos 48.16° = \mathbf{0.667\ lagging}$$

or $$F_p = \cos\theta = \frac{R}{Z_T} = \frac{10\ \Omega}{15\ \Omega} = \mathbf{0.667\ lagging}$$

c. The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta = (20 \text{ V})(1.33 \text{ A})(0.667) = \mathbf{17.74\ W}$$

d. The phasor diagram appears in Fig. 15.44.

e. The phasor sum of $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ is

$$\begin{aligned}\mathbf{E} &= \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C \\ &= 13.30 \text{ V} \angle -48.16° + 50.14 \text{ V} \angle 41.84° + 35.28 \text{ V} \angle -138.16° \\ \mathbf{E} &= 13.30 \text{ V} \angle -48.16° + 14.86 \text{ V} \angle 41.84°\end{aligned}$$

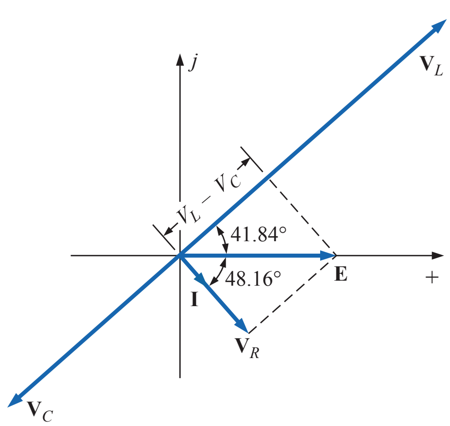


FIG. 15.44
Phasor diagram for the circuit of Fig. 15.42.

Therefore,

$$E = \sqrt{(13.30\text{ V})^2 + (14.86\text{ V})^2} = \mathbf{20\ V}$$

and $\qquad \theta_E = \mathbf{0°} \qquad$ (from phasor diagram)

and $\qquad \mathbf{E} = 20\ \angle 0°$

f. $$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_T} = \frac{(10\ \Omega\ \angle 0°)(20\text{ V}\ \angle 0°)}{15\ \Omega\ \angle 48.16°} = \frac{200\text{ V}\angle 0°}{15\ \angle 48.16°}$$

$$= \mathbf{13.3\ V\ \angle -48.16°}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(26.5\ \Omega\ \angle -90°)(20\text{ V}\ \angle 0°)}{15\ \Omega\ \angle 48.16°} = \frac{530.6\text{ V}\angle -90°}{15\ \angle 48.16°}$$

$$= \mathbf{35.37\ V\ \angle -138.16°}$$

15.5 FREQUENCY RESPONSE OF THE *R-C* CIRCUIT

Thus far, the analysis of series circuits has been limited to a particular frequency. We will now examine the effect of frequency on the response of an *R-C* series configuration such as that in Fig. 15.45. The magnitude of the source is fixed at 10 V, but the frequency range of analysis will extend from zero to 20 kHz.

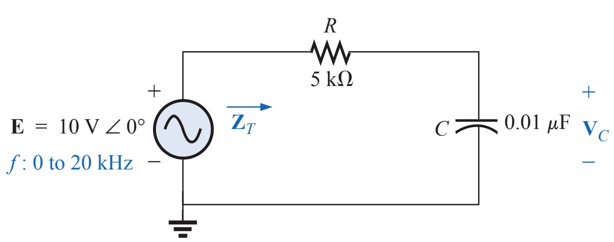


FIG. 15.45
Determining the frequency response of a series R-C circuit.

$\mathbf{Z}_T$ Let us first determine how the impedance of the circuit $\mathbf{Z}_T$ will vary with frequency for the specified frequency range of interest. Before getting into specifics, however, let us first develop a sense for what we should expect by noting the impedance-versus-frequency curve of each element, as drawn in Fig. 15.46.

At low frequencies the reactance of the capacitor will be quite high and considerably more than the level of the resistance R, suggesting that the total impedance will be primarily capacitive in nature. At high frequencies the reactance X_C will drop below the $R = 5\text{-k}\Omega$ level, and the network will start to shift toward one of a purely resistive nature (at 5 kΩ). The frequency at which $X_C = R$ can be determined in the following manner:

$$X_C = \frac{1}{2\pi f_1 C} = R$$

and
$$\boxed{f_1 = \frac{1}{2\pi RC}} \quad {}_{X_C = R} \qquad \textbf{(15.11)}$$

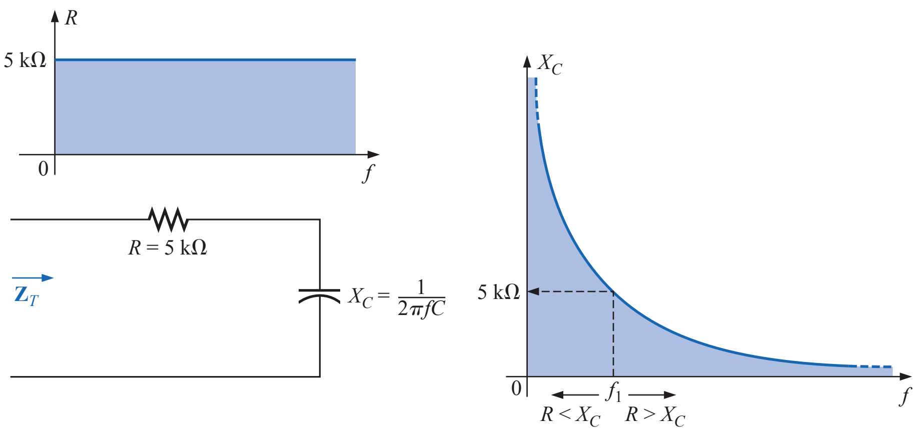


FIG. 15.46
The frequency response of the individual elements of a series R-C circuit.

which for the network of interest is

$$f_1 = \frac{1}{2\pi(5\text{ k}\Omega)(0.01\ \mu\text{F})} \cong \mathbf{3183.1\ Hz}$$

For frequencies less than f_1, $X_C > R$, and for frequencies greater than f_1, $R > X_C$, as shown in Fig. 15.46.

Now for the details. The total impedance is determined by the following equation:

$$\mathbf{Z}_T = R - j\,X_C$$

and

$$\boxed{\mathbf{Z}_T = Z_T \angle \theta_T = \sqrt{R^2 + X_C^2}\ \angle -\tan^{-1}\frac{X_C}{R}} \qquad \textbf{(15.12)}$$

The magnitude and angle of the total impedance can now be found at any frequency of interest by simply substituting into Eq. (15.12). The presence of the capacitor suggests that we start from a low frequency (100 Hz) and then open the spacing until we reach the upper limit of interest (20 kHz).

***f* = 100 Hz**

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(100\text{ Hz})(0.01\ \mu\text{F})} = 159.16\text{ k}\Omega$$

and $Z_T = \sqrt{R^2 + X_C^2} = \sqrt{(5\text{ k}\Omega)^2 + (159.16\text{ k}\Omega)^2} = 159.24\text{ k}\Omega$

with $\theta_T = -\tan^{-1}\dfrac{X_C}{R} = -\tan^{-1}\dfrac{159.16\text{ k}\Omega}{5\text{ k}\Omega} = -\tan^{-1} 31.83$
$= -88.2°$

and $$\mathbf{Z}_T = \mathbf{159.24\ k\Omega \angle -88.2°}$$

which compares very closely with $\mathbf{Z}_C = 159.16\text{ k}\Omega \angle -90°$ if the circuit were purely capacitive ($R = 0\ \Omega$). Our assumption that the circuit is primarily capacitive at low frequencies is therefore confirmed.

f = 1 kHz

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1\text{ kHz})(0.01\ \mu\text{F})} = 15.92\text{ k}\Omega$$

and $\quad Z_T = \sqrt{R^2 + X_C^2} = \sqrt{(5\text{ k}\Omega)^2 + (15.92\text{ k}\Omega)^2} = 16.69\text{ k}\Omega$

with
$$\theta_T = -\tan^{-1}\frac{X_C}{R} = -\tan^{-1}\frac{15.92\text{ k}\Omega}{5\text{ k}\Omega}$$
$$= -\tan^{-1} 3.18 = -72.54°$$

and $\quad \mathbf{Z}_T = \mathbf{16.69\ k\Omega\ \angle -72.54°}$

A noticeable drop in the magnitude has occurred, and the impedance angle has dropped almost 17° from the purely capacitive level.

Continuing:

$$f = 5\text{ kHz:}\quad \mathbf{Z}_T = \mathbf{5.93\ k\Omega\ \angle -32.48°}$$
$$f = 10\text{ kHz:}\quad \mathbf{Z}_T = \mathbf{5.25\ k\Omega\ \angle -17.66°}$$
$$f = 15\text{ kHz:}\quad \mathbf{Z}_T = \mathbf{5.11\ k\Omega\ \angle -11.98°}$$
$$f = 20\text{ kHz:}\quad \mathbf{Z}_T = \mathbf{5.06\ k\Omega\ \angle -9.04°}$$

Note how close the magnitude of Z_T at f = 20 kHz is to the resistance level of 5 kΩ. In addition, note how the phase angle is approaching that associated with a pure resistive network (0°).

A plot of Z_T versus frequency in Fig. 15.47 completely supports our assumption based on the curves of Fig. 15.46. The plot of θ_T versus frequency in Fig. 15.48 further suggests the fact that the total impedance made a transition from one of a capacitive nature ($\theta_T = -90°$) to one with resistive characteristics ($\theta_T = 0°$).

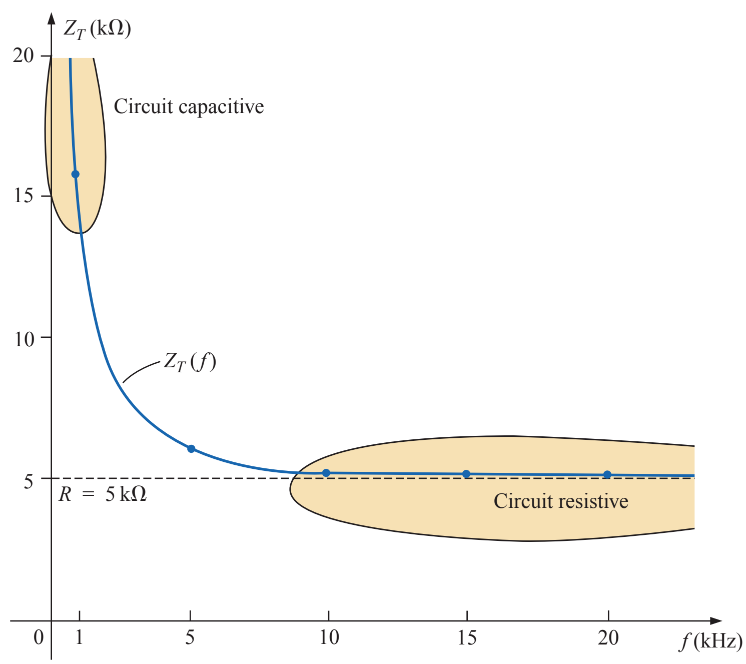


FIG. 15.47

The magnitude of the input impedance versus frequency for the circuit of Fig. 15.45.

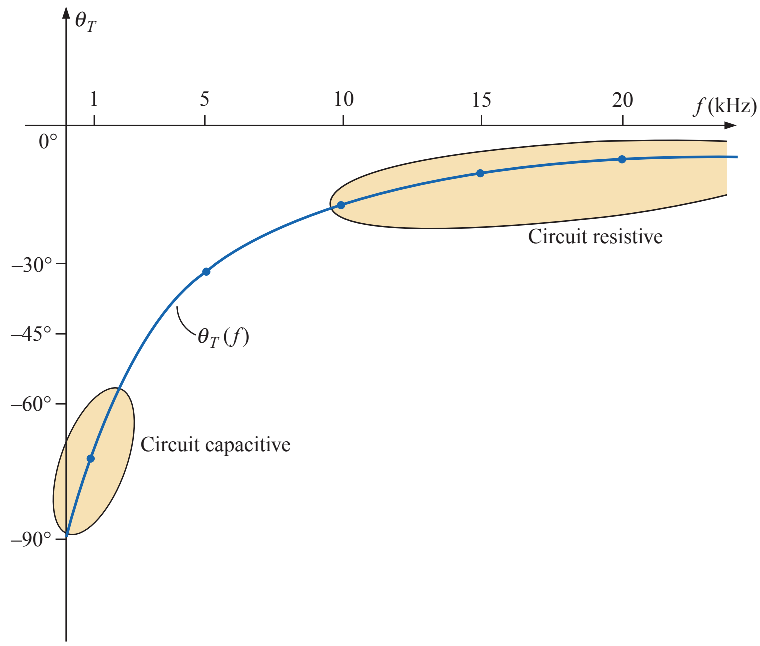


FIG. 15.48
The phase angle of the input impedance versus frequency for the circuit of Fig. 15.45.

Applying the voltage divider rule to determine the voltage across the capacitor in phasor form yields

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_R + \mathbf{Z}_C}$$

$$= \frac{(X_C \angle -90°)(E \angle 0°)}{R - j X_C} = \frac{X_C E \angle -90°}{R - j X_C}$$

$$= \frac{X_C E \angle -90°}{\sqrt{R^2 + X_C^2}\ \angle -\tan^{-1} X_C/R}$$

or $$\mathbf{V}_C = V_C \angle \theta_C = \frac{X_C E}{\sqrt{R^2 + X_C^2}} \angle -90° + \tan^{-1}(X_C/R)$$

The magnitude of $\mathbf{V}_C$ is therefore determined by

$$\boxed{V_C = \frac{X_C E}{\sqrt{R^2 + X_C^2}}} \tag{15.13}$$

and the phase angle θ_C by which $\mathbf{V}_C$ leads $\mathbf{E}$ is given by

$$\boxed{\theta_C = -90° + \tan^{-1}\frac{X_C}{R} = -\tan^{-1}\frac{R}{X_C}} \tag{15.14}$$

To determine the frequency response, X_C must be calculated for each frequency of interest and inserted into Eqs. (15.13) and (15.14).

To begin our analysis, it makes good sense to consider the case of $f = 0$ Hz (dc conditions).

***f* = 0 Hz**

$$X_C = \frac{1}{2\pi(0)C} = \frac{1}{0} \Rightarrow \text{very large value}$$

Applying the open-circuit equivalent for the capacitor based on the above calculation will result in the following:

$$\mathbf{V}_C = \mathbf{E} = 10\text{ V}\angle 0°$$

If we apply Eq. (15.13), we find

$$X_C^2 >> R^2$$

and $$\sqrt{R^2 + X_C^2} \cong \sqrt{X_C^2} = X_C$$

and $$V_C = \frac{X_C E}{\sqrt{R^2 + X_C^2}} = \frac{X_C E}{X_C} = E$$

with $$\theta_C = -\tan^{-1}\frac{R}{X_C} = -\tan^{-1} 0 = 0°$$

verifying the above conclusions.

***f* = 1 kHz** Applying Eq. (15.13):

$$X_C = \frac{1}{2\pi fC} = \frac{1}{(2\pi)(1 \times 10^3\text{ Hz})(0.01 \times 10^{-6}\text{ F})} \cong \mathbf{15.92\ k\Omega}$$

$$\sqrt{R^2 + X_C^2} = \sqrt{(5\text{ k}\Omega)^2 + (15.92\text{ k}\Omega)^2} \cong 16.69\text{ k}\Omega$$

and $$V_C = \frac{X_C E}{\sqrt{R^2 + X_C^2}} = \frac{(15.92\text{ k}\Omega)(10)}{16.69\text{ k}\Omega} = \mathbf{9.54\ V}$$

Applying Eq. (15.14):

$$\theta_C = -\tan^{-1}\frac{R}{X_C} = -\tan^{-1}\frac{5\text{ k}\Omega}{15.9\text{ k}\Omega}$$

$$= -\tan^{-1} 0.314 = \mathbf{-17.46°}$$

and $$\mathbf{V}_C = \mathbf{9.53\ V\angle -17.46°}$$

As expected, the high reactance of the capacitor at low frequencies has resulted in the major part of the applied voltage appearing across the capacitor.

If we plot the phasor diagrams for $f = 0$ Hz and $f = 1$ kHz, as shown in Fig. 15.49, we find that $\mathbf{V}_C$ is beginning a clockwise rotation with an increase in frequency that will increase the angle θ_C and decrease the phase angle between **I** and **E.** Recall that for a purely capacitive net-

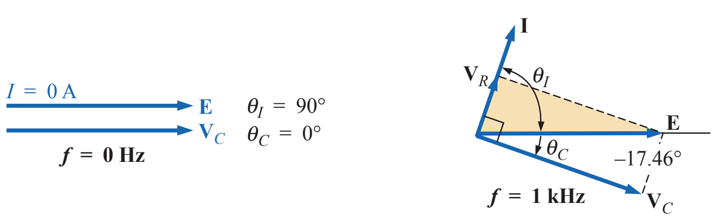


FIG. 15.49
The phasor diagram for the circuit of Fig. 15.45 for f = 0 Hz and 1 kHz.

work, **I** leads **E** by 90°. As the frequency increases, therefore, the capacitive reactance is decreasing, and eventually $R >> X_C$ with $\theta_C = -90°$, and the angle between **I** and **E** will approach 0°. Keep in mind as we proceed through the other frequencies that θ_C is the phase angle between $\mathbf{V}_C$ and **E** and that the magnitude of the angle by which **I** leads **E** is determined by

$$\boxed{|\theta_I| = 90° - |\theta_C|} \tag{15.15}$$

***f* = 5 kHz** Applying Eq. (15.13):

$$X_C = \frac{1}{2\pi fC} = \frac{1}{(2\pi)(5 \times 10^3 \text{ Hz})(0.01 \times 10^{-6} \text{ F})} \cong \mathbf{3.18\ k\Omega}$$

Note the dramatic drop in X_C from 1 kHz to 5 kHz. In fact, X_C is now less than the resistance R of the network, and the phase angle determined by $\tan^{-1}(X_C/R)$ must be less than 45°. Here,

$$V_C = \frac{X_C E}{\sqrt{R^2 + X_C^2}} = \frac{(3.18 \text{ k}\Omega)(10 \text{ V})}{\sqrt{(5 \text{ k}\Omega)^2 + (3.18 \text{ k}\Omega)^2}} = \mathbf{5.37\ V}$$

with $$\theta_C = -\tan^{-1}\frac{R}{X_C} = -\tan^{-1}\frac{5 \text{ k}\Omega}{3.2 \text{ k}\Omega}$$
$$= -\tan^{-1} 1.56 = \mathbf{-57.38°}$$

***f* = 10 kHz**

$$X_C \cong \mathbf{1.59\ k\Omega} \qquad V_C = \mathbf{3.03\ V} \qquad \theta_C = \mathbf{-72.34°}$$

***f* = 15 kHz**

$$X_C \cong \mathbf{1.06\ k\Omega} \qquad V_C = \mathbf{2.07\ V} \qquad \theta_C = \mathbf{-78.02°}$$

***f* = 20 kHz**

$$X_C \cong \mathbf{795.78\ \Omega} \qquad V_C = \mathbf{1.57\ V} \qquad \theta_C = \mathbf{-80.96°}$$

The phasor diagrams for $f = 5$ kHz and $f = 20$ kHz appear in Fig. 15.50 to show the continuing rotation of the $\mathbf{V}_C$ vector.

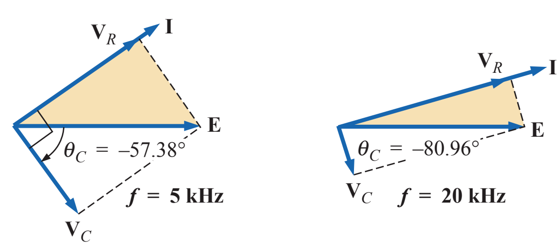


FIG. 15.50
The phasor diagram for the circuit of Fig. 15.45 for f = 5 kHz and 20 kHz.

Note also from Figs. 15.49 and 15.50 that the vector $\mathbf{V}_R$ and the current **I** have grown in magnitude with the reduction in the capacitive reactance. Eventually, at very high frequencies X_C will approach zero ohms and the short-circuit equivalent can be applied, resulting in $V_C \cong 0$ V and $\theta_C \cong -90°$, and producing the phasor diagram of Fig. 15.51. The network is then resistive, the phase angle between **I** and **E** is essentially zero degrees, and V_R and I are their maximum values.

A plot of V_C versus frequency appears in Fig. 15.52. At low frequencies $X_C >> R$, and V_C is very close to E in magnitude. As the applied frequency increases, X_C decreases in magnitude along with V_C as V_R captures more of the applied voltage. A plot of θ_C versus frequency is provided in Fig. 15.53. At low frequencies the phase angle

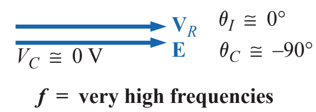


FIG. 15.51

The phasor diagram for the circuit of Fig. 15.45 at very high frequencies.

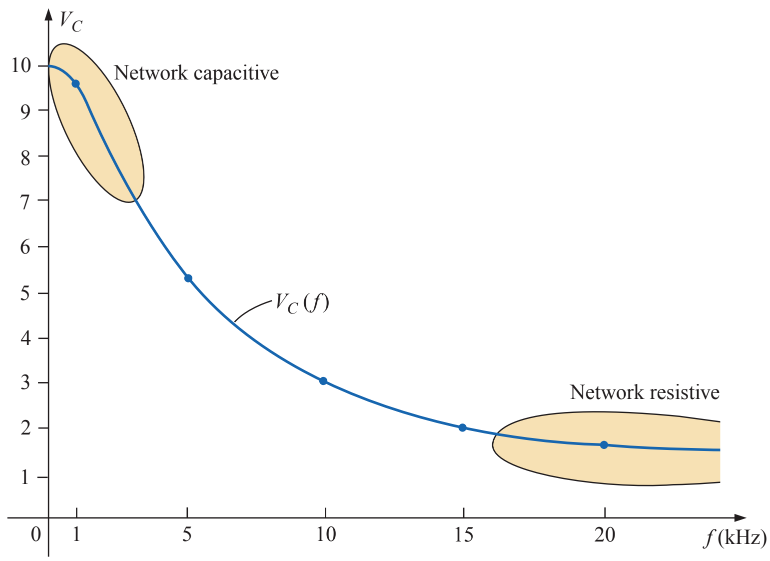


FIG. 15.52

The magnitude of the voltage V_C versus frequency for the circuit of Fig. 15.45.

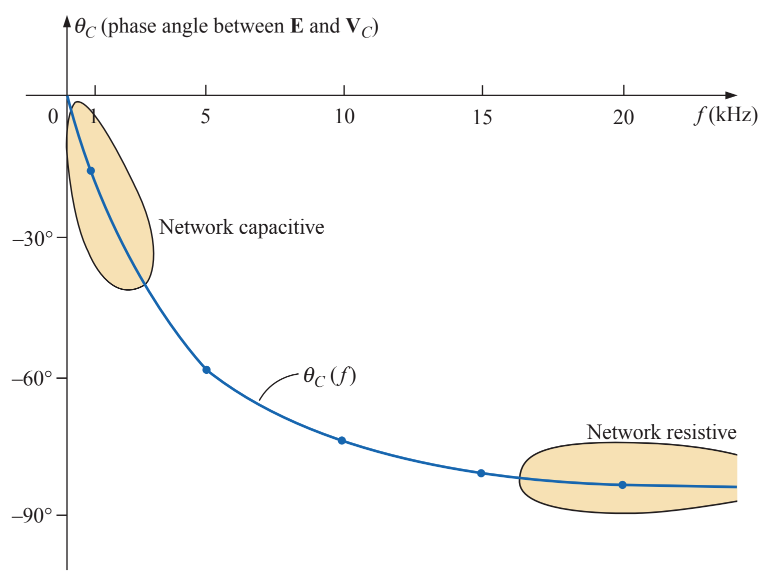


FIG. 15.53

*The phase angle between **E** and $\mathbf{V}_C$ versus frequency for the circuit of Fig. 15.45.*

between $\mathbf{V}_C$ and $\mathbf{E}$ is very small since $\mathbf{V}_C \cong \mathbf{E}$. Recall that if two phasors are equal, they must have the same angle. As the applied frequency increases, the network becomes more resistive and the phase angle between $\mathbf{V}_C$ and $\mathbf{E}$ approaches 90°. Keep in mind that, at high frequencies, $\mathbf{I}$ and $\mathbf{E}$ are approaching an in-phase situation and the angle between $\mathbf{V}_C$ and $\mathbf{E}$ will approach that between $\mathbf{V}_C$ and $\mathbf{I}$, which we know must be 90° ($\mathbf{I}_C$ leading $\mathbf{V}_C$).

A plot of V_R versus frequency would approach E volts from zero volts with an increase in frequency, but remember $V_R \neq E - V_C$ due to the vector relationship. The phase angle between $\mathbf{I}$ and $\mathbf{E}$ could be plotted directly from Fig. 15.53 using Eq. (15.15).

In Chapter 23, the analysis of this section will be extended to a much wider frequency range using a log axis for frequency. It will be demonstrated that an *R-C* circuit such as that in Fig. 15.45 can be used as a filter to determine which frequencies will have the greatest impact on the stage to follow. From our current analysis, it is obvious that any network connected across the capacitor will receive the greatest potential level at low frequencies and be effectively "shorted out" at very high frequencies.

The analysis of a series *R-L* circuit would proceed in much the same manner, except that X_L and V_L would increase with frequency and the angle between $\mathbf{I}$ and $\mathbf{E}$ would approach 90° (voltage leading the current) rather than 0°. If $\mathbf{V}_L$ were plotted versus frequency, $\mathbf{V}_L$ would approach $\mathbf{E}$, and X_L would eventually attain a level at which the open-circuit equivalent would be appropriate.

15.6 SUMMARY: SERIES ac CIRCUITS

The following is a review of important conclusions that can be derived from the discussion and examples of the previous sections. The list is not all-inclusive, but it does emphasize some of the conclusions that should be carried forward in the future analysis of ac systems.

For series ac circuits with reactive elements:

1. ***The total impedance will be frequency dependent.***
2. ***The impedance of any one element can be greater than the total impedance of the network.***
3. ***The inductive and capacitive reactances are always in direct opposition on an impedance diagram.***
4. ***Depending on the frequency applied, the same circuit can be either predominantly inductive or predominantly capacitive.***
5. ***At lower frequencies the capacitive elements will usually have the most impact on the total impedance, while at high frequencies the inductive elements will usually have the most impact.***
6. ***The magnitude of the voltage across any one element can be greater than the applied voltage.***
7. ***The magnitude of the voltage across an element compared to the other elements of the circuit is directly related to the magnitude of its impedance; that is, the larger the impedance of an element, the larger the magnitude of the voltage across the element.***
8. ***The voltages across a coil or capacitor are always in direct opposition on a phasor diagram.***
9. ***The current is always in phase with the voltage across the resistive elements, lags the voltage across all the inductive***

elements by 90°, and leads the voltage across all the capacitive elements by 90°.

10. ***The larger the resistive element of a circuit compared to the net reactive impedance, the closer the power factor is to unity.***

PARALLEL ac CIRCUITS

15.7 ADMITTANCE AND SUSCEPTANCE

The discussion for **parallel ac circuits** will be very similar to that for dc circuits. In dc circuits, *conductance* (G) was defined as being equal to $1/R$. The total conductance of a parallel circuit was then found by adding the conductance of each branch. The total resistance R_T is simply $1/G_T$.

In ac circuits, we define **admittance** ($\mathbf{Y}$) as being equal to $1/\mathbf{Z}$. The unit of measure for admittance as defined by the SI system is *siemens*, which has the symbol S. Admittance is a measure of how well an ac circuit will *admit*, or allow, current to flow in the circuit. The larger its value, therefore, the heavier the current flow for the same applied potential. The total admittance of a circuit can also be found by finding the sum of the parallel admittances. The total impedance $\mathbf{Z}_T$ of the circuit is then $1/\mathbf{Y}_T$; that is, for the network of Fig. 15.54:

$$\boxed{\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \cdots + \mathbf{Y}_N} \tag{15.16}$$

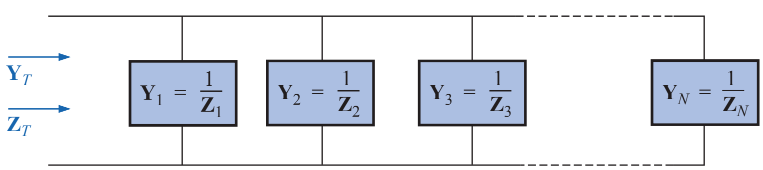


FIG. 15.54
Parallel ac network.

or, since $\mathbf{Z} = 1/\mathbf{Y}$,

$$\boxed{\frac{1}{\mathbf{Z}_T} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \cdots + \frac{1}{\mathbf{Z}_N}} \tag{15.17}$$

For two impedances in parallel,

$$\frac{1}{\mathbf{Z}_T} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}$$

If the manipulations used in Chapter 6 to find the total resistance of two parallel resistors are now applied, the following similar equation will result:

$$\boxed{\mathbf{Z}_T = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}} \tag{15.18}$$

For three parallel impedances,

$$\boxed{\mathbf{Z}_T = \frac{\mathbf{Z}_1\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3}} \tag{15.19}$$

As pointed out in the introduction to this section, conductance is the reciprocal of resistance, and

$$\boxed{\mathbf{Y}_R = \frac{1}{\mathbf{Z}_R} = \frac{1}{R \angle 0^\circ} = G \angle 0^\circ} \tag{15.20}$$

The reciprocal of reactance ($1/X$) is called **susceptance** and is a measure of how *susceptible* an element is to the passage of current through it. Susceptance is also measured in *siemens* and is represented by the capital letter B.

For the inductor,

$$\boxed{\mathbf{Y}_L = \frac{1}{\mathbf{Z}_L} = \frac{1}{X_L \angle 90^\circ} = \frac{1}{X_L} \angle -90^\circ} \tag{15.21}$$

Defining $$\boxed{B_L = \frac{1}{X_L}} \quad \text{(siemens, S)} \tag{15.22}$$

we have $$\boxed{\mathbf{Y}_L = B_L \angle -90^\circ} \tag{15.23}$$

Note that for inductance, an increase in frequency or inductance will result in a decrease in susceptance or, correspondingly, in admittance.

For the capacitor,

$$\boxed{\mathbf{Y}_C = \frac{1}{\mathbf{Z}_C} = \frac{1}{X_C \angle -90^\circ} = \frac{1}{X_C} \angle 90^\circ} \tag{15.24}$$

Defining $$\boxed{B_C = \frac{1}{X_C}} \quad \text{(siemens, S)} \tag{15.25}$$

we have $$\boxed{\mathbf{Y}_C = B_C \angle 90^\circ} \tag{15.26}$$

For the capacitor, therefore, an increase in frequency or capacitance will result in an increase in its susceptibility.

For parallel ac circuits, the **admittance diagram** is used with the three admittances, represented as shown in Fig. 15.55.

Note in Fig. 15.55 that the conductance (like resistance) is on the positive real axis, whereas inductive and capacitive susceptances are in direct opposition on the imaginary axis.

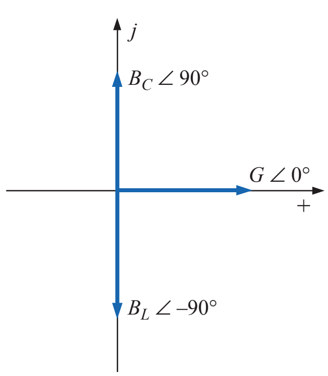


FIG. 15.55
Admittance diagram.

For any configuration (series, parallel, series-parallel, etc.), the angle associated with the total admittance is the angle by which the source current leads the applied voltage. For inductive networks, θ_T is negative, whereas for capacitive networks, θ_T is positive.

EXAMPLE 15.12 For the network of Fig. 15.56:
a. Find the admittance of each parallel branch.
b. Determine the input admittance.
c. Calculate the input impedance.
d. Draw the admittance diagram.

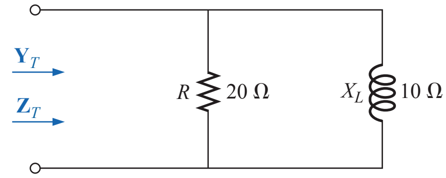


FIG. 15.56
Example 15.12.

Solutions:

a. $\mathbf{Y}_R = G \angle 0° = \dfrac{1}{R} \angle 0° = \dfrac{1}{20\ \Omega} \angle 0°$

$= \mathbf{0.05\ S \angle 0° = 0.05\ S} + \boldsymbol{j}\ \mathbf{0}$

$\mathbf{Y}_L = B_L \angle -90° = \dfrac{1}{X_L} \angle -90° = \dfrac{1}{10\ \Omega} \angle -90°$

$= \mathbf{0.1\ S \angle -90° = 0} - \boldsymbol{j}\ \mathbf{0.1\ S}$

b. $\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L = (0.05\ \text{S} + j\,0) + (0 - j\,0.1\ \text{S})$

$= \mathbf{0.05\ S} - \boldsymbol{j}\ \mathbf{0.1\ S} = G - jB_L$

c. $\mathbf{Z}_T = \dfrac{1}{\mathbf{Y}_T} = \dfrac{1}{0.05\ \text{S} - j\,0.1\ \text{S}} = \dfrac{1}{0.112\ \text{S} \angle -63.43°}$

$= \mathbf{8.93\ \Omega \angle 63.43°}$

or Eq. (15.17):

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(20\ \Omega \angle 0°)(10\ \Omega \angle 90°)}{20\ \Omega + j\,10\ \Omega}$$

$$= \frac{200\ \Omega \angle 90°}{22.36 \angle 26.57°} = \mathbf{8.93\ \Omega \angle 63.43°}$$

d. The admittance diagram appears in Fig. 15.57.

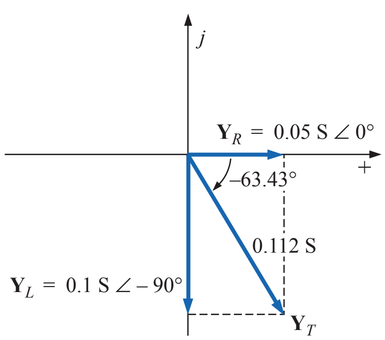


FIG. 15.57
Admittance diagram for the network of Fig. 15.56.

EXAMPLE 15.13 Repeat Example 15.12 for the parallel network of Fig. 15.58.

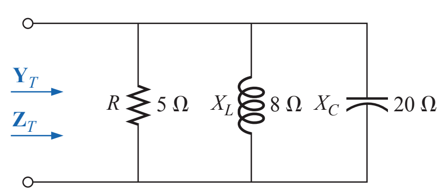


FIG. 15.58
Example 15.13.

Solutions:

a. $\mathbf{Y}_R = G \angle 0° = \dfrac{1}{R} \angle 0° = \dfrac{1}{5\ \Omega} \angle 0°$

$= \mathbf{0.2\ S \angle 0° = 0.2\ S} + \boldsymbol{j}\ \mathbf{0}$

$$\mathbf{Y}_L = B_L \angle -90^\circ = \frac{1}{X_L} \angle -90^\circ = \frac{1}{8\ \Omega} \angle -90^\circ$$
$$= \mathbf{0.125\ S \angle -90^\circ = 0 - \textit{j}\ 0.125\ S}$$
$$\mathbf{Y}_C = B_C \angle 90^\circ = \frac{1}{X_C} \angle 90^\circ = \frac{1}{20\ \Omega} \angle 90^\circ$$
$$= \mathbf{0.050\ S \angle +90^\circ = 0 + \textit{j}\ 0.050\ S}$$

b. $\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C$
$$= (0.2\ \text{S} + j\ 0) + (0 - j\ 0.125\ \text{S}) + (0 + j\ 0.050\ \text{S})$$
$$= 0.2\ \text{S} - j\ 0.075\ \text{S} = \mathbf{0.2136\ S \angle -20.56^\circ}$$

c. $$\mathbf{Z}_T = \frac{1}{0.2136\ \text{S} \angle -20.56^\circ} = \mathbf{4.68\ \Omega \angle 20.56^\circ}$$

or

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L\mathbf{Z}_C}{\mathbf{Z}_R\mathbf{Z}_L + \mathbf{Z}_L\mathbf{Z}_C + \mathbf{Z}_R\mathbf{Z}_C}$$
$$= \frac{(5\ \Omega \angle 0^\circ)(8\ \Omega \angle 90^\circ)(20\ \Omega \angle -90^\circ)}{(5\ \Omega \angle 0^\circ)(8\ \Omega \angle 90^\circ) + (8\ \Omega \angle 90^\circ)(20\ \Omega \angle -90^\circ) + (5\ \Omega \angle 0^\circ)(20\ \Omega \angle -90^\circ)}$$
$$= \frac{800\ \Omega \angle 0^\circ}{40 \angle 90^\circ + 160 \angle 0^\circ + 100 \angle -90^\circ}$$
$$= \frac{800\ \Omega}{160 + j\ 40 - j\ 100} = \frac{800\ \Omega}{160 - j\ 60}$$
$$= \frac{800\ \Omega}{170.88 \angle -20.56^\circ}$$
$$= \mathbf{4.68\ \Omega \angle 20.56^\circ}$$

d. The admittance diagram appears in Fig. 15.59.

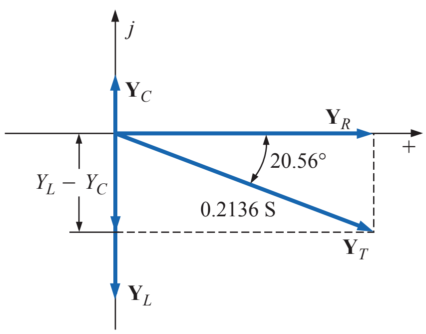


FIG. 15.59
Admittance diagram for the network of Fig. 15.58.

On many occasions, the inverse relationship $\mathbf{Y}_T = 1/\mathbf{Z}_T$ or $\mathbf{Z}_T = 1/\mathbf{Y}_T$ will require that we divide the number 1 by a complex number having a real and an imaginary part. This division, if not performed in the polar form, requires that we multiply the numerator and denominator by the conjugate of the denominator, as follows:

$$\mathbf{Y}_T = \frac{1}{\mathbf{Z}_T} = \frac{1}{4\ \Omega + j\ 6\ \Omega} = \left(\frac{1}{4\ \Omega + j\ 6\ \Omega}\right)\left(\frac{(4\ \Omega - j\ 6\ \Omega)}{(4\ \Omega - j\ 6\ \Omega)}\right) = \frac{4 - j\ 6}{4^2 + 6^2}$$

and
$$\mathbf{Y}_T = \frac{4}{52}\ \text{S} - j\ \frac{6}{52}\ \text{S}$$

To avoid this laborious task each time we want to find the reciprocal of a complex number in rectangular form, a format can be developed using the following complex number, which is symbolic of any impedance or admittance in the first or fourth quadrant:

$$\frac{1}{a_1 \pm j\ b_1} = \left(\frac{1}{a_1 \pm j\ b_1}\right)\left(\frac{a_1 \mp j\ b_1}{a_1 \mp j\ b_1}\right) = \frac{a_1 \mp j\ b_1}{a_1^2 + b_1^2}$$

or
$$\boxed{\frac{1}{a_1 \pm j\ b_1} = \frac{a_1}{a_1^2 + b_1^2} \mp j\ \frac{b_1}{a_1^2 + b_1^2}} \qquad \textbf{(15.27)}$$

Note that the denominator is simply the sum of the squares of each term. The sign is inverted between the real and imaginary parts. A few examples will develop some familiarity with the use of this equation.

EXAMPLE 15.14 Find the admittance of each set of series elements in Fig. 15.60.

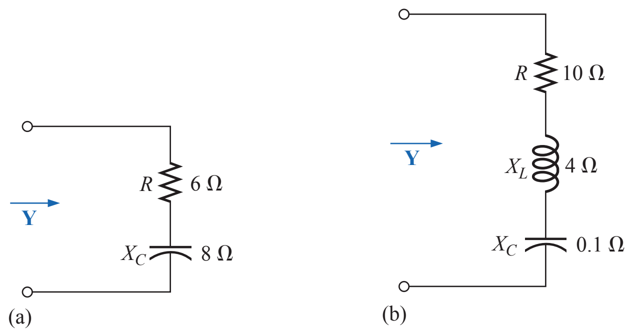


FIG. 15.60
Example 15.14.

Solutions:

a. $\mathbf{Z} = R - j\,X_C = 6\ \Omega - j\,8\ \Omega$

Eq. (15.27):

$$\mathbf{Y} = \frac{1}{6\ \Omega - j\,8\ \Omega} = \frac{6}{(6)^2 + (8)^2} + j\frac{8}{(6)^2 + (8)^2}$$

$$= \frac{\mathbf{6}}{\mathbf{100}}\ \mathbf{S} + \boldsymbol{j}\frac{\mathbf{8}}{\mathbf{100}}\ \mathbf{S}$$

b. $\mathbf{Z} = 10\ \Omega + j\,4\ \Omega + (-j\,0.1\ \Omega) = 10\ \Omega + j\,3.9\ \Omega$

Eq. (15.27):

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{1}{10\ \Omega + j\,3.9\ \Omega} = \frac{10}{(10)^2 + (3.9)^2} - j\frac{3.9}{(10)^2 + (3.9)^2}$$

$$= \frac{10}{115.21} - j\frac{3.9}{115.21} = \mathbf{0.087\ S} - \boldsymbol{j}\,\mathbf{0.034\ S}$$

15.8 PARALLEL ac NETWORKS

For the representative parallel ac network of Fig. 15.61, the total impedance or admittance is determined as described in the previous section, and the source current is determined by Ohm's law as follows:

$$\boxed{\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T} \qquad \textbf{(15.28)}$$

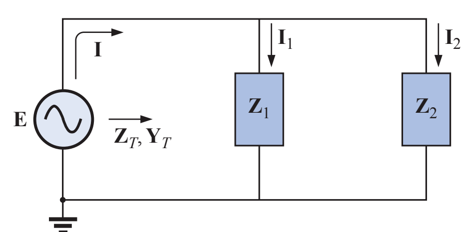


FIG. 15.61
Parallel ac network.

Since the voltage is the same across parallel elements, the current through each branch can then be found through another application of Ohm's law:

$$\boxed{\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \mathbf{E}\mathbf{Y}_1} \qquad \textbf{(15.29a)}$$

$$\boxed{\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \mathbf{E}\mathbf{Y}_2} \qquad \textbf{(15.29b)}$$

Kirchhoff's current law can then be applied in the same manner as employed for dc networks. However, keep in mind that we are now dealing with the algebraic manipulation of quantities that have both magnitude and direction.

$$\mathbf{I} - \mathbf{I}_1 - \mathbf{I}_2 = 0$$

or

$$\boxed{\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2} \tag{15.30}$$

The power to the network can be determined by

$$\boxed{P = EI\cos\theta_T} \tag{15.31}$$

where θ_T is the phase angle between **E** and **I**.

Let us now look at a few examples carried out in great detail for the first exposure.

R-L

Refer to Fig. 15.62.

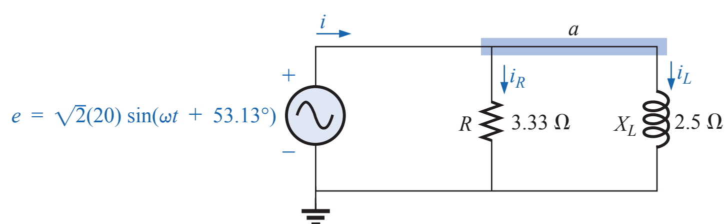


FIG. 15.62
Parallel R-L network.

Phasor Notation As shown in Fig. 15.63.

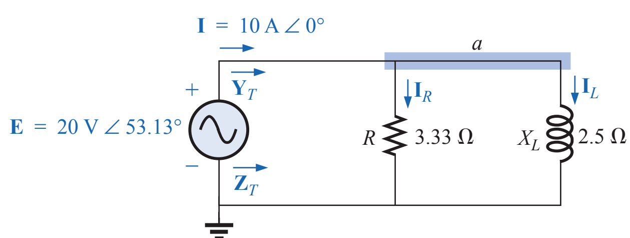


FIG. 15.63
Applying phasor notation to the network of Fig. 15.62.

$\mathbf{Y}_T$ and $\mathbf{Z}_T$

$$\begin{aligned}
\mathbf{Y}_T &= \mathbf{Y}_R + \mathbf{Y}_L \\
&= G\angle 0° + B_L\angle -90° = \frac{1}{3.33\ \Omega}\angle 0° + \frac{1}{2.5\ \Omega}\angle -90° \\
&= 0.3\text{ S}\angle 0° + 0.4\text{ S}\angle -90° = 0.3\text{ S} - j\,0.4\text{ S} \\
&= \mathbf{0.5\ S\angle -53.13°}
\end{aligned}$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.5\text{ S}\angle -53.13°} = \mathbf{2\ \Omega\angle 53.13°}$$

Admittance diagram: As shown in Fig. 15.64.

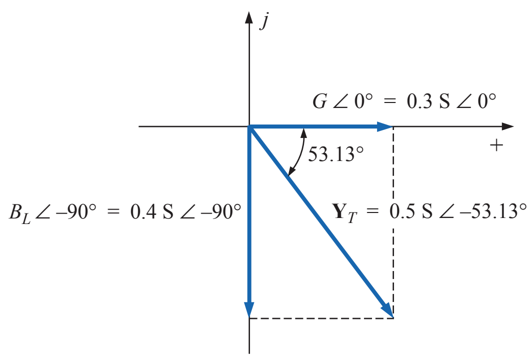


FIG. 15.64
Admittance diagram for the parallel R-L network of Fig. 15.62.

I

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T = (20\text{ V} \angle 53.13°)(0.5\text{ S} \angle -53.13°) = \mathbf{10\text{ A} \angle 0°}$$

I_R and I_L

$$\begin{aligned}\mathbf{I}_R &= \frac{E \angle \theta}{R \angle 0°} = (E \angle \theta)(G \angle 0°)\\ &= (20\text{ V} \angle 53.13°)(0.3\text{ S} \angle 0°) = \mathbf{6\text{ A} \angle 53.13°}\\ \mathbf{I}_L &= \frac{E \angle \theta}{X_L \angle 90°} = (E \angle \theta)(B_L \angle -90°)\\ &= (20\text{ V} \angle 53.13°)(0.4\text{ S} \angle -90°)\\ &= \mathbf{8\text{ A} \angle -36.87°}\end{aligned}$$

Kirchhoff's current law: At node *a*,

$$\mathbf{I} - \mathbf{I}_R - \mathbf{I}_L = 0$$

or

$$\begin{aligned}\mathbf{I} &= \mathbf{I}_R + \mathbf{I}_L\\ 10\text{ A} \angle 0° &= 6\text{ A} \angle 53.13° + 8\text{ A} \angle -36.87°\\ 10\text{ A} \angle 0° &= (3.60\text{ A} + j\,4.80\text{ A}) + (6.40\text{ A} - j\,4.80\text{ A}) = 10\text{ A} + j\,0\end{aligned}$$

and $$\mathbf{10\text{ A} \angle 0° = 10\text{ A} \angle 0°} \qquad \text{(checks)}$$

Phasor diagram: The phasor diagram of Fig. 15.65 indicates that the applied voltage $\mathbf{E}$ is in phase with the current $\mathbf{I}_R$ and leads the current $\mathbf{I}_L$ by 90°.

Power: The total power in watts delivered to the circuit is

$$\begin{aligned}P_T &= EI\cos\theta_T\\ &= (20\text{ V})(10\text{ A})\cos 53.13° = (200\text{ W})(0.6)\\ &= \mathbf{120\text{ W}}\end{aligned}$$

or $$P_T = I^2R = \frac{V_R^2}{R} = V_R^2G = (20\text{ V})^2(0.3\text{ S}) = \mathbf{120\text{ W}}$$

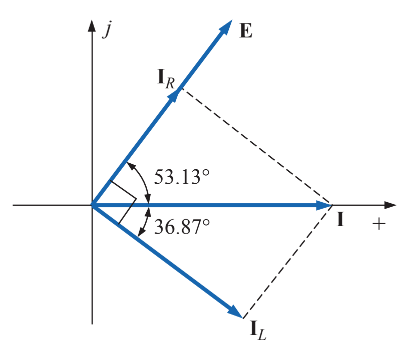


FIG. 15.65
Phasor diagram for the parallel R-L network of Fig. 15.62.

or, finally,

$$
\begin{aligned}
P_T &= P_R + P_L = EI_R \cos\theta_R + EI_L \cos\theta_L \\
&= (20 \text{ V})(6 \text{ A}) \cos 0° + (20 \text{ V})(8 \text{ A}) \cos 90° = 120 \text{ W} + 0 \\
&= \mathbf{120\ W}
\end{aligned}
$$

Power factor: The power factor of the circuit is

$$F_p = \cos\theta_T = \cos 53.13° = \mathbf{0.6\ lagging}$$

or, through an analysis similar to that employed for a series ac circuit,

$$\cos\theta_T = \frac{P}{EI} = \frac{E^2/R}{EI} = \frac{EG}{I} = \frac{G}{I/V} = \frac{G}{Y_T}$$

and

$$\boxed{F_p = \cos\theta_T = \frac{G}{Y_T}} \tag{15.32}$$

where G and Y_T are the magnitudes of the total conductance and admittance of the parallel network. For this case,

$$F_p = \cos\theta_T = \frac{0.3 \text{ S}}{0.5 \text{ S}} = \mathbf{0.6\ lagging}$$

Impedance approach: The current **I** can also be found by first finding the total impedance of the network:

$$
\begin{aligned}
\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L}{\mathbf{Z}_R + \mathbf{Z}_L} &= \frac{(3.33\ \Omega \angle 0°)(2.5\ \Omega \angle 90°)}{3.33\ \Omega \angle 0° + 2.5\ \Omega \angle 90°} \\
&= \frac{8.325 \angle 90°}{4.164 \angle 36.87°} = \mathbf{2\ \Omega \angle 53.13°}
\end{aligned}
$$

And then, using Ohm's law, we obtain

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{20 \text{ V} \angle 53.13°}{2\ \Omega \angle 53.13°} = \mathbf{10\ A \angle 0°}$$

R-C

Refer to Fig. 15.66.

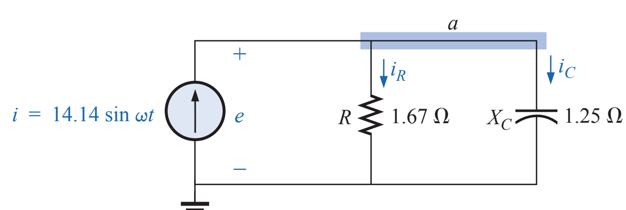


FIG. 15.66
Parallel R-C network.

Phasor Notation As shown in Fig. 15.67.

$\mathbf{Y}_T$ and $\mathbf{Z}_T$

$$
\begin{aligned}
\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_C = G \angle 0° + B_C \angle 90° &= \frac{1}{1.67\ \Omega} \angle 0° + \frac{1}{1.25\ \Omega} \angle 90° \\
&= 0.6 \text{ S} \angle 0° + 0.8 \text{ S} \angle 90° = 0.6 \text{ S} + j\,0.8 \text{ S} = \mathbf{1.0\ S \angle 53.13°}
\end{aligned}
$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{1.0 \text{ S} \angle 53.13°} = \mathbf{1\ \Omega \angle -53.13°}$$

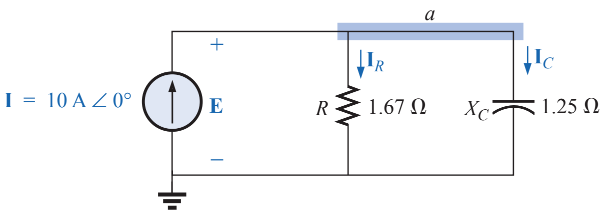


FIG. 15.67
Applying phasor notation to the network of Fig. 15.66.

Admittance diagram: As shown in Fig. 15.68.

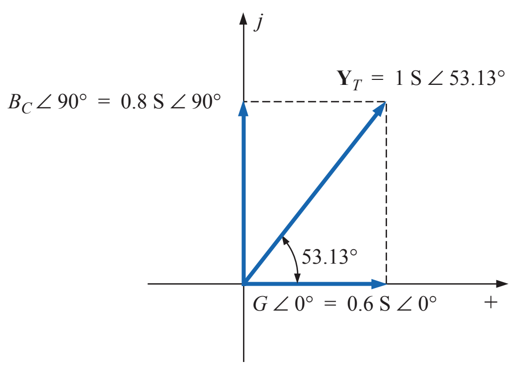


FIG. 15.68
Admittance diagram for the parallel R-C network of Fig. 15.66.

E

$$\mathbf{E} = \mathbf{I}\mathbf{Z}_T = \frac{\mathbf{I}}{\mathbf{Y}_T} = \frac{10\text{ A }\angle 0^\circ}{1\text{ S }\angle 53.13^\circ} = \mathbf{10\ V\ \angle{-53.13^\circ}}$$

I_R and I_C

$$\begin{aligned}\mathbf{I}_R &= (E\angle\theta)(G\angle 0^\circ)\\ &= (10\text{ V }\angle{-53.13^\circ})(0.6\text{ S }\angle 0^\circ) = \mathbf{6\ A\ \angle{-53.13^\circ}}\\ \mathbf{I}_C &= (E\angle\theta)(B_C\angle 90^\circ)\\ &= (10\text{ V }\angle{-53.13^\circ})(0.8\text{ S }\angle 90^\circ) = \mathbf{8\ A\ \angle 36.87^\circ}\end{aligned}$$

Kirchhoff's current law: At node *a*,

$$\mathbf{I} - \mathbf{I}_R - \mathbf{I}_C = 0$$

or

$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_C$$

which can also be verified (as for the *R-L* network) through vector algebra.

Phasor diagram: The phasor diagram of Fig. 15.69 indicates that **E** is in phase with the current through the resistor $\mathbf{I}_R$ and lags the capacitive current $\mathbf{I}_C$ by 90°.

Time domain:

$$\begin{aligned}e &= \sqrt{2}(10)\sin(\omega t - 53.13^\circ) = \mathbf{14.14\sin(\omega t - 53.13^\circ)}\\ i_R &= \sqrt{2}(6)\sin(\omega t - 53.13^\circ) = \mathbf{8.48\sin(\omega t - 53.13^\circ)}\\ i_C &= \sqrt{2}(8)\sin(\omega t + 36.87^\circ) = \mathbf{11.31\sin(\omega t + 36.87^\circ)}\end{aligned}$$

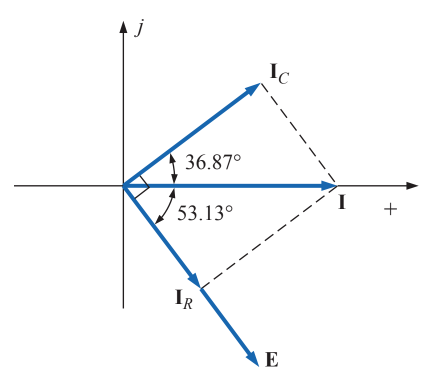


FIG. 15.69
Phasor diagram for the parallel R-C network of Fig. 15.66.

A plot of all of the currents and the voltage appears in Fig. 15.70. Note that e and i_R are in phase and e lags i_C by 90°.

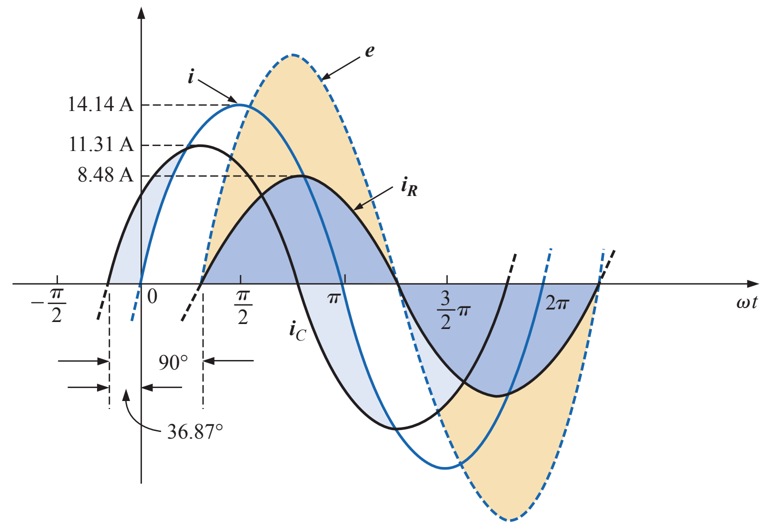


FIG. 15.70
Waveforms for the parallel R-C network of Fig. 15.66.

Power:

$$P_T = EI\cos\theta = (10\text{ V})(10\text{ A})\cos 53.13° = (10)^2(0.6)$$
$$= \mathbf{60\ W}$$

or
$$P_T = E^2G = (10\text{ V})^2(0.6\text{ S}) = \mathbf{60\ W}$$

or, finally,

$$P_T = P_R + P_C = EI_R\cos\theta_R + EI_C\cos\theta_C$$
$$= (10\text{ V})(6\text{ A})\cos 0° + (10\text{ V})(8\text{ A})\cos 90°$$
$$= \mathbf{60\ W}$$

Power factor: The power factor of the circuit is

$$F_p = \cos 53.13° = \mathbf{0.6\ leading}$$

Using Eq. (15.32), we have

$$F_p = \cos\theta_T = \frac{G}{Y_T} = \frac{0.6\text{ S}}{1.0\text{ S}} = \mathbf{0.6\ leading}$$

Impedance approach: The voltage **E** can also be found by first finding the total impedance of the circuit:

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_C}{\mathbf{Z}_R + \mathbf{Z}_C} = \frac{(1.67\ \Omega\ \angle 0°)(1.25\ \Omega\ \angle -90°)}{1.67\ \Omega\ \angle 0° + 1.25\ \Omega\ \angle -90°}$$
$$= \frac{2.09\ \angle -90°}{2.09\ \angle -36.81°} = \mathbf{1\ \Omega\ \angle -53.19°}$$

and then, using Ohm's law, we find

$$\mathbf{E} = \mathbf{IZ}_T = (10\text{ A}\ \angle 0°)(1\ \Omega\ \angle -53.19°) = \mathbf{10\ V\ \angle -53.19°}$$

R-L-C

Refer to Fig. 15.71.

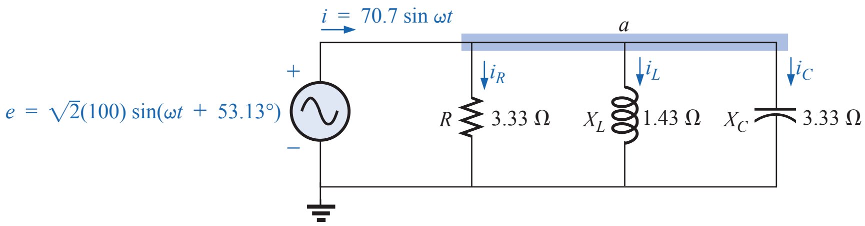


FIG. 15.71
Parallel R-L-C ac network.

Phasor notation: As shown in Fig. 15.72.

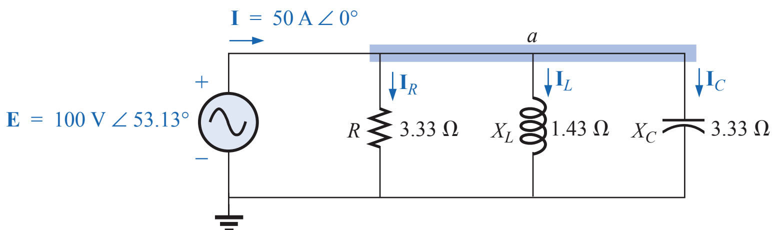


FIG. 15.72
Applying phasor notation to the network of Fig. 15.71.

$\mathbf{Y}_T$ and $\mathbf{Z}_T$

$$\begin{aligned}
\mathbf{Y}_T &= \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C = G\angle 0° + B_L\angle -90° + B_C\angle 90° \\
&= \frac{1}{3.33\ \Omega}\angle 0° + \frac{1}{1.43\ \Omega}\angle -90° + \frac{1}{3.33\ \Omega}\angle 90° \\
&= 0.3\ \text{S}\angle 0° + 0.7\ \text{S}\angle -90° + 0.3\ \text{S}\angle 90° \\
&= 0.3\ \text{S} - j\,0.7\ \text{S} + j\,0.3\ \text{S} \\
&= 0.3\ \text{S} - j\,0.4\ \text{S} = \mathbf{0.5\ S\angle -53.13°}
\end{aligned}$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.5\ \text{S}\angle -53.13°} = \mathbf{2\ \Omega\angle 53.13°}$$

Admittance diagram: As shown in Fig. 15.73.

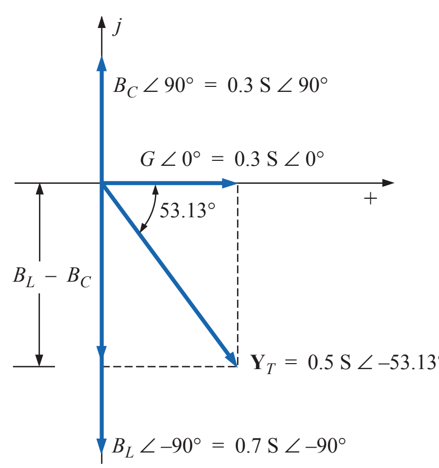


FIG. 15.73
Admittance diagram for the parallel R-L-C network of Fig. 15.71.

I

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T = (100\ \text{V}\angle 53.13°)(0.5\ \text{S}\angle -53.13°) = \mathbf{50\ A\angle 0°}$$

$\mathbf{I}_R$, $\mathbf{I}_L$, and $\mathbf{I}_C$

$$\begin{aligned}
\mathbf{I}_R &= (E\angle\theta)(G\angle 0°) \\
&= (100\ \text{V}\angle 53.13°)(0.3\ \text{S}\angle 0°) = \mathbf{30\ A\angle 53.13°} \\
\mathbf{I}_L &= (E\angle\theta)(B_L\angle -90°) \\
&= (100\ \text{V}\angle 53.13°)(0.7\ \text{S}\angle -90°) = \mathbf{70\ A\angle -36.87°} \\
\mathbf{I}_C &= (E\angle\theta)(B_C\angle 90°) \\
&= (100\ \text{V}\angle 53.13°)(0.3\ \text{S}\angle +90°) = \mathbf{30\ A\angle 143.13°}
\end{aligned}$$

Kirchhoff's current law: At node *a*,

$$\mathbf{I} - \mathbf{I}_R - \mathbf{I}_L - \mathbf{I}_C = 0$$

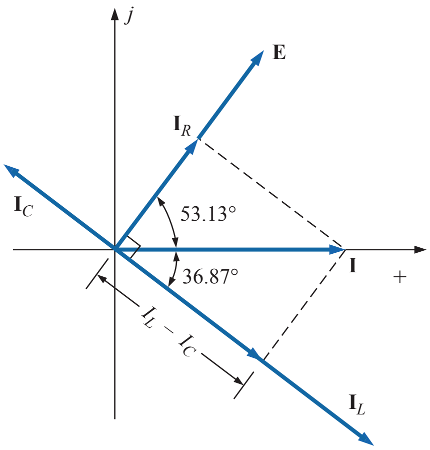


FIG. 15.74
Phasor diagram for the parallel R-L-C network of Fig. 15.71.

or
$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_L + \mathbf{I}_C$$

Phasor diagram: The phasor diagram of Fig. 15.74 indicates that the impressed voltage **E** is in phase with the current $\mathbf{I}_R$ through the resistor, leads the current $\mathbf{I}_L$ through the inductor by 90°, and lags the current $\mathbf{I}_C$ of the capacitor by 90°.

Time domain:

$$i = \sqrt{2}(50)\sin\omega t = \mathbf{70.70\ sin}\ \boldsymbol{\omega t}$$
$$i_R = \sqrt{2}(30)\sin(\omega t + 53.13°) = \mathbf{42.42\ sin(\boldsymbol{\omega t} + 53.13°)}$$
$$i_L = \sqrt{2}(70)\sin(\omega t - 36.87°) = \mathbf{98.98\ sin(\boldsymbol{\omega t} - 36.87°)}$$
$$i_C = \sqrt{2}(30)\sin(\omega t + 143.13°) = \mathbf{42.42\ sin(\boldsymbol{\omega t} + 143.13°)}$$

A plot of all of the currents and the impressed voltage appears in Fig. 15.75.

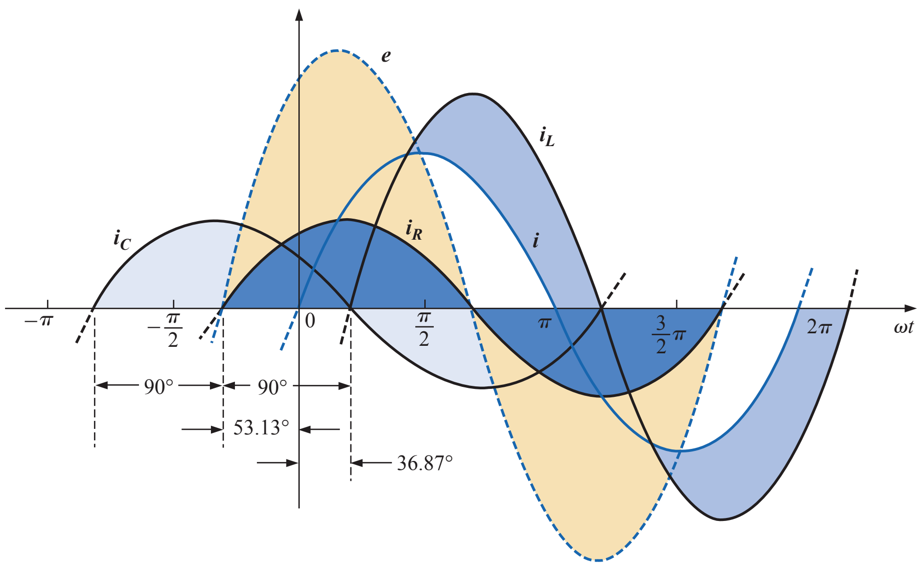


FIG. 15.75
Waveforms for the parallel R-L-C network of Fig. 15.71.

Power: The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta = (100\text{ V})(50\text{ A})\cos 53.13° = (5000)(0.6)$$
$$= \mathbf{3000\ W}$$

or
$$P_T = E^2G = (100\text{ V})^2(0.3\text{ S}) = \mathbf{3000\ W}$$

or, finally,

$$\begin{aligned} P_T &= P_R + P_L + P_C \\ &= EI_R\cos\theta_R + EI_L\cos\theta_L + EL_C\cos\theta_C \\ &= (100\text{ V})(30\text{ A})\cos 0° + (100\text{ V})(70\text{ A})\cos 90° \\ &\qquad + (100\text{ V})(30\text{ A})\cos 90° \\ &= 3000\text{ W} + 0 + 0 \\ &= \mathbf{3000\ W} \end{aligned}$$

Power factor: The power factor of the circuit is

$$F_p = \cos\theta_T = \cos 53.13° = \mathbf{0.6\ lagging}$$

Using Eq. (15.32), we obtain

$$F_p = \cos\theta_T = \frac{G}{Y_T} = \frac{0.3\text{ S}}{0.5\text{ S}} = \mathbf{0.6\ lagging}$$

Impedance approach: The input current **I** can also be determined by first finding the total impedance in the following manner:

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L\mathbf{Z}_C}{\mathbf{Z}_R\mathbf{Z}_L + \mathbf{Z}_L\mathbf{Z}_C + \mathbf{Z}_R\mathbf{Z}_C} = \mathbf{2\ \Omega\ \angle 53.13^\circ}$$

and, applying Ohm's law, we obtain

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\text{ V}\angle 53.13^\circ}{2\ \Omega\angle 53.13^\circ} = \mathbf{50\ A\ \angle 0^\circ}$$

15.9 CURRENT DIVIDER RULE

The basic format for the **current divider rule** in ac circuits is exactly the same as that for dc circuits; that is, for two parallel branches with impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$ as shown in Fig. 15.76,

$$\boxed{\mathbf{I}_1 = \frac{\mathbf{Z}_2\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} \quad\text{or}\quad \mathbf{I}_2 = \frac{\mathbf{Z}_1\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2}} \tag{15.33}$$

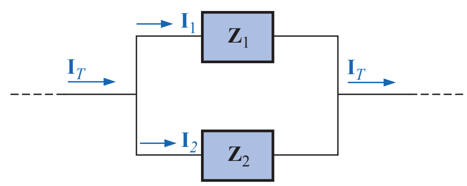


FIG. 15.76
Applying the current divider rule.

EXAMPLE 15.15 Using the current divider rule, find the current through each impedance of Fig. 15.77.

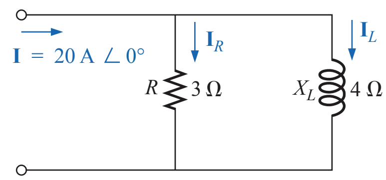


FIG. 15.77
Example 15.15.

Solution:

$$\mathbf{I}_R = \frac{\mathbf{Z}_L\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(4\ \Omega\angle 90^\circ)(20\text{ A}\angle 0^\circ)}{3\ \Omega\angle 0^\circ + 4\ \Omega\angle 90^\circ} = \frac{80\text{ A}\angle 90^\circ}{5\angle 53.13^\circ}$$
$$= \mathbf{16\ A\ \angle 36.87^\circ}$$

$$\mathbf{I}_L = \frac{\mathbf{Z}_R\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(3\ \Omega\angle 0^\circ)(20\text{ A}\angle 0^\circ)}{5\ \Omega\angle 53.13^\circ} = \frac{60\text{ A}\angle 0^\circ}{5\angle 53.13^\circ}$$
$$= \mathbf{12\ A\ \angle -53.13^\circ}$$

EXAMPLE 15.16 Using the current divider rule, find the current through each parallel branch of Fig. 15.78.

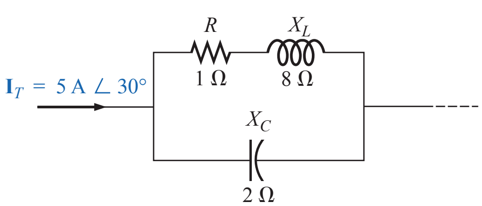


FIG. 15.78
Example 15.16.

Solution:

$$\mathbf{I}_{R\text{-}L} = \frac{\mathbf{Z}_C\mathbf{I}_T}{\mathbf{Z}_C + \mathbf{Z}_{R\text{-}L}} = \frac{(2\ \Omega\angle -90^\circ)(5\text{ A}\angle 30^\circ)}{-j\,2\ \Omega + 1\ \Omega + j\,8\ \Omega} = \frac{10\text{ A}\angle -60^\circ}{1 + j\,6}$$
$$= \frac{10\text{ A}\angle -60^\circ}{6.083\angle 80.54^\circ} \cong \mathbf{1.644\ A\ \angle -140.54^\circ}$$

$$\mathbf{I}_C = \frac{\mathbf{Z}_{R\text{-}L}\mathbf{I}_T}{\mathbf{Z}_{R\text{-}L} + \mathbf{Z}_C} = \frac{(1\ \Omega + j\ 8\ \Omega)(5\ \text{A}\ \angle 30°)}{6.08\ \Omega\ \angle 80.54°}$$

$$= \frac{(8.06\ \angle 82.87°)(5\ \text{A}\ \angle 30°)}{6.08\ \angle 80.54°} = \frac{40.30\ \text{A}\ \angle 112.87°}{6.083\ \angle 80.54°}$$

$$= \mathbf{6.625\ A\ \angle 32.33°}$$

15.10 FREQUENCY RESPONSE OF THE PARALLEL *R-L* NETWORK

In Section 15.5 the frequency response of a series *R-C* circuit was analyzed. Let us now note the impact of frequency on the total impedance and inductive current for the parallel *R-L* network of Fig. 15.79 for a frequency range of zero through 40 kHz.

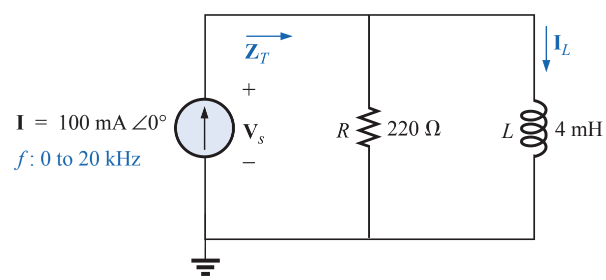


FIG. 15.79
Determining the frequency response of a parallel R-L network.

$\mathbf{Z}_T$ Before getting into specifics, let us first develop a "sense" for the impact of frequency on the network of Fig. 15.79 by noting the impedance-versus-frequency curves of the individual elements, as shown in Fig. 15.80. The fact that the elements are now in parallel requires that we consider their characteristics in a different manner than occurred for the series *R-C* circuit of Section 15.5. Recall that for parallel elements, the element with the smallest impedance will have the greatest impact

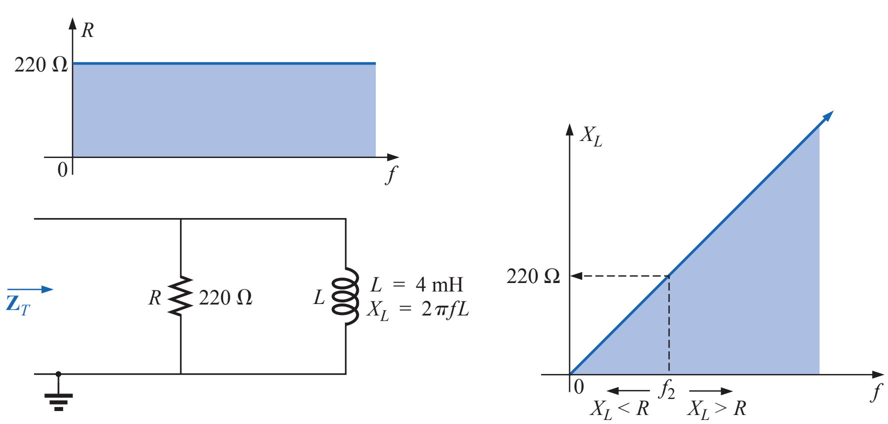


FIG. 15.80
The frequency response of the individual elements of a parallel R-L network.

on the total impedance at that frequency. In Fig. 15.80, for example, X_L is very small at low frequencies compared to R, establishing X_L as the predominant factor in this frequency range. In other words, at low frequencies the network will be primarily inductive, and the angle associated with the total impedance will be close to 90°, as with a pure inductor. As the frequency increases, X_L will increase until it equals the impedance of the resistor (220 Ω). The frequency at which this situation occurs can be determined in the following manner:

$$X_L = 2\pi f_2 L = R$$

and

$$\boxed{f_2 = \frac{R}{2\pi L}} \tag{15.34}$$

which for the network of Fig. 15.79 is

$$f_2 = \frac{R}{2\pi L} = \frac{220\ \Omega}{2\pi(4 \times 10^{-3}\ \text{H})}$$
$$\cong \mathbf{8.75\ kHz}$$

which falls within the frequency range of interest.

For frequencies less than f_2, $X_L < R$, and for frequencies greater than f_2, $X_L > R$, as shown in Fig. 15.80. A general equation for the total impedance in vector form can be developed in the following manner:

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L}{\mathbf{Z}_R + \mathbf{Z}_L}$$
$$= \frac{(R\ \angle 0°)(X_L\ \angle 90°)}{R + j\,X_L} = \frac{RX_L\ \angle 90°}{\sqrt{R^2 + X_L^2}\ \angle \tan^{-1} X_L/R}$$

and

$$\mathbf{Z}_T = \frac{RX_L}{\sqrt{R^2 + X_L^2}}\ \underline{/90° - \tan^{-1} X_L/R}$$

so that

$$\boxed{Z_T = \frac{RX_L}{\sqrt{R^2 + X_L^2}}} \tag{15.35}$$

and

$$\boxed{\theta_T = 90° - \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{R}{X_L}} \tag{15.36}$$

The magnitude and angle of the total impedance can now be found at any frequency of interest simply by substituting Eqs. (15.35) and (15.36).

***f* = 1 kHz**

$$X_L = 2\pi fL = 2\pi(1\ \text{kHz})(4 \times 10^{-3}\ \text{H}) = 25.12\ \Omega$$

and

$$Z_T = \frac{RX_L}{\sqrt{R^2 + X_L^2}} = \frac{(220\ \Omega)(25.12\ \Omega)}{\sqrt{(220\ \Omega)^2 + (25.12\ \Omega)^2}} = \mathbf{24.96\ \Omega}$$

with

$$\theta_T = \tan^{-1}\frac{R}{X_L} = \tan^{-1}\frac{220\ \Omega}{25.12\ \Omega}$$
$$= \tan^{-1} 8.76 = 83.49°$$

and $\mathbf{Z}_T = \mathbf{24.96\ \Omega\ \angle 83.49°}$

This value compares very closely with $X_L = 25.12\ \Omega\ \angle 90°$, which it would be if the network were purely inductive ($R = \infty\ \Omega$). Our assumption that the network is primarily inductive at low frequencies is therefore confirmed.

Continuing:

$$f = 5\text{ kHz:}\quad \mathbf{Z}_T = \mathbf{109.1\ \Omega\ \angle 60.23°}$$
$$f = 10\text{ kHz:}\quad \mathbf{Z}_T = \mathbf{165.5\ \Omega\ \angle 41.21°}$$
$$f = 15\text{ kHz:}\quad \mathbf{Z}_T = \mathbf{189.99\ \Omega\ \angle 30.28°}$$
$$f = 20\text{ kHz:}\quad \mathbf{Z}_T = \mathbf{201.53\ \Omega\ \angle 23.65°}$$
$$f = 30\text{ kHz:}\quad \mathbf{Z}_T = \mathbf{211.19\ \Omega\ \angle 16.27°}$$
$$f = 40\text{ kHz:}\quad \mathbf{Z}_T = \mathbf{214.91\ \Omega\ \angle 12.35°}$$

At $f = 40$ kHz, note how closely the magnitude of Z_T has approached the resistance level of 220 Ω and how the associated angle with the total impedance is approaching zero degrees. The result is a network with terminal characteristics that are becoming more and more resistive as the frequency increases, which further confirms the earlier conclusions developed by the curves of Fig. 15.80.

Plots of Z_T versus frequency in Fig. 15.81 and θ_T in Fig. 15.82 clearly reveal the transition from an inductive network to one that has resistive characteristics. Note that the transition frequency of 8.75 kHz occurs right in the middle of the knee of the curves for both Z_T and θ_T.

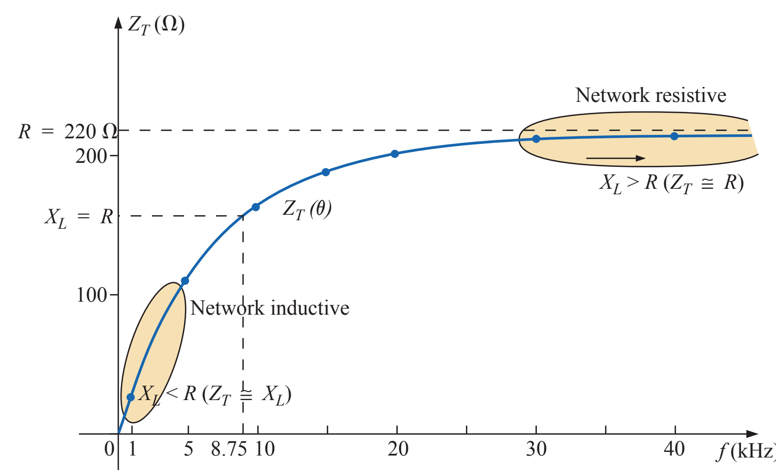


FIG. 15.81
The magnitude of the input impedance versus frequency for the network of Fig. 15.79.

A review of Figs. 15.47 and 15.81 will reveal that a series *R-C* and a parallel *R-L* network will have an impedance level that approaches the resistance of the network at high frequencies. The capacitive circuit approaches the level from above, whereas the inductive network does the same from below. For the series *R-L* circuit and the parallel *R-C* network, the total impedance will begin at the resistance level and then display the characteristics of the reactive elements at high frequencies.

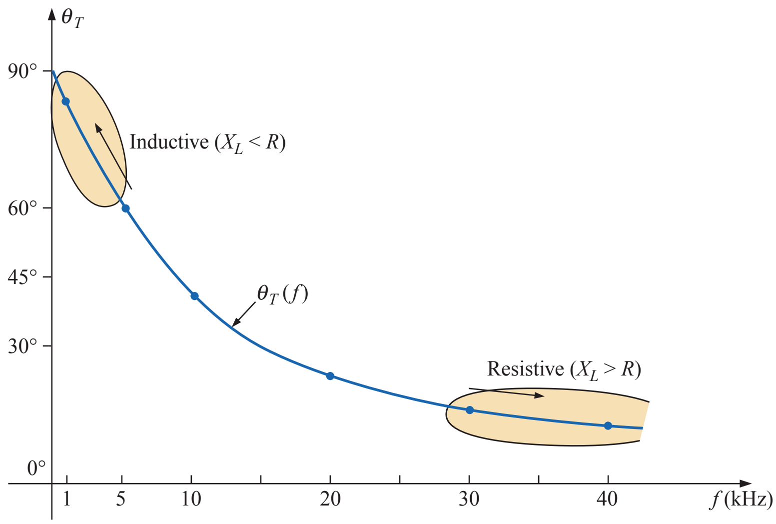


FIG. 15.82
The phase angle of the input impedance versus frequency for the network of Fig. 15.79.

I_L Applying the current divider rule to the network of Fig. 15.79 will result in the following:

$$\mathbf{I}_L = \frac{\mathbf{Z}_R\mathbf{I}}{\mathbf{Z}_R + \mathbf{Z}_L}$$

$$= \frac{(R\angle 0°)(I\angle 0°)}{R + jX_L} = \frac{RI\angle 0°}{\sqrt{R^2+X_L^2}\,\angle \tan^{-1} X_L/R}$$

and $$\mathbf{I}_L = I_L\angle\theta_L = \frac{RI}{\sqrt{R^2+X_L^2}}\angle -\tan^{-1} X_L/R$$

The magnitude of I_L is therefore determined by

$$\boxed{I_L = \frac{RI}{\sqrt{R^2+X_L^2}}} \tag{15.37}$$

and the phase angle θ_L, by which $\mathbf{I}_L$ leads $\mathbf{I}$, is given by

$$\boxed{\theta_L = -\tan^{-1}\frac{X_L}{R}} \tag{15.38}$$

Because θ_L is always negative, the magnitude of θ_L is, in actuality, the angle by which $\mathbf{I}_L$ lags $\mathbf{I}$.

To begin our analysis, let us first consider the case of $f = 0$ Hz (dc conditions).

$f = 0$ Hz

$$X_L = 2\pi fL = 2\pi(0\text{ Hz})L = 0\ \Omega$$

Applying the short-circuit equivalent for the inductor in Fig. 15.79 would result in

$$\mathbf{I}_L = \mathbf{I} = 100\text{ mA}\angle 0°$$

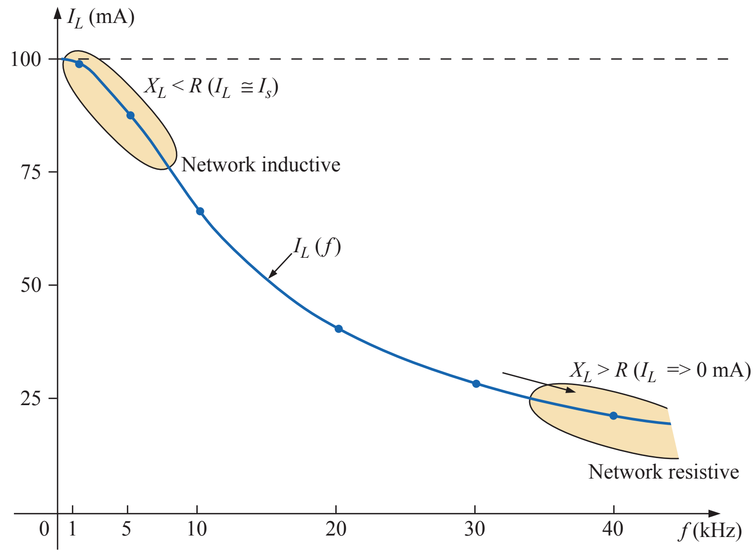


FIG. 15.83
The magnitude of the current $\mathbf{I}_L$ versus frequency for the parallel R-L network of Fig. 15.79.

as appearing in Figs. 15.83 and 15.84.

***f* = 1 kHz** Applying Eq. (15.37):

$$X_L = 2\pi fL = 2\pi(1 \text{ kHz})(4 \text{ mH}) = 25.12 \ \Omega$$

and
$$\sqrt{R^2 + X_L^2} = \sqrt{(220 \ \Omega)^2 + (25.12 \ \Omega)^2} = 221.43 \ \Omega$$

and
$$I_L = \frac{RI}{\sqrt{R^2 + X_L^2}} = \frac{(220 \ \Omega)(100 \text{ mA})}{221.43 \ \Omega} = \mathbf{99.35 \text{ mA}}$$

with

$$\theta_L = \tan^{-1}\frac{X_L}{R} = -\tan^{-1}\frac{25.12 \ \Omega}{220 \ \Omega} = -\tan^{-1} 0.114 = \mathbf{-6.51^\circ}$$

and
$$\mathbf{I}_L = \mathbf{99.35 \text{ mA} \angle -6.51^\circ}$$

The result is a current $\mathbf{I}_L$ that is still very close to the source current **I** in both magnitude and phase.

Continuing:

$$f = 5 \text{ kHz:} \quad \mathbf{I}_L = \mathbf{86.84 \text{ mA} \angle -29.72^\circ}$$
$$f = 10 \text{ kHz:} \quad \mathbf{I}_L = \mathbf{65.88 \text{ mA} \angle -48.79^\circ}$$
$$f = 15 \text{ kHz:} \quad \mathbf{I}_L = \mathbf{50.43 \text{ mA} \angle -59.72^\circ}$$
$$f = 20 \text{ kHz:} \quad \mathbf{I}_L = \mathbf{40.11 \text{ mA} \angle -66.35^\circ}$$
$$f = 30 \text{ kHz:} \quad \mathbf{I}_L = \mathbf{28.02 \text{ mA} \angle -73.73^\circ}$$
$$f = 40 \text{ kHz:} \quad \mathbf{I}_L = \mathbf{21.38 \text{ mA} \angle -77.65^\circ}$$

The plot of the magnitude of I_L versus frequency is provided in Fig. 15.83 and reveals that the current through the coil dropped from its maximum of 100 mA to almost 20 mA at 40 kHz. As the reactance of the coil increased with frequency, more of the source current chose the

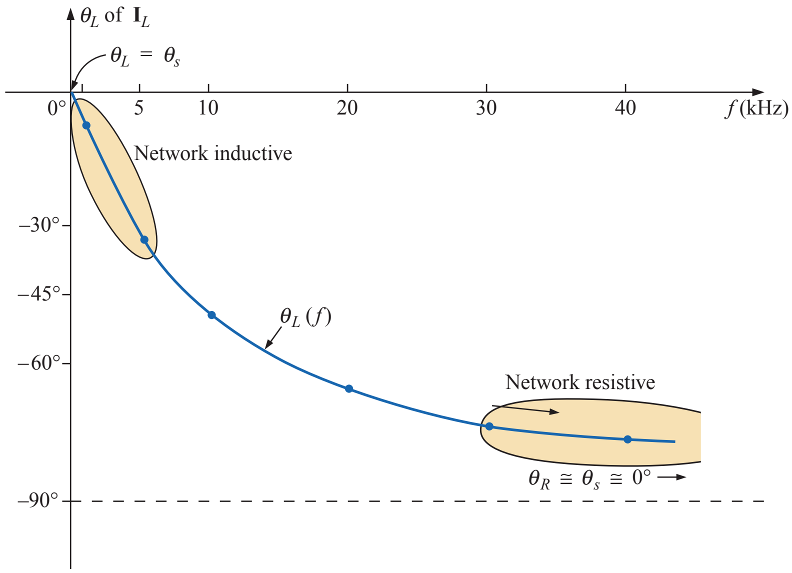


FIG. 15.84
The phase angle of the current $\mathbf{I}_L$ versus frequency for the parallel R-L network of Fig. 15.79.

lower-resistance path of the resistor. The magnitude of the phase angle between $\mathbf{I}_L$ and $\mathbf{I}$ is approaching 90° with an increase in frequency, as shown in Fig. 15.84, leaving its initial value of zero degrees at $f = 0$ Hz far behind.

At $f = 1$ kHz, the phasor diagram of the network appears as shown in Fig. 15.85. First note that the magnitude and the phase angle of $\mathbf{I}_L$ are very close to those of $\mathbf{I}$. Since the voltage across a coil must lead the current through a coil by 90°, the voltage $\mathbf{V}_s$ appears as shown. The voltage across a resistor is in phase with the current through the resistor, resulting in the direction of $\mathbf{I}_R$ shown in Fig. 15.85. Of course, at this frequency $R > X_L$, and the current I_R is relatively small in magnitude.

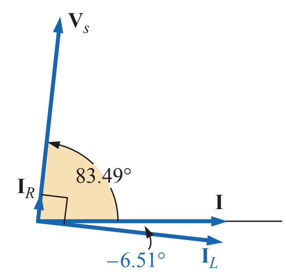


FIG. 15.85
The phasor diagram for the parallel R-L network of Fig. 15.79 at $f = 1$ kHz.

At $f = 40$ kHz, the phasor diagram changes to that appearing in Fig. 15.86. Note that now $\mathbf{I}_R$ and $\mathbf{I}$ are close in magnitude and phase because $X_L > R$. The magnitude of $\mathbf{I}_L$ has dropped to very low levels, and the phase angle associated with $\mathbf{I}_L$ is approaching −90°. The network is now more "resistive" compared to its "inductive" characteristics at low frequencies.

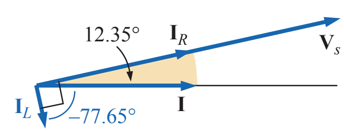


FIG. 15.86
The phasor diagram for the parallel R-L network of Fig. 15.79 at $f = 40$ kHz.

The analysis of a parallel *R-C* or *R-L-C* network would proceed in much the same manner, with the inductive impedance predominating at low frequencies and the capacitive reactance predominating at high frequencies.

15.11 SUMMARY: PARALLEL ac NETWORKS

The following is a review of important conclusions that can be derived from the discussion and examples of the previous sections. The list is not all-inclusive, but it does emphasize some of the conclusions that should be carried forward in the future analysis of ac systems.

For parallel ac networks with reactive elements:

1. ***The total admittance (impedance) will be frequency dependent.***
2. ***The impedance of any one element can be less than the total impedance (recall that for dc circuits the total resistance must always be less than the smallest parallel resistor).***
3. ***The inductive and capacitive susceptances are in direct opposition on an admittance diagram.***
4. ***Depending on the frequency applied, the same network can be either predominantly inductive or predominantly capacitive.***
5. ***At lower frequencies the inductive elements will usually have the most impact on the total impedance, while at high frequencies the capacitive elements will usually have the most impact.***
6. ***The magnitude of the current through any one branch can be greater than the source current.***
7. ***The magnitude of the current through an element, compared to the other elements of the network, is directly related to the magnitude of its impedance; that is, the smaller the impedance of an element, the larger the magnitude of the current through the element.***
8. ***The current through a coil is always in direct opposition with the current through a capacitor on a phasor diagram.***
9. ***The applied voltage is always in phase with the current through the resistive elements, leads the voltage across all the inductive elements by 90°, and lags the current through all capacitive elements by 90°.***
10. ***The smaller the resistive element of a network compared to the net reactive susceptance, the closer the power factor is to unity.***

15.12 EQUIVALENT CIRCUITS

In a series ac circuit, the total impedance of two or more elements in series is often equivalent to an impedance that can be achieved with fewer elements of different values, the elements and their values being determined by the frequency applied. This is also true for parallel circuits. For the circuit of Fig. 15.87(a),

$$\mathbf{Z}_T = \frac{\mathbf{Z}_C\mathbf{Z}_L}{\mathbf{Z}_C + \mathbf{Z}_L} = \frac{(5\ \Omega \angle -90^\circ)(10\ \Omega \angle 90^\circ)}{5\ \Omega \angle -90^\circ + 10\ \Omega \angle 90^\circ} = \frac{50 \angle 0^\circ}{5 \angle 90^\circ}$$
$$= 10\ \Omega \angle -90^\circ$$

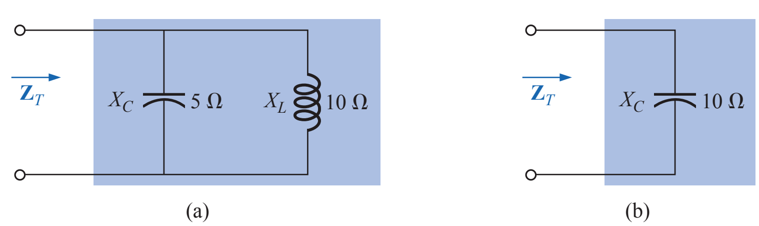


FIG. 15.87
Defining the equivalence between two networks at a specific frequency.

The total impedance at the frequency applied is equivalent to a capacitor with a reactance of 10 Ω, as shown in Fig. 15.87(b). Always keep in mind that this equivalence is true only at the applied frequency. If the frequency changes, the reactance of each element changes, and the equivalent circuit will change—perhaps from capacitive to inductive in the above example.

Another interesting development appears if the impedance of a parallel circuit, such as the one of Fig. 15.88(a), is found in rectangular form. In this case,

$$\mathbf{Z}_T = \frac{\mathbf{Z}_L \mathbf{Z}_R}{\mathbf{Z}_L + \mathbf{Z}_R} = \frac{(4\ \Omega \angle 90^\circ)(3\ \Omega \angle 0^\circ)}{4\ \Omega \angle 90^\circ + 3\ \Omega \angle 0^\circ}$$
$$= \frac{12 \angle 90^\circ}{5 \angle 53.13^\circ} = 2.40\ \Omega \angle 36.87^\circ$$
$$= 1.920\ \Omega + j\,1.440\ \Omega$$

which is the impedance of a series circuit with a resistor of 1.92 Ω and an inductive reactance of 1.44 Ω, as shown in Fig. 15.88(b).

The current **I** will be the same in each circuit of Fig. 15.87 or Fig. 15.88 if the same input voltage **E** is applied. For a parallel circuit of one resistive element and one reactive element, the series circuit with the same input impedance will always be composed of one resistive and one reactive element. The impedance of each element of the series circuit will be different from that of the parallel circuit, but the reactive elements will always be of the same type; that is, an *R-L* circuit and an *R-C* parallel circuit will have an equivalent *R-L* and *R-C* series circuit, respectively. The same is true when converting from a series to a parallel circuit. In the discussion to follow, keep in mind that

the term* equivalent *refers only to the fact that for the same applied potential, the same impedance and input current will result.

To formulate the equivalence between the series and parallel circuits, the equivalent series circuit for a resistor and reactance in parallel can be found by determining the total impedance of the circuit in rectangular form; that is, for the circuit of Fig. 15.89(a),

$$\mathbf{Y}_p = \frac{1}{R_p} + \frac{1}{\pm j\,X_p} = \frac{1}{R_P} \mp j\frac{1}{X_p}$$

and

$$\mathbf{Z}_p = \frac{1}{\mathbf{Y}_p} = \frac{1}{(1/R_p) \mp j\,(1/X_p)}$$
$$= \frac{1/R_p}{(1/R_p)^2 + (1/X_p)^2} \pm j\frac{1/X_p}{(1/R_p)^2 + (1/X_p)^2}$$

Multiplying the numerator and denominator of each term by $R_p^2 X_p^2$ results in

$$\mathbf{Z}_p = \frac{R_p X_p^2}{X_p^2 + R_p^2} \pm j\frac{R_p^2 X_p}{X_p^2 + R_p^2}$$
$$= R_s \pm j\,X_s \qquad \text{[Fig. 15.89(b)]}$$

and

$$\boxed{R_s = \frac{R_p X_p^2}{X_p^2 + R_p^2}} \qquad \textbf{(15.39)}$$

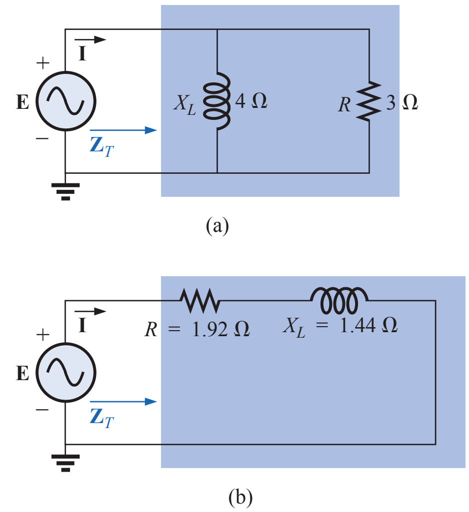


FIG. 15.88
Finding the series equivalent circuit for a parallel R-L network.

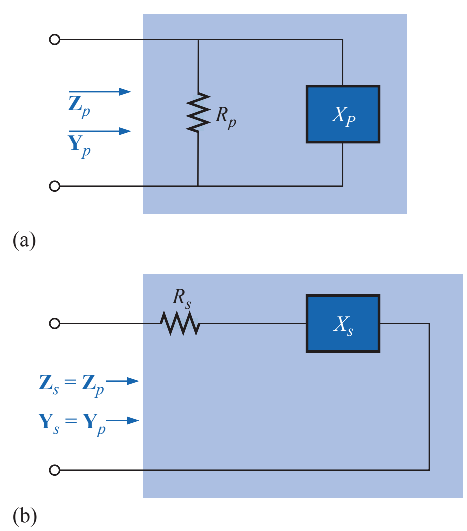


FIG. 15.89
Defining the parameters of equivalent series and parallel networks.

with
$$\boxed{X_s = \frac{R_p^2 X_p}{X_p^2 + R_p^2}} \qquad \textbf{(15.40)}$$

For the network of Fig. 15.88,

$$R_s = \frac{R_p X_p^2}{X_p^2 + R_p^2} = \frac{(3\ \Omega)(4\ \Omega)^2}{(4\ \Omega)^2 + (3\ \Omega)^2} = \frac{48\ \Omega}{25} = \mathbf{1.920\ \Omega}$$

and

$$X_s = \frac{R_p^2 X_p}{X_p^2 + R_p^2} = \frac{(3\ \Omega)^2(4\ \Omega)}{(4\ \Omega)^2 + (3\ \Omega)^2} = \frac{36\ \Omega}{25} = \mathbf{1.440\ \Omega}$$

which agrees with the previous result.

The equivalent parallel circuit for a circuit with a resistor and reactance in series can be found by simply finding the total admittance of the system in rectangular form; that is, for the circuit of Fig. 15.89(b),

$$\mathbf{Z}_s = R_s \pm j X_s$$

$$\mathbf{Y}_s = \frac{1}{\mathbf{Z}_s} = \frac{1}{R_s \pm j X_s} = \frac{R_s}{R_s^2 + X_s^2} \mp j \frac{X_s}{R_s^2 + X_s^2}$$

$$= G_p \mp j B_p = \frac{1}{R_p} \mp j \frac{1}{X_p} \qquad \text{[Fig. 15.89(a)]}$$

or
$$\boxed{R_p = \frac{R_s^2 + X_s^2}{R_s}} \qquad \textbf{(15.41)}$$

with
$$\boxed{X_p = \frac{R_s^2 + X_s^2}{X_s}} \qquad \textbf{(15.42)}$$

For the above example,

$$R_p = \frac{R_s^2 + X_s^2}{R_s} = \frac{(1.92\ \Omega)^2 + (1.44\ \Omega)^2}{1.92\ \Omega} = \frac{5.76\ \Omega}{1.92} = \mathbf{3.0\ \Omega}$$

and
$$X_p = \frac{R_s^2 + X_s^2}{X_s} = \frac{5.76\ \Omega}{1.44} = \mathbf{4.0\ \Omega}$$

as shown in Fig. 15.88(a).

EXAMPLE 15.17 Determine the series equivalent circuit for the network of Fig. 15.90.

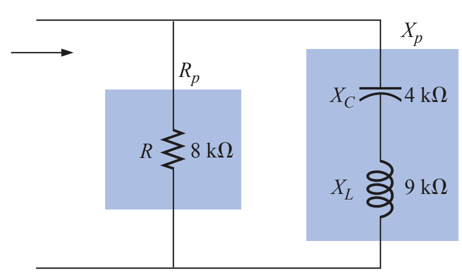


FIG. 15.90
Example 15.17.

Solution:

$$R_p = 8\text{ k}\Omega$$
$$X_p\text{ (resultant)} = |X_L - X_C| = |9\text{ k}\Omega - 4\text{ k}\Omega|$$
$$= 5\text{ k}\Omega$$

and

$$R_s = \frac{R_pX_p^2}{X_p^2 + R_p^2} = \frac{(8\text{ k}\Omega)(5\text{ k}\Omega)^2}{(5\text{ k}\Omega)^2 + (8\text{ k}\Omega)^2} = \frac{200\text{ k}\Omega}{89} = \mathbf{2.247\text{ k}\Omega}$$

with

$$X_s = \frac{R_p^2X_p}{X_p^2 + R_p^2} = \frac{(8\text{ k}\Omega)^2(5\text{ k}\Omega)}{(5\text{ k}\Omega)^2 + (8\text{ k}\Omega)^2} = \frac{320\text{ k}\Omega}{89}$$
$$= \mathbf{3.596\text{ k}\Omega} \qquad \textbf{(inductive)}$$

The equivalent series circuit appears in Fig. 15.91.

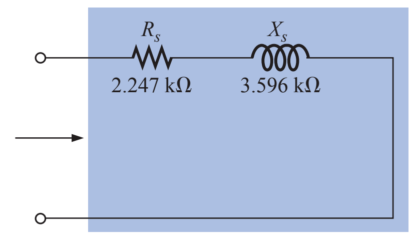


FIG. 15.91
The equivalent series circuit for the parallel network of Fig. 15.90.

EXAMPLE 15.18 For the network of Fig. 15.92:

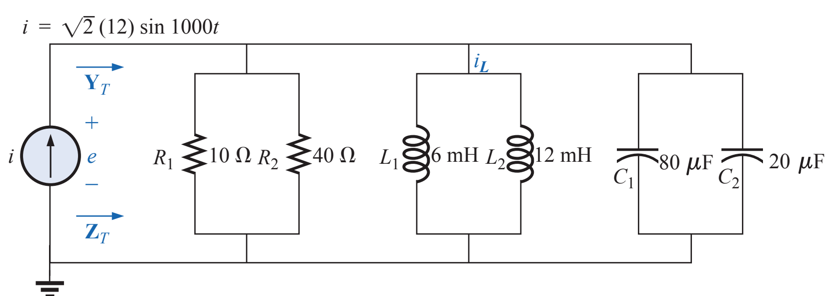


FIG. 15.92
Example 15.18.

a. Determine $\mathbf{Y}_T$.
b. Sketch the admittance diagram.
c. Find $\mathbf{E}$ and $\mathbf{I}_L$.
d. Compute the power factor of the network and the power delivered to the network.
e. Determine the equivalent series circuit as far as the terminal characteristics of the network are concerned.
f. Using the equivalent circuit developed in part (e), calculate **E**, and compare it with the result of part (c).
g. Determine the power delivered to the network, and compare it with the solution of part (d).
h. Determine the equivalent parallel network from the equivalent series circuit, and calculate the total admittance $\mathbf{Y}_T$. Compare the result with the solution of part (a).

Solutions:

a. Combining common elements and finding the reactance of the inductor and capacitor, we obtain

$$R_T = 10\ \Omega \parallel 40\ \Omega = 8\ \Omega$$
$$L_T = 6\text{ mH} \parallel 12\text{ mH} = 4\text{ mH}$$
$$C_T = 80\ \mu\text{F} + 20\ \mu\text{F} = 100\ \mu\text{F}$$

$$X_L = \omega L = (1000 \text{ rad/s})(4 \text{ mH}) = 4\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(1000 \text{ rad/s})(100\ \mu\text{F})} = 10\ \Omega$$

The network is redrawn in Fig. 15.93 with phasor notation. The total admittance is

$$\begin{aligned}
\mathbf{Y}_T &= \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C \\
&= G\angle 0° + B_L\angle -90° + B_C\angle +90° \\
&= \frac{1}{8\ \Omega}\angle 0° + \frac{1}{4\ \Omega}\angle -90° + \frac{1}{10\ \Omega}\angle +90° \\
&= 0.125 \text{ S}\angle 0° + 0.25 \text{ S}\angle -90° + 0.1 \text{ S}\angle +90° \\
&= 0.125 \text{ S} - j\,0.25 \text{ S} + j\,0.1 \text{ S} \\
&= 0.125 \text{ S} - j\,0.15 \text{ S} = \mathbf{0.195\ S\angle -50.194°}
\end{aligned}$$

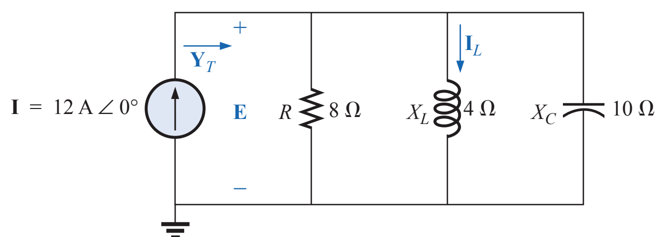


FIG. 15.93
Applying phasor notation to the network of Fig. 15.92.

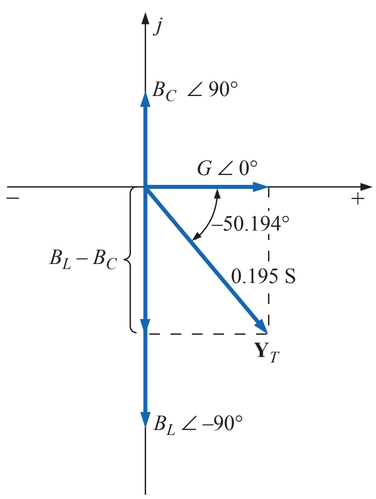


FIG. 15.94
Admittance diagram for the parallel R-L-C network of Fig. 15.92.

b. See Fig. 15.94.

c. $\mathbf{E} = \mathbf{I}\mathbf{Z}_T = \dfrac{\mathbf{I}}{\mathbf{Y}_T} = \dfrac{12 \text{ A}\angle 0°}{0.195 \text{ S}\angle -50.194°} = \mathbf{61.538\ V\angle 50.194°}$

$\mathbf{I}_L = \dfrac{\mathbf{V}_L}{\mathbf{Z}_L} = \dfrac{\mathbf{E}}{\mathbf{Z}_L} = \dfrac{61.538 \text{ V}\angle 50.194°}{4\ \Omega\angle 90°} = \mathbf{15.385\ A\angle -39.81°}$

d. $F_p = \cos\theta = \dfrac{G}{Y_T} = \dfrac{0.125 \text{ S}}{0.195 \text{ S}} = \mathbf{0.641\ lagging}$ (**E** leads **I**)

$P = EI\cos\theta = (61.538 \text{ V})(12 \text{ A})\cos 50.194°$
$= \mathbf{472.75\ W}$

e. $\mathbf{Z}_T = \dfrac{1}{\mathbf{Y}_T} = \dfrac{1}{0.195 \text{ S}\angle -50.194°} = 5.128\ \Omega\angle +50.194°$
$= 3.283\ \Omega + j\,3.939\ \Omega$
$= R + j\,X_L$

$X_L = 3.939\ \Omega = \omega L$

$L = \dfrac{3.939\ \Omega}{\omega} = \dfrac{3.939\ \Omega}{1000 \text{ rad/s}} = \mathbf{3.939\ mH}$

The series equivalent circuit appears in Fig. 15.95.

f. $\mathbf{E} = \mathbf{I}\mathbf{Z}_T = (12 \text{ A}\angle 0°)(5.128\ \Omega\angle 50.194°)$
$= \mathbf{61.536\ V\angle 50.194°}$ (as above)

g. $P = I^2R = (12 \text{ A})^2(3.283\ \Omega) = \mathbf{472.75\ W}$ (as above)

h. $R_p = \dfrac{R_s^2 + X_s^2}{R_s} = \dfrac{(3.283\ \Omega)^2 + (3.939\ \Omega)^2}{3.283\ \Omega} = \mathbf{8\ \Omega}$

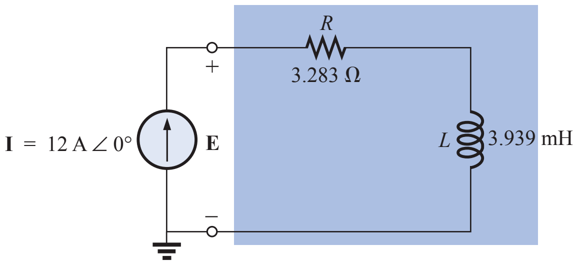


FIG. 15.95
Series equivalent circuit for the parallel R-L-C network of Fig. 15.92 with ω = 1000 rad/s.

$$X_p = \frac{R_s^2 + X_s^2}{X_s} = \frac{(3.283\ \Omega)^2 + (3.939\ \Omega)^2}{3.939\ \Omega} = \mathbf{6.675\ \Omega}$$

The parallel equivalent circuit appears in Fig. 15.96.

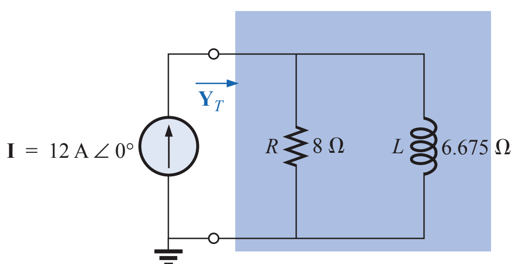


FIG. 15.96
Parallel equivalent of the circuit of Fig. 15.95.

$$\begin{aligned}\mathbf{Y}_T = G \angle 0° + B_L \angle -90° &= \frac{1}{8\ \Omega} \angle 0° + \frac{1}{6.675\ \Omega} \angle -90° \\ &= 0.125\ \text{S} \angle 0° + 0.15\ \text{S} \angle -90° \\ &= 0.125\ \text{S} - j\,0.15\ \text{S} = \mathbf{0.195\ S \angle -50.194°} \qquad \text{(as above)}\end{aligned}$$

15.13 PHASE MEASUREMENTS (DUAL-TRACE OSCILLOSCOPE)

The phase shift between the voltages of a network or between the voltages and currents of a network can be found using a dual-trace (two signals displayed at the same time) oscilloscope. Phase-shift measurements can also be performed using a single-trace oscilloscope by properly interpreting the resulting Lissajous patterns obtained on the screen. This latter approach, however, will be left for the laboratory experience.

In Fig. 15.97, channel 1 of the dual-trace oscilloscope is hooked up to display the applied voltage *e*. Channel 2 is connected to display the voltage across the inductor v_L. Of particular importance is the fact that the ground of the scope is connected to the ground of the oscilloscope for both channels. In other words, there is only one common ground for the circuit and oscilloscope. The resulting waveforms may appear as shown in Fig. 15.98.

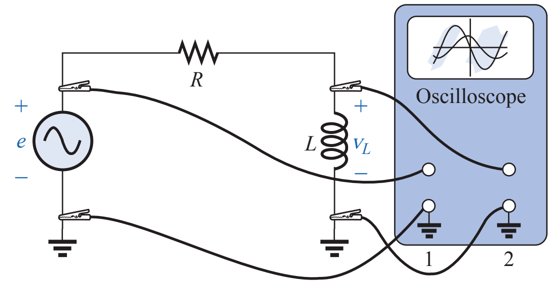


FIG. 15.97
Determining the phase relationship between e and v_L.

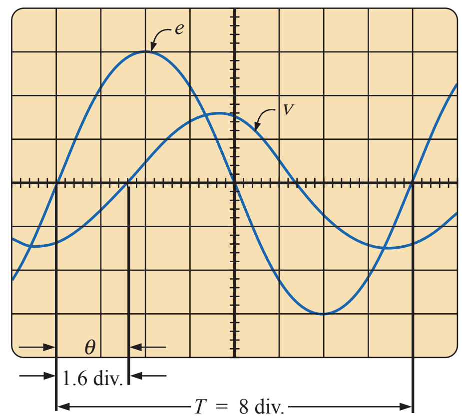


FIG. 15.98
Determining the phase angle between e and v_L.

For the chosen horizontal sensitivity, each waveform of Fig. 15.98 has a period T defined by eight horizontal divisions, and the phase angle between the two waveforms is defined by $1\frac{1}{2}$ divisions. Using the fact that each period of a sinusoidal waveform encompasses 360°, the following ratios can be set up to determine the phase angle θ:

$$\frac{8 \text{ div.}}{360^\circ} = \frac{1.6 \text{ div.}}{\theta}$$

and

$$\theta = \left(\frac{1.6}{8}\right)360^\circ = \mathbf{72^\circ}$$

In general,

$$\boxed{\theta = \frac{(\text{div. for } \theta)}{(\text{div. for } T)} \times 360^\circ} \qquad \textbf{(15.43)}$$

If the phase relationship between e and v_R is required, the oscilloscope *must not* be hooked up as shown in Fig. 15.99. Points a and b have a common ground that will establish a zero-volt drop between the two points; this drop will have the same effect as a short-circuit connection between a and b. The resulting short circuit will "short out" the inductive element, and the current will increase due to the drop in impedance for the circuit. A dangerous situation can arise if the inductive element has a high impedance and the resistor has a relatively low

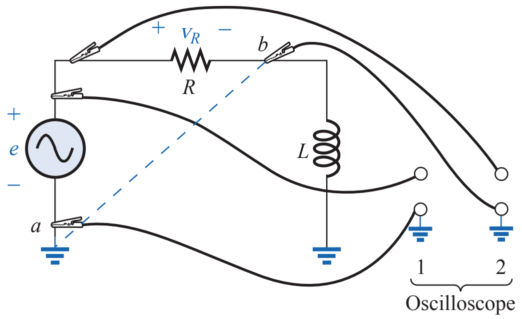


FIG. 15.99
An improper phase-measurement connection.

impedance. The current, controlled solely by the resistance R, could jump to dangerous levels and damage the equipment.

The phase relationship between e and v_R can be determined by simply interchanging the positions of the coil and resistor or by introducing a sensing resistor, as shown in Fig. 15.100. A sensing resistor is exactly that: introduced to "sense" a quantity without adversely affecting the behavior of the network. In other words, the sensing resistor must be small enough compared to the other impedances of the network not to cause a significant change in the voltage and current levels or phase relationships. Note that the sensing resistor is introduced in a way that will result in one end being connected to the common ground of the network. In Fig. 15.100, channel 2 will display the voltage v_{R_s}, which is in phase with the current i. However, the current i is also in phase with the voltage v_R across the resistor R. The net result is that the voltages v_{R_s} and v_R are in phase and the phase relationship between e and v_R can be determined from the waveforms e and v_{R_s}. Since v_{R_s} and i are in phase, the above procedure will also determine the phase angle between the applied voltage e and the source current i. If the magnitude of R_s is sufficiently small compared to R or X_L, the phase measurements of Fig. 15.97 can be performed with R_s in place. That is, channel 2 can be connected to the top of the inductor and to ground, and the effect of R_s can be ignored. In the above application, the sensing resistor will not reveal the magnitude of the voltage v_R but simply the phase relationship between e and v_R.

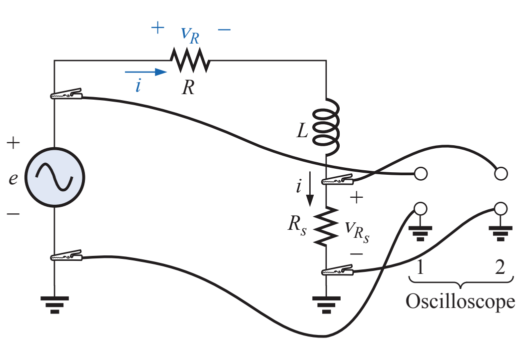


FIG. 15.100
Determining the phase relationship between e and v_R or e and i using a sensing resistor.

For the parallel network of Fig. 15.101, the phase relationship between two of the branch currents, i_R and i_L, can be determined using a sensing resistor, as shown in the figure. Channel 1 will display the voltage v_R, and channel 2 will display the voltage v_{R_s}. Since v_R is in phase with i_R, and v_{R_s} is in phase with the current i_L, the phase relationship between v_R and v_{R_s} will be the same as that between i_R and i_L. In this case, the magnitudes of the current levels can be determined using Ohm's law and the resistance levels R and R_s, respectively.

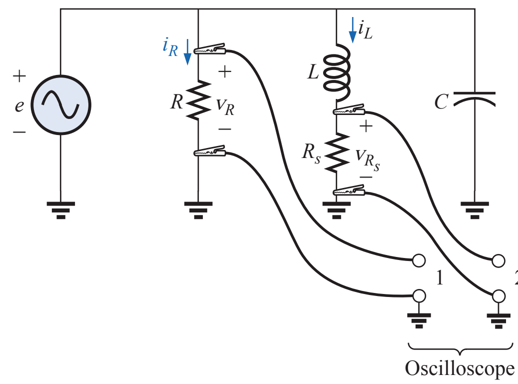


FIG. 15.101
Determining the phase relationship between i_R and i_L.

If the phase relationship between e and i_s of Fig. 15.101 is required, a sensing resistor can be employed, as shown in Fig. 15.102.

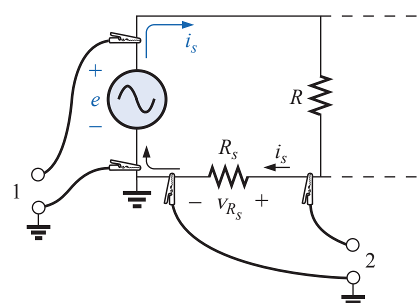


FIG. 15.102
Determining the phase relationship between e and i_s.

In general, therefore, for dual-trace measurements of phase relationships, be particularly careful of the grounding arrangement, and fully utilize the in-phase relationship between the voltage and current of a resistor.

15.14 APPLICATIONS

Home Wiring

An expanded view of house wiring is provided in Fig. 15.103 to permit a discussion of the entire system. The house panel has been included with the "feed" and the important grounding mechanism. In addition, a

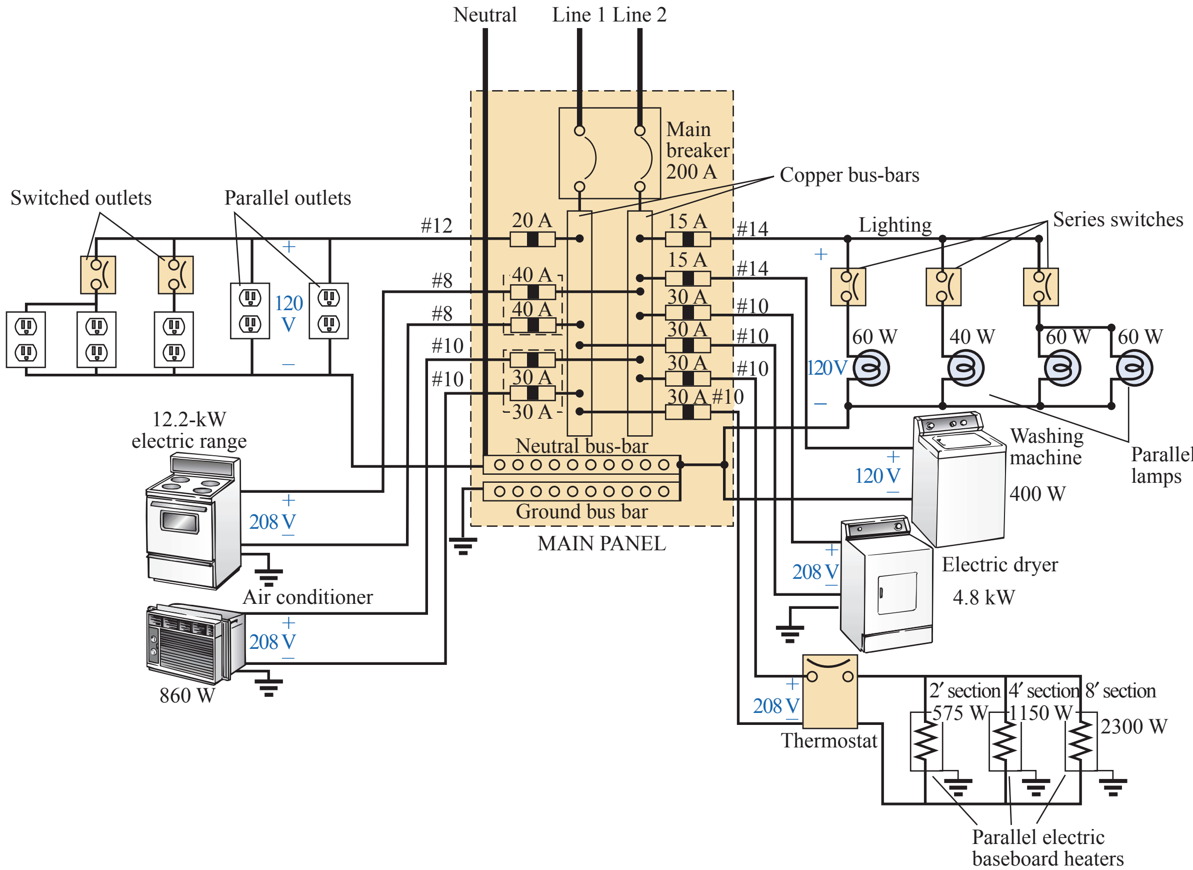


FIG. 15.103

Home wiring diagram.

number of typical circuits found in the home have been included to provide a sense for the manner in which the total power is distributed.

First note how the copper bars in the panel are laid out to provide both 120 V and 208 V. Between any one bar and ground is the single-phase 120-V supply. However, the bars have been arranged so that 208 V can be obtained between two vertical adjacent bars using a double-gang circuit breaker. When time permits, examine your own panel (but do not remove the cover), and note the dual circuit breaker arrangement for the 208-V supply.

For appliances such as fixtures and heaters that have a metal casing, the ground wire is connected to the metal casing to provide a direct path to ground path for a “shorting” or errant current as described in Section 7.7. For outlets and such that do not have a conductive casing, the ground lead is connected to a point on the outlet that distributes to all important points of the outlet.

Note the series arrangement between the thermostat and the heater but the parallel arrangement between heaters on the same circuit. In addition, note the series connection of switches to lights in the upper-right corner but the parallel connection of lights and outlets. Due to high current demand the air conditioner, heaters, and electric stove have 30-A breakers. Keep in mind that the total current does not equal the product of the two (or 60 A) since each breaker is in a line and the same current will flow through each breaker.

In general, you now have a surface understanding of the general wiring in your home. You may not be a qualified, licensed electrician, but at least you should now be able to converse with some intelligence about the system.

Speaker Systems

The best reproduction of sound is obtained using a different speaker for the low-, mid-, and high-frequency regions. Although the typical audio range for the human ear is from about 100 Hz to 20 kHz, speakers are available from 20 Hz to 40 kHz. For the low-frequency range usually extending from about 20 Hz to 300 Hz, a speaker referred to as a *woofer* is used. Of the three speakers, it is normally the largest. The mid-range speaker is typically smaller in size and covers the range from about 100 Hz to 5 kHz. The *tweeter,* as it is normally called, is usually the smallest of the three speakers and typically covers the range from about 2 kHz to 25 kHz. There is an overlap of frequencies to ensure that frequencies aren't lost in those regions where the response of one drops off and the other takes over. A great deal more about the range of each speaker and their dB response (a term you may have heard when discussing speaker response) will be covered in detail in Chapter 23.

One popular method for hooking up the three speakers is the *crossover* configuration of Fig. 15.104. Note that it is nothing more than a parallel network with a speaker in each branch and full applied voltage across each branch. The added elements (inductors and capacitors) were carefully chosen to set the range of response for each speaker. Note that each speaker is labeled with an impedance level and associated frequency. This type of information is typical when purchasing a quality speaker. It immediately identifies the type of speaker and reveals at which frequency it will have its maximum response. A detailed analysis of the same network will be included in Section 23.15. For now, however, it should prove interesting to determine the total impedance of each branch at specific frequencies to see if indeed the response of one will far outweigh the response of the other two. Since an amplifier with an output impedance of 8 Ω is to be employed, maximum

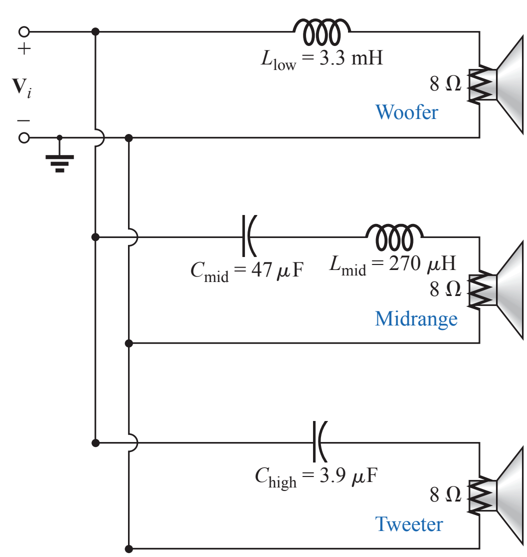


FIG. 15.104
Crossover speaker system.

transfer of power (see Section 18.5 for ac networks) to the speaker will result when the impedance of the branch is equal to or very close to 8 Ω.

Let us begin by examining the response of the frequencies to be carried primarily by the mid-range speaker since it represents the greatest portion of the human hearing range. Since the mid-range speaker branch is rated at 8 Ω at 1.4 kHz, let us test the effect of applying 1.4 kHz to all branches of the crossover network.

For the mid-range speaker:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1.4\text{ kHz})(47\ \mu\text{F})} = 2.42\ \Omega$$

$$X_L = 2\pi fL = 2\pi(1.4\text{ kHz})(270\ \mu\text{H}) = 2.78\ \Omega$$

$$R = 8\ \Omega$$

and
$$\begin{aligned}\mathbf{Z}_{\text{mid-range}} &= R + j(X_L - X_C) = 8\ \Omega + j(2.78\ \Omega - 2.42\ \Omega)\\ &= 8\ \Omega + j0.36\ \Omega\\ &= 8.008\ \Omega\ \angle -2.58^\circ \cong 8\ \Omega\ \angle 0^\circ = R\end{aligned}$$

In Fig. 15.105(a), the amplifier with the output impedance of 8 Ω has been applied across the mid-range speaker at a frequency of 1.4 kHz. Since the total reactance offered by the two series reactive elements is so small compared to the 8-Ω resistance of the speaker, we can essentially replace the series combination of the coil and capacitor by a short circuit of 0 Ω. We are then left with a situation where the load impedance is an exact match with the output impedance of the amplifier, and maximum power will be delivered to the speaker. Because of the equal series impedances, each will capture half the applied voltage or 6 V. The power to the speaker is then $V^2/R = (6\text{ V})^2/8\ \Omega = 4.5$ W.

At a frequency of 1.4 kHz we would expect the woofer and tweeter to have minimum impact on the generated sound. We will now test the validity of this statement by determining the impedance of each branch at 1.4 kHz.

For the woofer:

$$X_L = 2\pi fL = 2\pi(1.4\text{ kHz})(3.3\text{ mH}) = 29.03\ \Omega$$

and
$$\begin{aligned}\mathbf{Z}_{\text{woofer}} &= R + jX_L = 8\ \Omega + j29.03\ \Omega\\ &= 30.11\ \Omega\ \angle 74.59^\circ\end{aligned}$$

which is a poor match with the output impedance of the amplifier. The resulting network is shown in Fig. 15.105(b).

The total load on the source of 12 V is

$$\begin{aligned}\mathbf{Z}_T &= 8\ \Omega + 8\ \Omega + j29.03\ \Omega = 16\ \Omega + j29.03\ \Omega\\ &= 33.15\ \Omega\ \angle 61.14^\circ\end{aligned}$$

and the current is

$$\begin{aligned}\mathbf{I} &= \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{12\text{ V}\ \angle 0^\circ}{33.15\ \Omega\ \angle 61.14^\circ}\\ &= 362\text{ mA}\ \angle -61.14^\circ\end{aligned}$$

The power to the 8-Ω speaker is then

$$P_{\text{woofer}} = I^2R = (362\text{ mA})^2 8\ \Omega = \mathbf{1.048\ W}$$

or about 1 W.

Consequently, the sound generated by the mid-range speaker will far outweigh the response of the woofer (as it should).

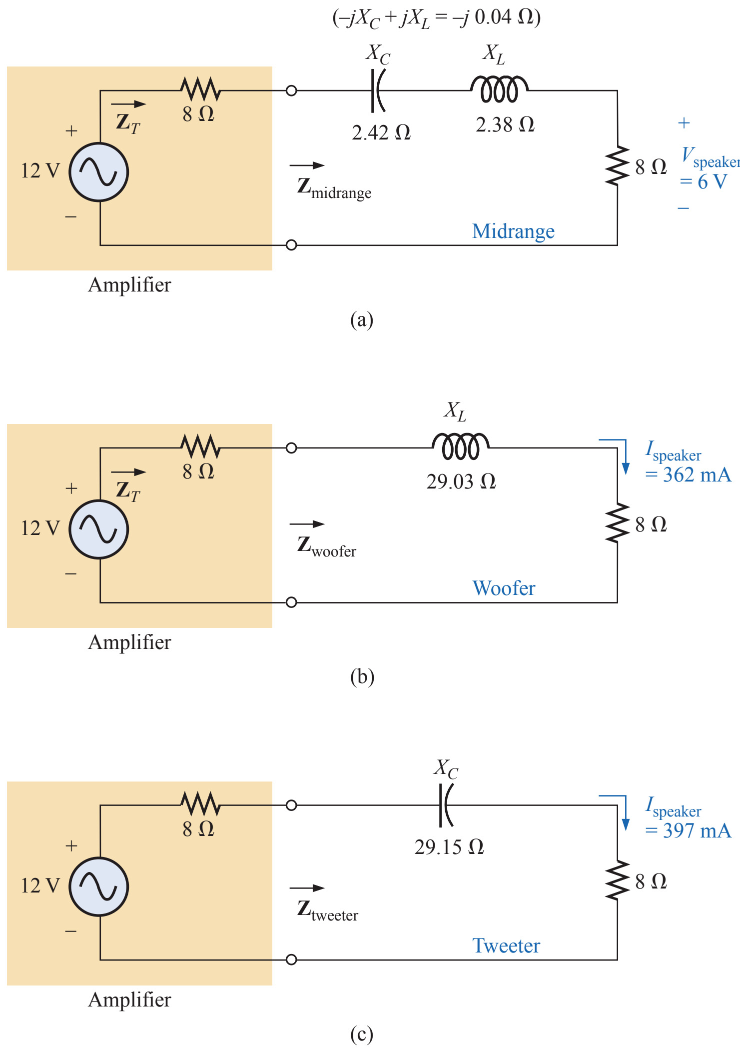


FIG. 15.105

Crossover network: (a) mid-range speaker at 1.4 kHz; (b) woofer at 1.4 kHz; (c) tweeter.

For the tweeter:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1.4\text{ kHz})(3.9\ \mu\text{F})} = 29.15\ \Omega$$

and

$$\mathbf{Z}_{\text{tweeter}} = R - jX_C = 8\ \Omega - j29.15\ \Omega$$
$$= 30.23\ \Omega\ \angle -74.65^\circ$$

which, as for the woofer, is a poor match with the output impedance of the amplifier. The current

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{12\text{ V}\ \angle 0^\circ}{30.23\ \Omega\ \angle -74.65^\circ}$$
$$= 397\text{ mA}\ \angle 74.65^\circ$$

The power to the 8-Ω speaker is then

$$P_{\text{tweeter}} = I^2R = (397\text{ mA})^2(8\ \Omega) = \mathbf{1.261\ W}$$

or about 1.3 W.

Consequently, the sound generated by the mid-range speaker will far outweigh the response of the tweeter also.

All in all, the mid-range speaker predominates at a frequency of 1.4 kHz for the crossover network of Fig. 15.104.

Just for interest sake, let us now determine the impedance of the tweeter at 20 kHz and the impact of the woofer at this frequency.

For the tweeter:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(20\text{ kHz})(3.9\ \mu\text{F})} = 2.04\ \Omega$$

with $$\mathbf{Z}_{\text{tweeter}} = 8\ \Omega - j\,2.04\ \Omega = 8.26\ \Omega \angle -14.31°$$

Even though the magnitude of the impedance of the branch is not exactly 8 Ω, it is very close, and the speaker will receive a high level of power (actually 4.43 W).

For the woofer:

$$X_L = 2\pi fL = 2\pi(20\text{ kHz})(3.3\text{ mH}) = 414.69\ \Omega$$

with $$\mathbf{Z}_{\text{woofer}} = 8\ \Omega - j\,414.69\ \Omega = 414.77\ \Omega \angle 88.9°$$

which is a terrible match with the output impedance of the amplifier. Therefore, the speaker will receive a very low level of power (6.69 mW ≅ 0.007 W).

For all the calculations, note that the capacitive elements predominate at low frequencies, and the inductive elements at high frequencies. For the low frequencies, the reactance of the coil will be quite small, permitting a full transfer of power to the speaker. For the high-frequency tweeter, the reactance of the capacitor is quite small, providing a direct path for power flow to the speaker.

Phase-Shift Power Control

In Chapter 12 the internal structure of a light dimmer was examined and its basic operation described. We can now turn our attention to how the power flow to the bulb is controlled.

If the dimmer were composed of simply resistive elements, all the voltages of the network would be in phase as shown in Fig. 15.106(a). If we assume that 20 V are required to turn on the triac of Fig. 12.49, then the power will be distributed to the bulb for the period highlighted by the blue area of Fig. 15.106(a). For this situation, the bulb is close to full brightness since the applied voltage is available to the bulb for almost the entire cycle. To reduce the power to the bulb (and therefore reduce its brightness), the controlling voltage would have to have a lower peak voltage as shown in Fig. 15.106(b). In fact, the waveform of Fig. 15.106(b) is such that the turn-on voltage is not reached until the peak value occurs. In this case power is delivered to the bulb for only half the cycle, and the brightness of the bulb will be reduced. The problem with using only resistive elements in a dimmer now becomes apparent: The bulb can be made no dimmer than the situation depicted by Fig. 15.106(b). Any further reduction in the controlling voltage would reduce its peak value below the trigger level, and the bulb would never turn on.

This dilemma can be resolved by using a series combination of elements such as shown in Fig. 15.107(a) from the dimmer of Fig. 12.49. Note that the controlling voltage is the voltage across the capacitor,

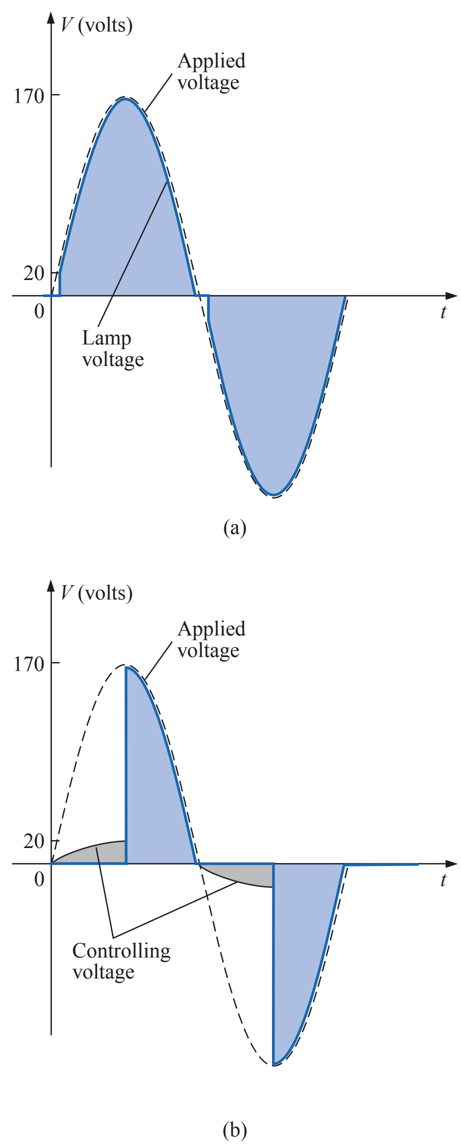


FIG. 15.106
Light dimmer: (a) with purely resistive elements; (b) half-cycle power distribution.

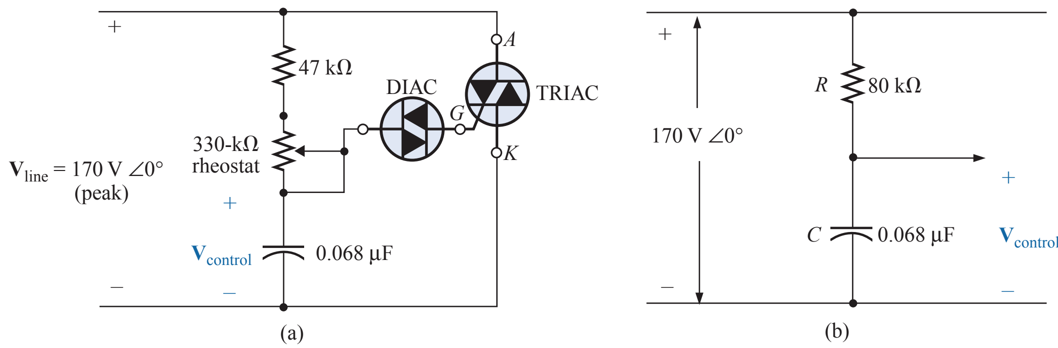


FIG. 15.107
Light dimmer: (a) from Fig. 12.49; (b) with rheostat set at 33 kΩ.

while the full line voltage of 120 V rms, 170 V peak, is across the entire branch. To describe the behavior of the network, let us examine the case defined by setting the potentiometer (used as a rheostat) to 1/10 its maximum value, or 33 kΩ. Combining the 33 kΩ with the fixed resistance of 47 kΩ will result in a total resistance of 80 kΩ and the equivalent network of Fig. 15.107(b).

At 60 Hz, the reactance of the capacitor is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(60\text{ Hz})(62\ \mu\text{F})} = 42.78\text{ k}\Omega$$

Applying the voltage divider rule:

$$\begin{aligned}\mathbf{V}_{\text{control}} &= \frac{\mathbf{Z}_C\mathbf{V}_s}{\mathbf{Z}_R + \mathbf{Z}_C}\\ &= \frac{(42.78\text{ k}\Omega\ \angle -90^\circ)(V_s\ \angle 0^\circ)}{80\text{ k}\Omega - j\,42.78\text{ k}\Omega} = \frac{42.78\text{ k}\Omega\ V_s\ \angle -90^\circ}{90.72\text{ k}\Omega\ \angle -28.14^\circ}\\ &= 0.472V_s\ \angle -61.86^\circ\end{aligned}$$

Using a peak value of 170 V:

$$\begin{aligned}\mathbf{V}_{\text{control}} &= 0.472(170\text{ V})\ \angle -61.86^\circ\\ &= 80.24\text{ V}\ \angle -61.86^\circ\end{aligned}$$

producing the waveform of Fig. 15.108(a). The result is a waveform with a phase shift of 61.86° (lagging the applied line voltage) and a relatively high peak value. The high peak value will result in a quick transition to the 20-V turn-on level, and power will be distributed to the bulb for the major portion of the applied signal. Recall from the discussion of Chapter 12 that the response in the negative region is a replica of that achieved in the positive region. If we reduced the potentiometer resistance further, the phase angle would be reduced, and the bulb would burn brighter. The situation is now very similar to that described for the response of Fig. 15.106(a). In other words, nothing has been gained thus far by using the capacitive element in the control network. However, let us now increase the potentiometer resistance to 200 kΩ and note the effect on the controlling voltage.

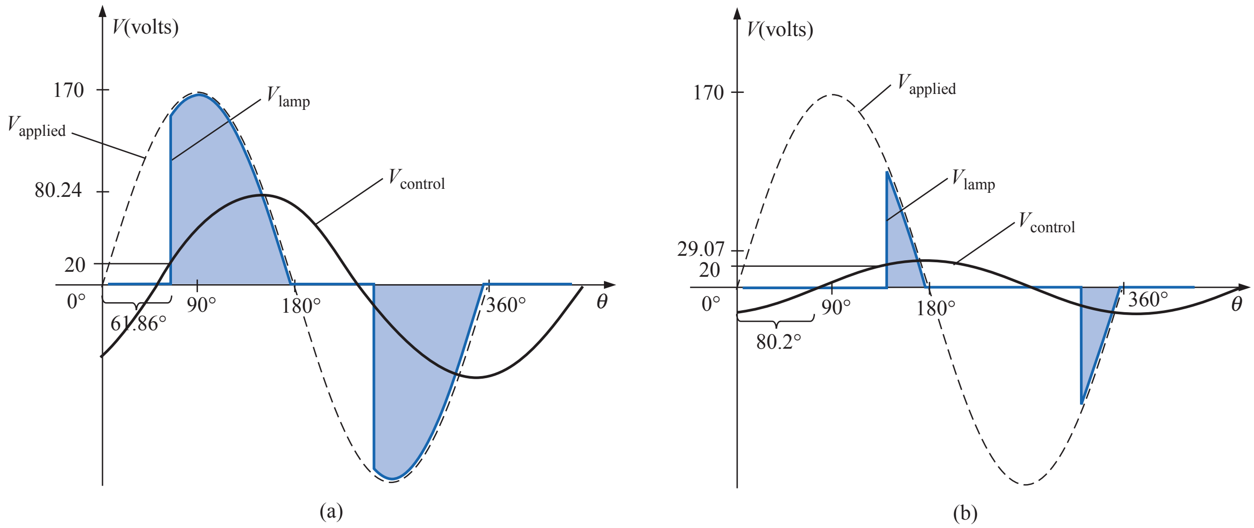


FIG. 15.108
Light dimmer of Fig. 12.49: (a) rheostat set at 33 kΩ; (b) rheostat set at 200 kΩ.

That is,

$$R_T = 200\text{ k}\Omega + 47\text{ k}\Omega = 247\text{ k}\Omega$$

$$\mathbf{V}_{\text{control}} = \frac{\mathbf{Z}_C\mathbf{V}_s}{\mathbf{Z}_R + \mathbf{Z}_C}$$

$$= \frac{(42.78\text{ k}\Omega\angle -90°)(V_s\angle 0°)}{247\text{ k}\Omega - j42.78\text{ k}\Omega} = \frac{42.78\text{ k}\Omega\ V_s\angle -90°}{250.78\text{ k}\Omega\angle -9.8°}$$

$$= 0.171V_s\angle -80.2°$$

and using a peak value of 170 V, we have

$$\mathbf{V}_{\text{control}} = 0.171(170\text{ V})\angle -80.2°$$
$$= 29.07\text{ V}\angle -80.2°$$

The peak value has been substantially reduced to only 29.07 V, and the phase-shift angle has increased to 80.2°. The result, as depicted by Fig. 15.108(b), is that the firing potential of 20 V is not reached until near the end of the positive region of the applied voltage. Power is delivered to the bulb for only a very short period of time, causing the bulb to be quite dim, significantly dimmer than obtained from the response of Fig. 15.106(b).

A conduction angle less than 90° is therefore possible due only to the phase shift introduced by the series *R-C* combination. Thus, it is possible to construct a network of some significance with a rather simple pair of elements.

15.15 COMPUTER ANALYSIS

PSpice

Series *R-L-C* Circuit The *R-L-C* network of Fig. 15.35 will now be analyzed using OrCAD Capture. Since the inductive and capacitive reactances cannot be entered onto the screen, the associated inductive and capacitive levels were first determined as follows:

$$X_L = 2\pi f L \Rightarrow L = \frac{X_L}{2\pi f} = \frac{7\ \Omega}{2\pi(1\text{ kHz})} = 1.114\text{ mH}$$

$$X_C = \frac{1}{2\pi f C} \Rightarrow C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(1\text{ kHz})3\ \Omega} = 53.05\ \mu\text{F}$$

The values were then entered into the schematic as shown in Fig. 15.109. For the ac source, the sequence is **Place part** icon-**SOURCE-VSIN-OK** with **VOFF** set at 0 V, **VAMPL** set at 70.7 V (the peak value of the applied sinusoidal source in Fig. 15.35), and **FREQ** = 1 kHz. If we double-click on the source symbol, the **Property Editor** will appear, confirming the above choices and showing that **DF** = 0 s, **PHASE** = 0°, and **TD** = 0 s as set by the default levels. We are now ready to do an analysis of the circuit for the fixed frequency of 1 kHz.

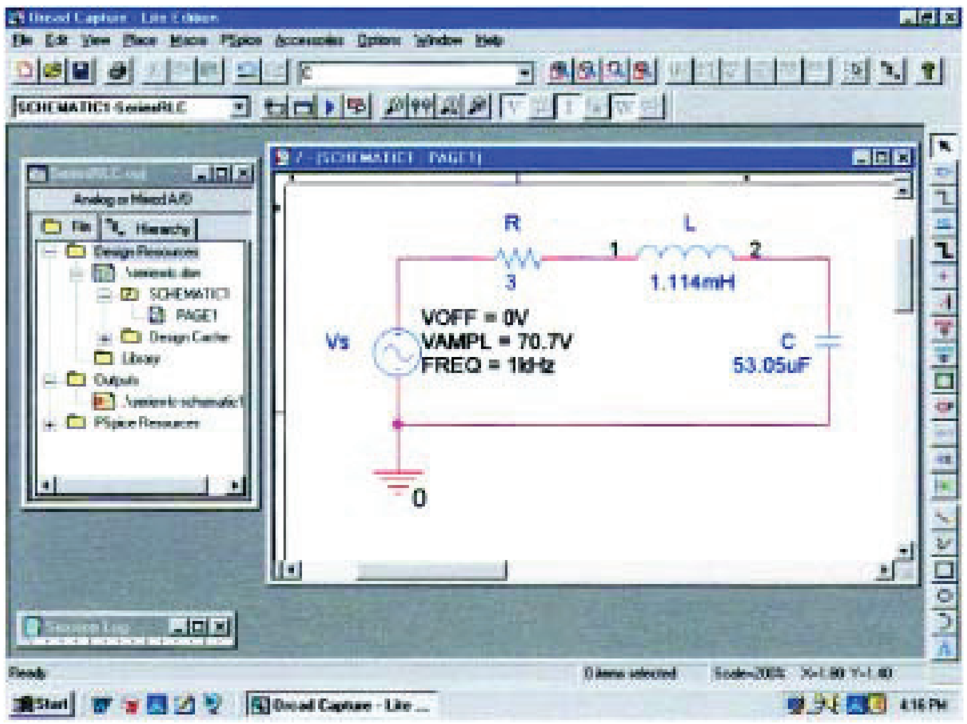


FIG. 15.109
Using PSpice to analyze a series R-L-C ac circuit.

The simulation process is initiated by first selecting the **New Simulation Profile** icon and inserting **SeriesRLC** as the **Name** followed by **Create.** The **Simulation Settings** dialog will now appear, and since we are continuing to plot the results against time, the **Time Domain(Transient)** option is selected under **Analysis type.** Since the period of each **cycle** of the applied source is 1 ms, the **Run to time** will be set at 5 ms so that five cycles will appear. The **Start saving data after** will be left at 0 s even though there will be an oscillatory period for the reactive elements before the circuit settles down. The **Maximum step size** will be set at 5 ms/1000 = 5 μs. Finally **OK** is selected followed by the

Run PSpice key. The result will be a blank screen with an *x*-axis extending from 0 s to 5 ms.

The first quantity of interest is the current through the circuit, so **Trace-Add-Trace** is selected followed by **I(R)** and **OK.** The resulting plot of Fig. 15.110 clearly shows that there is a period of storing and discharging of the reactive elements before a steady-state level is established. It would appear that after 3 ms, steady-state conditions have been essentially established. Select the **Toggle cursor** key, and left-click the mouse; a cursor will result that can be moved along the axis near the maximum value around 1.4 ms. In fact, the cursor reveals a maximum value of 16.4 A which exceeds the steady-state solution by over 2 A. A right click of the mouse will establish a second cursor on the screen that can be placed near the steady-state peak around 4.4 ms. The resulting peak value is about 14.15 A which is a match with the longhand solution for Fig. 15.35. We will therefore assume that steady-state conditions have been established for the circuit after 4 ms.

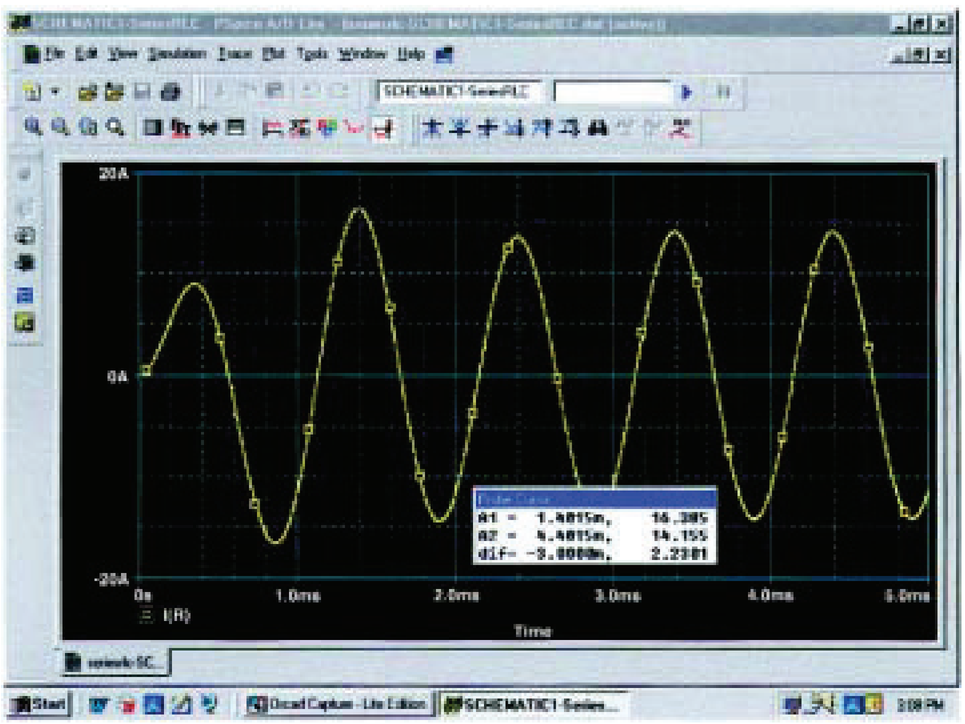


FIG. 15.110

A plot of the current for the circuit of Fig. 15.109 showing the transition from the transient state to the steady-state response.

Let us now add the source voltage through **Trace-Add Trace-V(Vs:+)-OK** to obtain the multiple plot at the bottom of Fig. 15.111. For the voltage across the coil, the sequence **Plot-Add Plot to Window-Trace-Add Trace-V(L:1)-V(L:2)** will result in the plot appearing at the top of Fig. 15.111. Take special note of the fact that the **Trace Expression** is **V(L:1)−V(L:2)** rather than just **V(L:1)** because **V(L:1)** would be the voltage from that point to ground which would include the voltage across the capacitor. In addition, the **−** sign between the two comes from the **Functions or Macros** list at right of the **Add Traces**

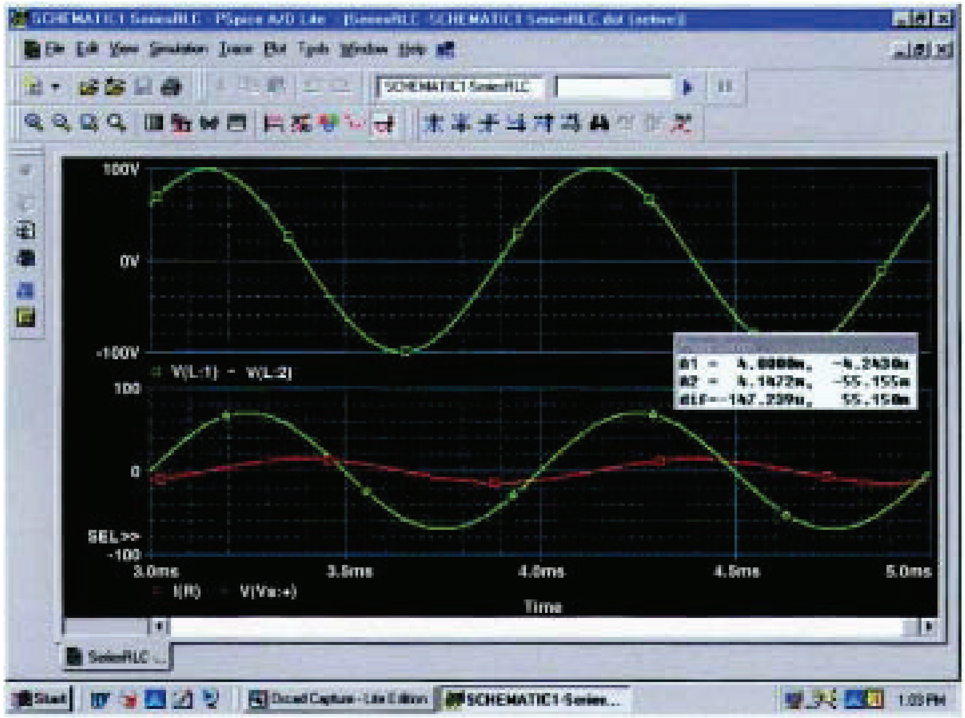


FIG. 15.111

A plot of the steady-state response ($t > 3$ ms) for v_L, v_s, and i for the circuit of Fig. 15.109.

dialog box. Finally, since we know that the waveforms are fairly steady after 3 ms, let us cut away the waveforms before 3 ms with **Plot-Axis Settings-X axis-User Defined-3ms to 5ms-OK** to obtain the two cycles of Fig. 15.111. Now you can clearly see that the peak value of the voltage across the coil is 100 V to match the analysis of Fig. 15.35. It is also clear that the applied voltage leads the input current by an angle that can be determined using the cursors. First activate the cursor option by selecting the cursor key (a red plot through the origin) in the second toolbar down from the menu bar. Then select **V(Vs:+)** at the bottom left of the screen with a left click of the mouse, and set it at that point where the applied voltage passes through the horizontal axis with a positive slope. The result is **A1** = 4 ms at $-4.243\ \mu$V $\cong$ 0 V. Then select **I(R)** at the bottom left of the screen with a right click of the mouse, and place it at the point where the current waveform passes through the horizontal axis with a positive slope. The result is **A2** = 4.15 ms at -55.15 mA = 0.55 A $\cong$ 0 A (compared to a peak value of 14.14 A). At the bottom of the **Probe Cursor** dialog box, the time difference is 147.24 μs.

Now set up the ratio

$$\frac{147.24\ \mu\text{s}}{1000\ \mu\text{s}} = \frac{\theta}{360^\circ}$$

$$\theta = 52.99^\circ$$

The phase angle by which the applied voltage leads the source is 52.99° which is very close to the theoretical solution of 53.13° obtained in Fig. 15.39. Increasing the number of data points for the plot would have increased the accuracy level and brought the results closer to 53.13°.

Electronics Workbench

We will now examine the response of a network versus frequency rather than time using the network of Fig. 15.79 which now appears on the schematic of Fig. 15.112. The ac current source appears as **AC_CURRENT_SOURCE** in the **Sources** tool bin next to the ac voltage source. Note that the current source was given an amplitude of 1 A to establish a magnitude match between the response of the voltage across the network and the impedance of the network. That is,

$$|Z_T| = \left|\frac{V_s}{I_s}\right| = \left|\frac{V_s}{1\text{ A}}\right| = |V_s|$$

Before applying computer methods, we should develop a rough idea of what to expect so that we have something to which to compare the computer solution. At very high frequencies such as 1 MHz, the impedance of the inductive element will be about 25 kΩ which when placed in parallel with the 220 Ω will look like an open circuit. The result is that as the frequency gets very high, we should expect the impedance of the network to approach the 220-Ω level of the resistor. In addition, since the network will take on resistive characteristics at very high frequencies, the angle associated with the input impedance should also approach 0 Ω. At very low frequencies the reactance of the inductive element will be much less than the 220 Ω of the resistor, and the network will take on inductive characteristics. In fact, at, say, 10 Hz, the reactance of the inductor is only about 0.25 Ω which is very close to a short-circuit equivalent compared to the parallel 220-Ω resistor. The result is that the impedance of the network is very close to 0 Ω at very low frequencies. Again, since the inductive effects are so strong at low

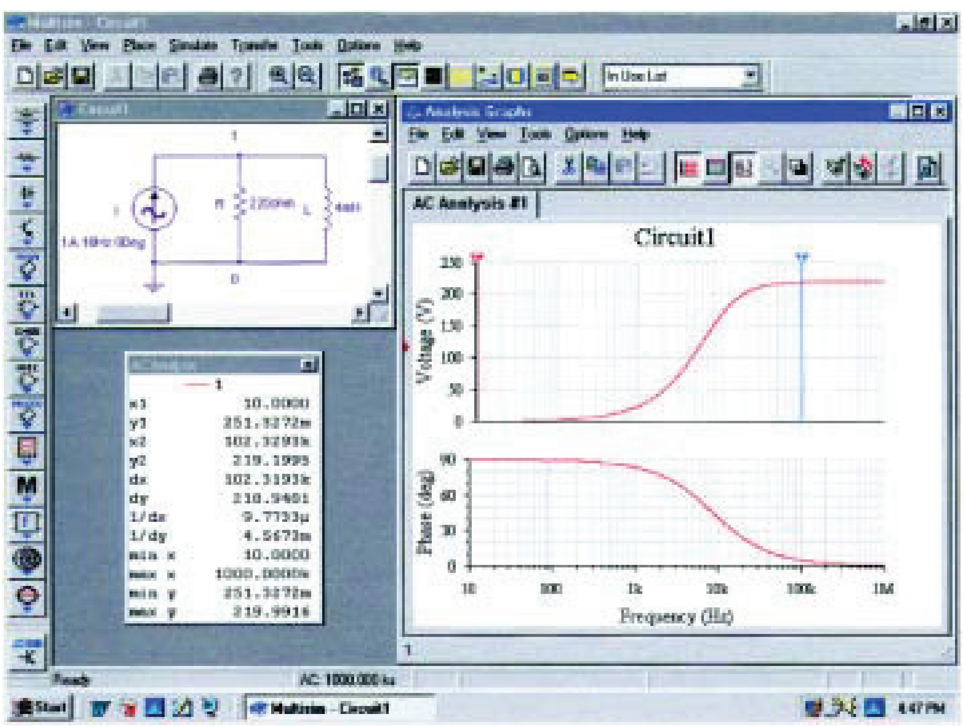


FIG. 15.112
Obtaining an impedance plot for a parallel R-L network using Electronics Workbench.

frequencies, the phase angle associated with the input impedance should be very close to 90°.

Now for the computer analysis. The current source, the resistor element, and the inductor are all placed and connected using procedures described in detail in earlier chapters. However, there is one big difference this time that the user must be aware of: Since the output will be plotted versus frequency, the **Analysis Setup** heading must be selected in the **AC Current** dialog box for the current source. When selected, the **AC Magnitude** must be set to the value of the ac source. In this case, the default level of **1A** matches that of the applied source, so we were set even if we failed to check the setting. In the future, however, a voltage or current source may be used that does not have an amplitude of 1, and proper entries must be made to this listing.

For the simulation the sequence **Simulate-Analyses-AC Analysis** is first applied to obtain the **AC Analysis** dialog box. The **Start frequency** will be set at **10 Hz** so that we have entries at very low frequencies, and the **Stop frequency** will be set at **1MHz** so that we have data points at the other end of the spectrum. The **Sweep type** can remain **Decade,** but the number of points per decade will be 1000 so that we obtain a detailed plot. The **Vertical scale** will be set on **Linear.** Within **Output variables** we find that only one node, **1,** is defined. Shifting it over to the **Selected variables for analysis** column using the **Plot during simulation** key pad and then hitting the **Simulate** key will result in the two plots of Fig. 15.112. The **Show/Hide Grid** key was selected to place the grid on the graph, and the **Show/Hide Cursors** key was selected to place the **AC Analysis** dialog box appearing in Fig. 15.112. Since two graphs are present, we must define the one we are working on by clicking on the **Voltage** or **Phase** heading on the left side of each plot. A small red arrow will appear when selected to keep us aware of the active plot. When setting up the cursors, be sure that you have activated the correct plot. When the red cursor is moved to 10 Hz **(x1),** we find that the voltage across the network is only 0.251 V **(y1),** resulting in an input impedance of only 0.25 Ω—quite small and matching our theoretical prediction. In addition, note that the phase angle is essentially at 90° in the other plot, confirming our other assumption above—a totally inductive network. If we set the blue cursor near 100 kHz (**x2** = 102.3 kHz), we find that the impedance at 219.2 Ω **(y2)** is closing in on the resistance of the parallel resistor of 220 Ω, again confirming the preliminary analysis above. As noted in the bottom of the **AC Analysis** box, the maximum value of the voltage is 219.99 Ω or essentially 220 Ω at 1 MHz. Before leaving the plot, note the advantages of using a log axis when you want a response over a wide frequency range.

PROBLEMS

SECTION 15.2 Impedance and the Phasor Diagram

1. Express the impedances of Fig. 15.113 in both polar and rectangular forms.

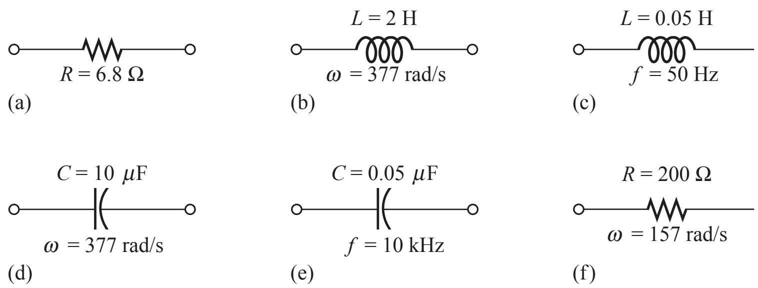


FIG. 15.113
Problem 1.

2. Find the current i for the elements of Fig. 15.114 using complex algebra. Sketch the waveforms for v and i on the same set of axes.

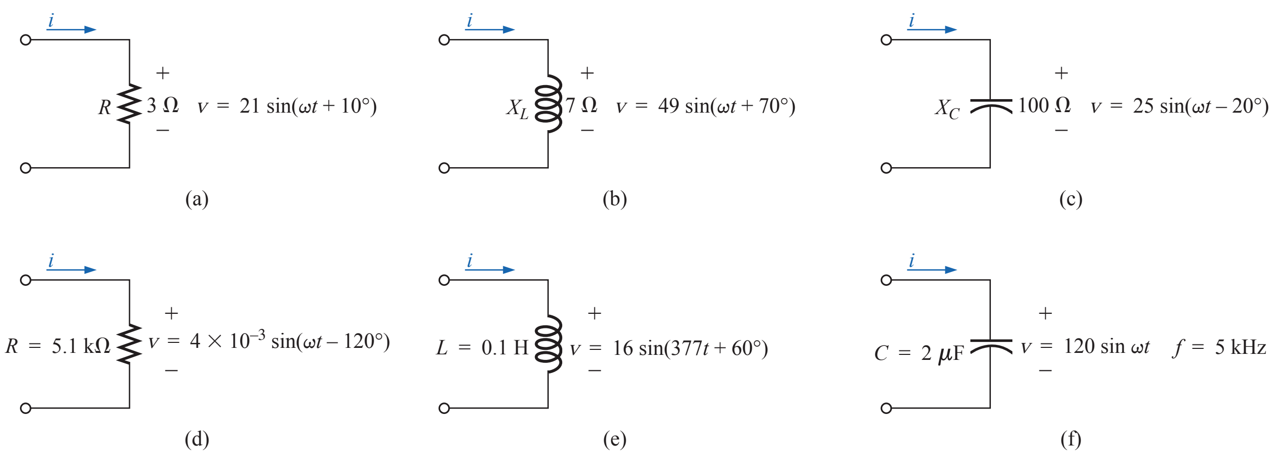


FIG. 15.114
Problem 2.

3. Find the voltage v for the elements of Fig. 15.115 using complex algebra. Sketch the waveforms of v and i on the same set of axes.

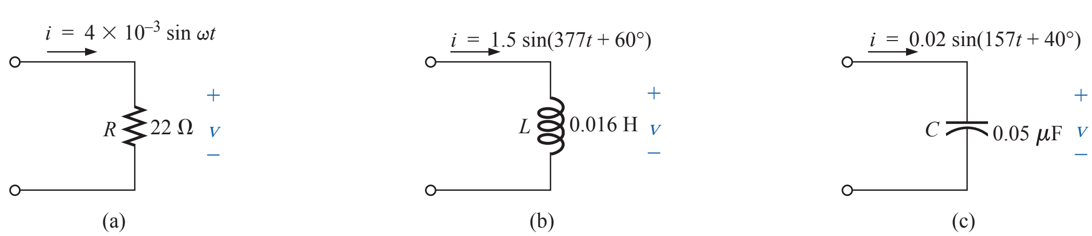


FIG. 15.115
Problem 3.

SECTION 15.3 Series Configuration

4. Calculate the total impedance of the circuits of Fig. 15.116. Express your answer in rectangular and polar forms, and draw the impedance diagram.

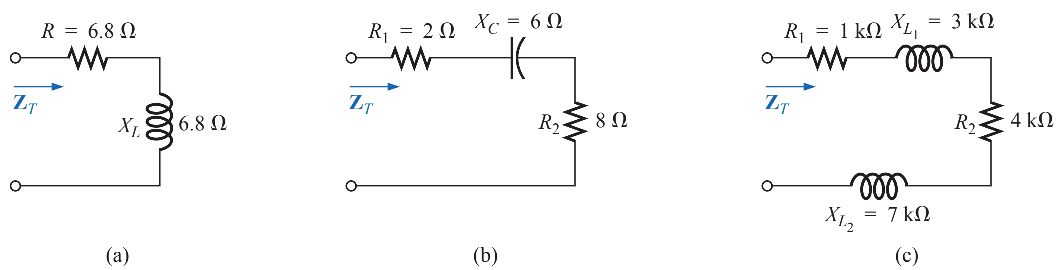


FIG. 15.116
Problem 4.

5. Calculate the total impedance of the circuits of Fig. 15.117. Express your answer in rectangular and polar forms, and draw the impedance diagram.

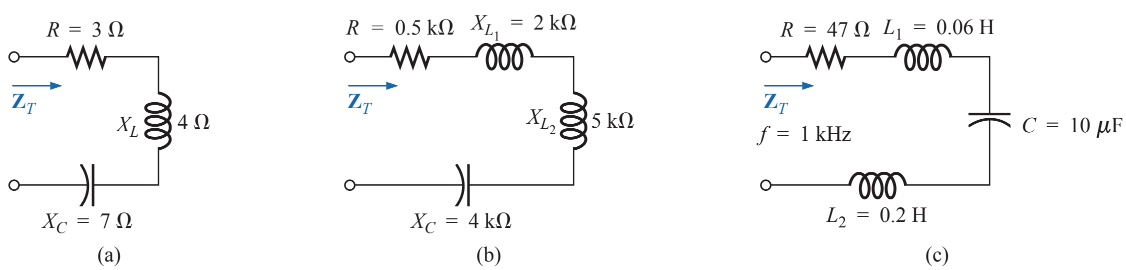


FIG. 15.117
Problem 5.

6. Find the type and impedance in ohms of the series circuit elements that must be in the closed container of Fig. 15.118 for the indicated voltages and currents to exist at the input terminals. (Find the simplest series circuit that will satisfy the indicated conditions.)

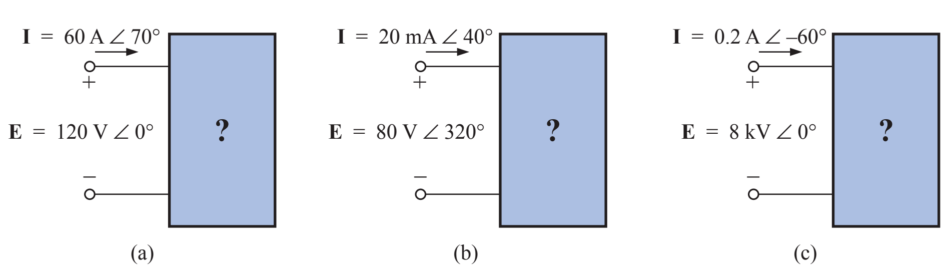


FIG. 15.118
Problems 6 and 26.

7. For the circuit of Fig. 15.119:

a. Find the total impedance $\mathbf{Z}_T$ in polar form.
b. Draw the impedance diagram.
c. Find the current $\mathbf{I}$ and the voltages $\mathbf{V}_R$ and $\mathbf{V}_L$ in phasor form.
d. Draw the phasor diagram of the voltages $\mathbf{E}$, $\mathbf{V}_R$, and $\mathbf{V}_L$, and the current $\mathbf{I}$.
e. Verify Kirchhoff's voltage law around the closed loop.
f. Find the average power delivered to the circuit.
g. Find the power factor of the circuit, and indicate whether it is leading or lagging.
h. Find the sinusoidal expressions for the voltages and current if the frequency is 60 Hz.
i. Plot the waveforms for the voltages and current on the same set of axes.

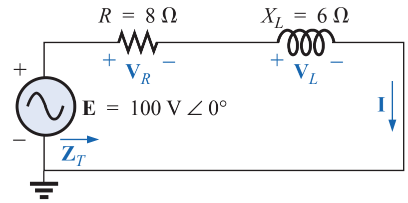


FIG. 15.119
Problems 7 and 47.

8. Repeat Problem 7 for the circuit of Fig. 15.120, replacing $\mathbf{V}_L$ with $\mathbf{V}_C$ in parts (c) and (d).

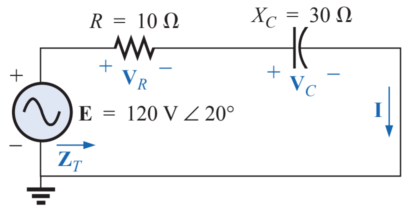


FIG. 15.120
Problem 8.

9. Given the network of Fig. 15.121:

a. Determine $\mathbf{Z}_T$.
b. Find $\mathbf{I}$.
c. Calculate $\mathbf{V}_R$ and $\mathbf{V}_L$.
d. Find P and F_p.

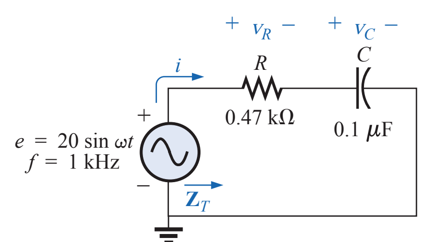


FIG. 15.121
Problems 9 and 49.

10. For the circuit of Fig. 15.122:

a. Find the total impedance $\mathbf{Z}_T$ in polar form.
b. Draw the impedance diagram.
c. Find the value of C in microfarads and L in henries.
d. Find the current $\mathbf{I}$ and the voltages $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ in phasor form.
e. Draw the phasor diagram of the voltages $\mathbf{E}$, $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$, and the current $\mathbf{I}$.
f. Verify Kirchhoff's voltage law around the closed loop.
g. Find the average power delivered to the circuit.
h. Find the power factor of the circuit, and indicate whether it is leading or lagging.
i. Find the sinusoidal expressions for the voltages and current.
j. Plot the waveforms for the voltages and current on the same set of axes.

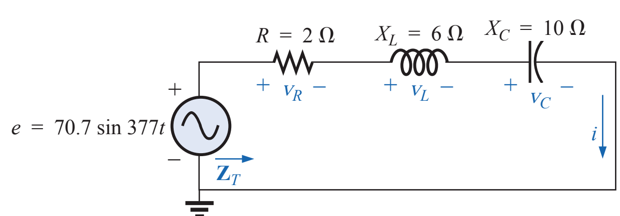


FIG. 15.122
Problem 10.

11. Repeat Problem 10 for the circuit of Fig. 15.123.

12. Using the oscilloscope reading of Fig. 15.124, determine the resistance R.

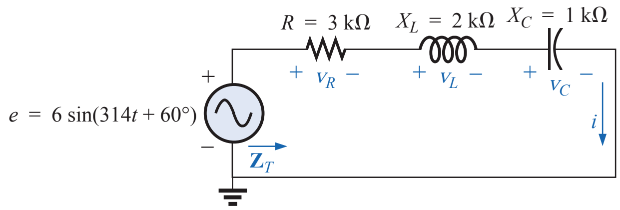


FIG. 15.123
Problem 11.

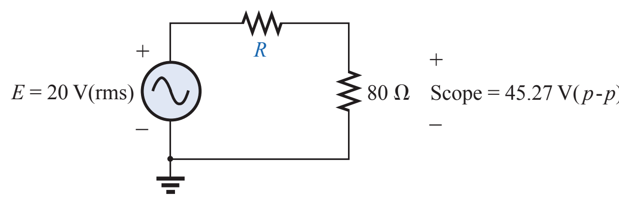


FIG. 15.124
Problem 12.

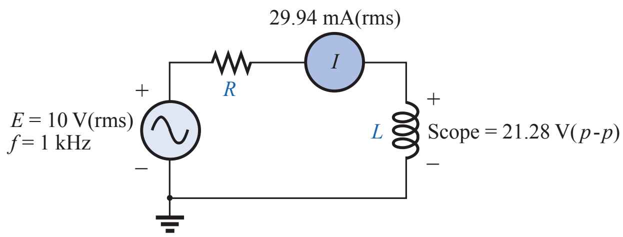


FIG. 15.125
Problem 13.

***13.** Using the DMM current reading and the oscilloscope measurement of Fig. 15.125:

a. Determine the inductance L.
b. Find the resistance R.

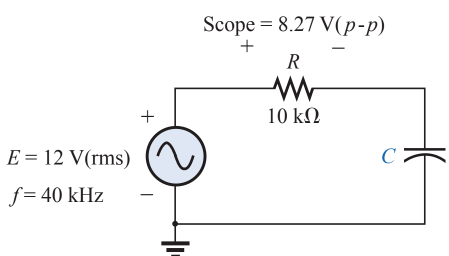


FIG. 15.126
Problem 14.

***14.** Using the oscilloscope reading of Fig. 15.126, determine the capacitance C.

SECTION 15.4 Voltage Divider Rule

15. Calculate the voltages $\mathbf{V}_1$ and $\mathbf{V}_2$ for the circuit of Fig. 15.127 in phasor form using the voltage divider rule.

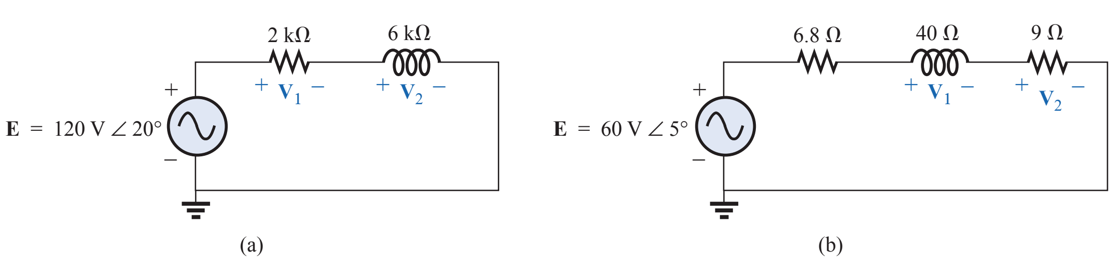


FIG. 15.127
Problem 15.

16. Calculate the voltages $\mathbf{V}_1$ and $\mathbf{V}_2$ for the circuit of Fig. 15.128 in phasor form using the voltage divider rule.

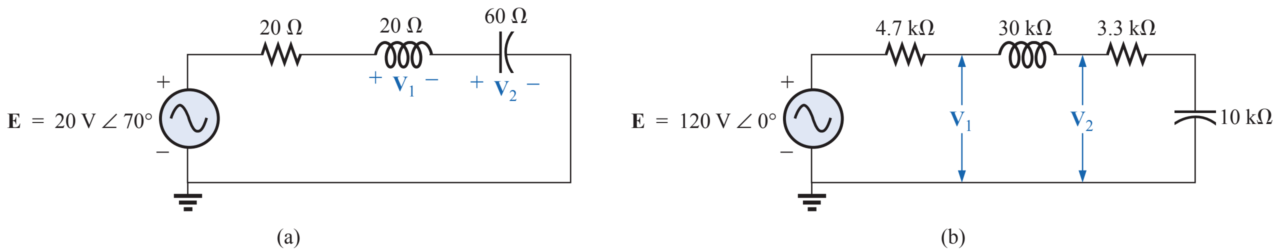


FIG. 15.128
Problem 16.

***17.** For the circuit of Fig. 15.129:

a. Determine $\mathbf{I}$, $\mathbf{V}_R$, and $\mathbf{V}_C$ in phasor form.
b. Calculate the total power factor, and indicate whether it is leading or lagging.
c. Calculate the average power delivered to the circuit.
d. Draw the impedance diagram.
e. Draw the phasor diagram of the voltages $\mathbf{E}$, $\mathbf{V}_R$, and $\mathbf{V}_C$, and the current $\mathbf{I}$.
f. Find the voltages $\mathbf{V}_R$ and $\mathbf{V}_C$ using the voltage divider rule, and compare them with the results of part (a) above.
g. Draw the equivalent series circuit of the above as far as the total impedance and the current i are concerned.

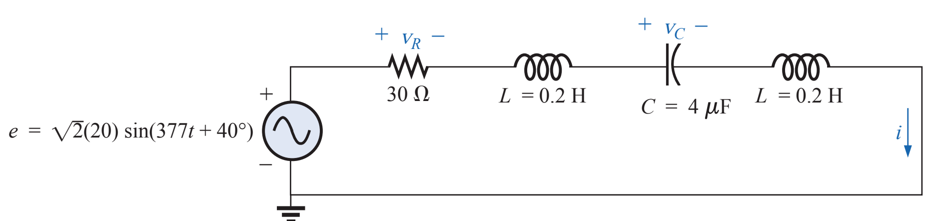


FIG. 15.129
Problems 17, 18, and 50.

***18.** Repeat Problem 17 if the capacitance is changed to 1000 μF.

19. An electrical load has a power factor of 0.8 lagging. It dissipates 8 kW at a voltage of 200 V. Calculate the impedance of this load in rectangular coordinates.

***20.** Find the series element or elements that must be in the enclosed container of Fig. 15.130 to satisfy the following conditions:

a. Average power to circuit = 300 W.

b. Circuit has a lagging power factor.

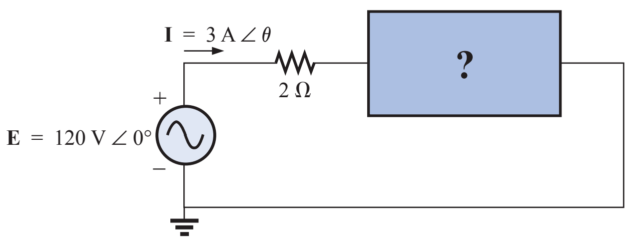


FIG. 15.130
Problem 20.

SECTION 15.5 Frequency Response of the *R-C* Circuit

***21.** For the circuit of Fig. 15.131:

a. Plot Z_T and θ_T versus frequency for a frequency range of zero to 20 kHz.

b. Plot V_L versus frequency for the frequency range of part (a).

c. Plot θ_L versus frequency for the frequency range of part (a).

d. Plot V_R versus frequency for the frequency range of part (a).

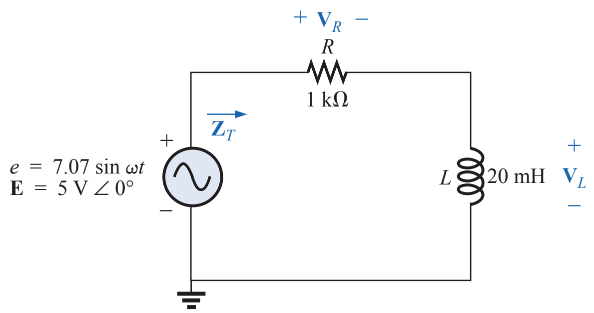


FIG. 15.131
Problem 21.

***22.** For the circuit of Fig. 15.132:

a. Plot Z_T and θ_T versus frequency for a frequency range of zero to 10 kHz.

b. Plot V_C versus frequency for the frequency range of part (a).

c. Plot θ_C versus frequency for the frequency range of part (a).

d. Plot V_R versus frequency for the frequency range of part (a).

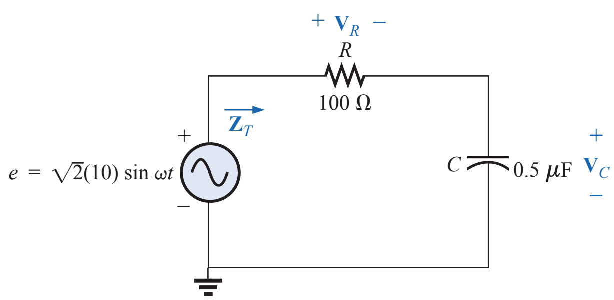


FIG. 15.132
Problem 22.

***23.** For the series *R-L-C* circuit of Fig. 15.133:

a. Plot Z_T and θ_T versus frequency for a frequency range of zero to 20 kHz in increments of 1 kHz.

b. Plot V_C (magnitude only) versus frequency for the same frequency range of part (a).

c. Plot I (magnitude only) versus frequency for the same frequency range of part (a).

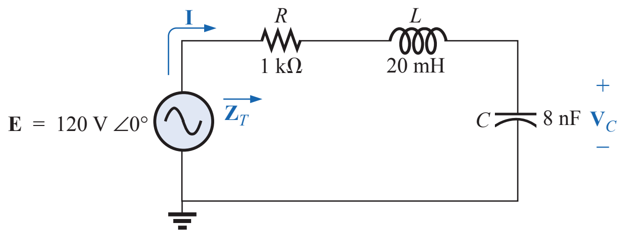


FIG. 15.133
Problem 23.

SECTION 15.7 Admittance and Susceptance

24. Find the total admittance and impedance of the circuits of Fig. 15.134. Identify the values of conductance and susceptance, and draw the admittance diagram.

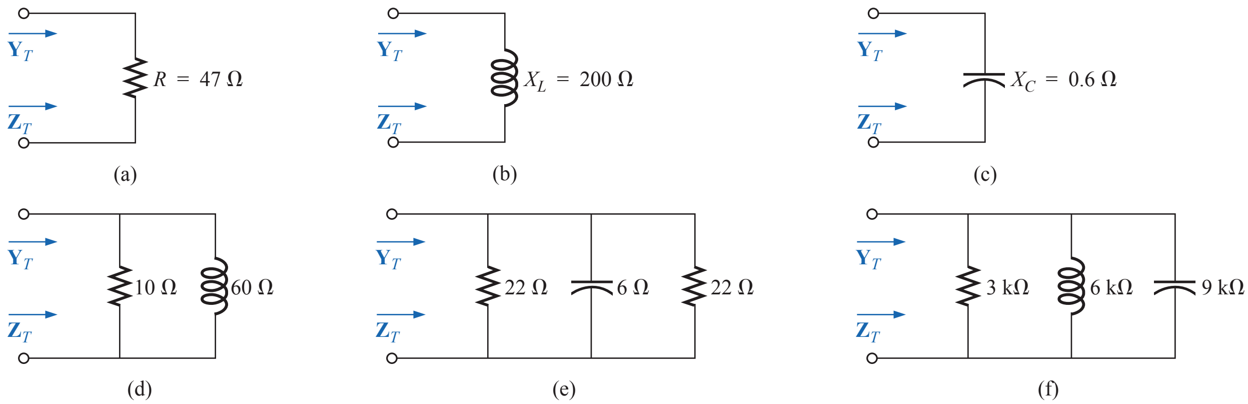


FIG. 15.134
Problem 24.

25. Find the total admittance and impedance of the circuits of Fig. 15.135. Identify the values of conductance and susceptance, and draw the admittance diagram.

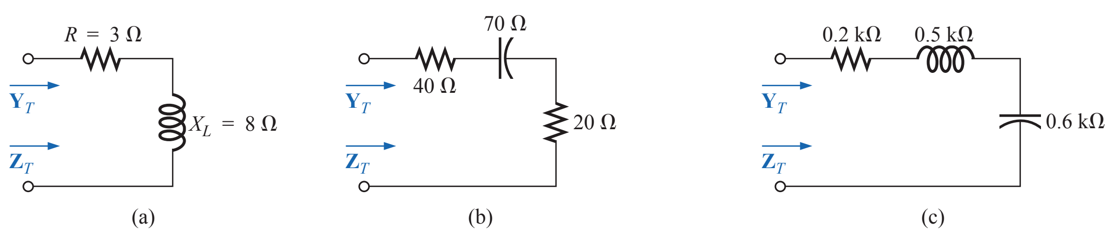


FIG. 15.135
Problem 25.

26. Repeat Problem 6 for the parallel circuit elements that must be in the closed container for the same voltage and current to exist at the input terminals. (Find the simplest parallel circuit that will satisfy the conditions indicated.)

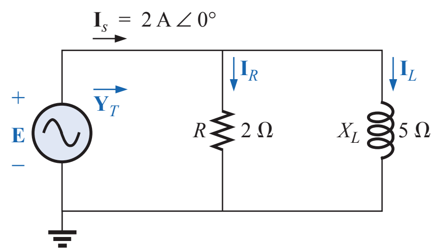


FIG. 15.136
Problem 27.

SECTION 15.8 Parallel ac Networks

27. For the circuit of Fig. 15.136:

a. Find the total admittance $\mathbf{Y}_T$ in polar form.
b. Draw the admittance diagram.
c. Find the voltage **E** and the currents $\mathbf{I}_R$ and $\mathbf{I}_L$ in phasor form.
d. Draw the phasor diagram of the currents $\mathbf{I}_s$, $\mathbf{I}_R$, and $\mathbf{I}_L$, and the voltage **E**.
e. Verify Kirchhoff's current law at one node.
f. Find the average power delivered to the circuit.
g. Find the power factor of the circuit, and indicate whether it is leading or lagging.
h. Find the sinusoidal expressions for the currents and voltage if the frequency is 60 Hz.
i. Plot the waveforms for the currents and voltage on the same set of axes.

28. Repeat Problem 27 for the circuit of Fig. 15.137, replacing $\mathbf{I}_L$ with $\mathbf{I}_C$ in parts (c) and (d).

29. Repeat Problem 27 for the circuit of Fig. 15.138, replacing **E** with $\mathbf{I}_s$ in part (c).

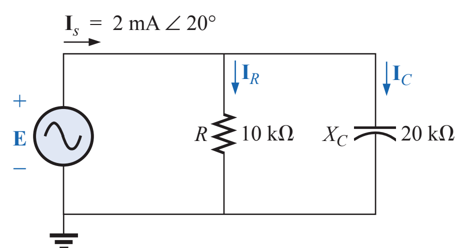


FIG. 15.137
Problem 28.

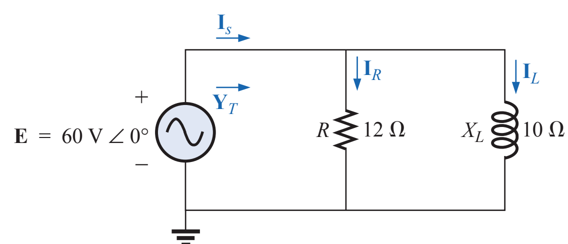


FIG. 15.138
Problems 29 and 48.

30. For the circuit of Fig. 15.139:

a. Find the total admittance $\mathbf{Y}_T$ in polar form.
b. Draw the admittance diagram.
c. Find the value of C in microfarads and L in henries.
d. Find the voltage **E** and currents $\mathbf{I}_R$, $\mathbf{I}_L$, and $\mathbf{I}_C$ in phasor form.
e. Draw the phasor diagram of the currents $\mathbf{I}_s$, $\mathbf{I}_R$, $\mathbf{I}_L$, and $\mathbf{I}_C$, and the voltage **E.**
f. Verify Kirchhoff's current law at one node.
g. Find the average power delivered to the circuit.
h. Find the power factor of the circuit, and indicate whether it is leading or lagging.
i. Find the sinusoidal expressions for the currents and voltage.
j. Plot the waveforms for the currents and voltage on the same set of axes.

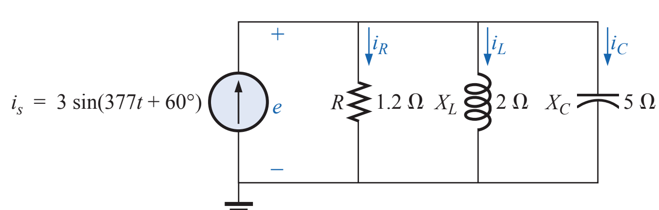


FIG. 15.139
Problem 30.

31. Repeat Problem 30 for the circuit of Fig. 15.140.

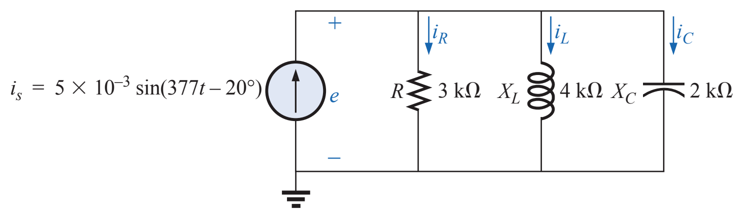


FIG. 15.140
Problem 31.

32. Repeat Problem 30 for the circuit of Fig. 15.141, replacing e with i_s in part (d).

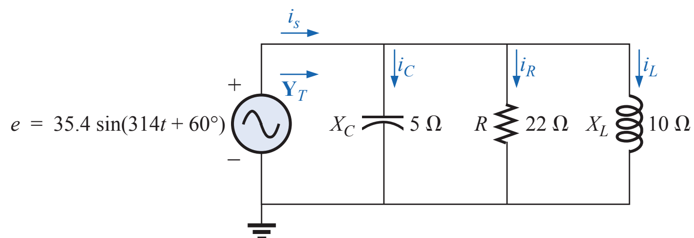


FIG. 15.141
Problem 32.

SECTION 15.9 Current Divider Rule

33. Calculate the currents $\mathbf{I}_1$ and $\mathbf{I}_2$ of Fig. 15.142 in phasor form using the current divider rule.

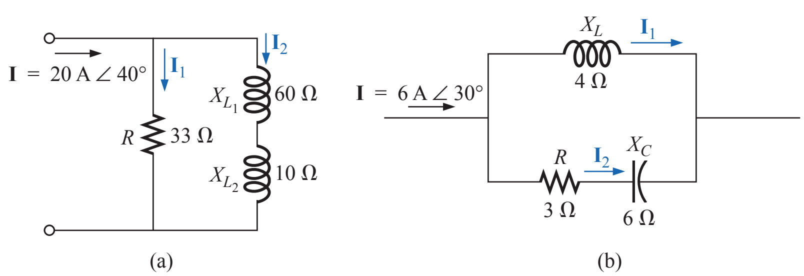


FIG. 15.142
Problem 33.

SECTION 15.10 Frequency Response of the Parallel *R-L* Network

***34.** For the parallel *R-C* network of Fig. 15.143:

a. Plot Z_T and θ_T versus frequency for a frequency range of zero to 20 kHz.

b. Plot V_C versus frequency for the frequency range of part (a).

c. Plot I_R versus frequency for the frequency range of part (a).

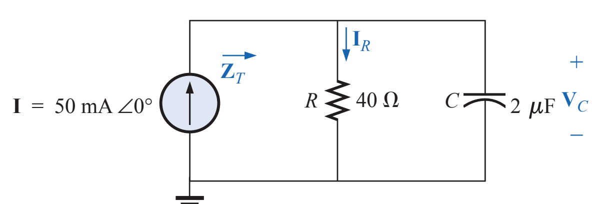


FIG. 15.143
Problems 34 and 36.

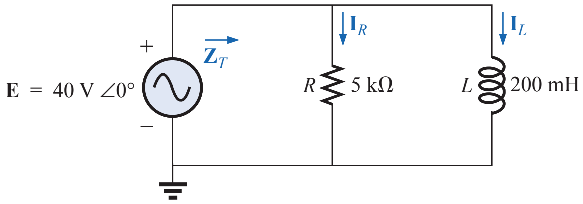


FIG. 15.144
Problems 35 and 37.

***35.** For the parallel *R-L* network of Fig. 15.144:
   a. Plot Z_T and θ_T versus frequency for a frequency range of zero to 10 kHz.
   b. Plot I_L versus frequency for the frequency range of part (a).
   c. Plot I_R versus frequency for the frequency range of part (a).

36. Plot Y_T and θ_T (of $\mathbf{Y}_T = Y_T \angle\theta_T$) for a frequency range of zero to 20 kHz for the network of Fig. 15.143.

37. Plot Y_T and θ_T (of $\mathbf{Y}_T = Y_T \angle\theta_T$) for a frequency range of zero to 10 kHz for the network of Fig. 15.144.

38. For the parallel *R-L-C* network of Fig. 15.145:
   a. Plot Y_T and θ_T (of $\mathbf{Y}_T = Y_T \angle\theta_T$) for a frequency range of zero to 20 kHz.
   b. Repeat part (a) for Z_T and θ_T (of $\mathbf{Z}_T = Z_T \angle\theta_T$).
   c. Plot V_C versus frequency for the frequency range of part (a).
   d. Plot I_L versus frequency for the frequency range of part (a).

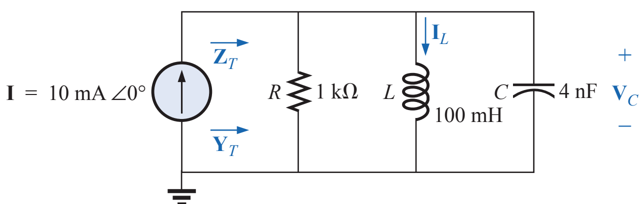


FIG. 15.145
Problem 38.

SECTION 15.12 Equivalent Circuits

39. For the series circuits of Fig. 15.146, find a parallel circuit that will have the same total impedance ($\mathbf{Z}_T$).

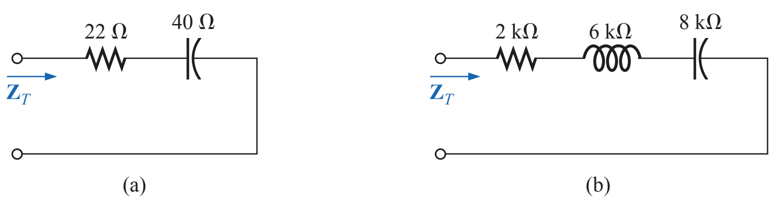


FIG. 15.146
Problem 39.

40. For the parallel circuits of Fig. 15.147, find a series circuit that will have the same total impedance.

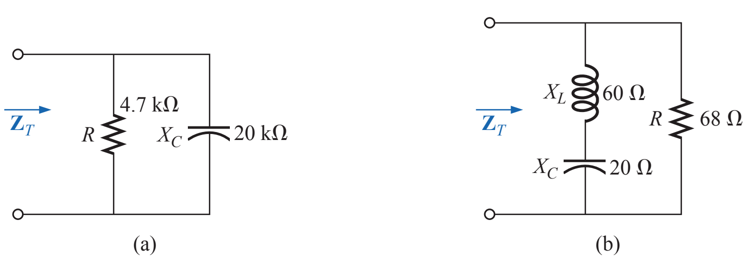


FIG. 15.147
Problem 40.

41. For the network of Fig. 15.148:

a. Calculate $\mathbf{E}$, $\mathbf{I}_R$, and $\mathbf{I}_L$ in phasor form.

b. Calculate the total power factor, and indicate whether it is leading or lagging.

c. Calculate the average power delivered to the circuit.

d. Draw the admittance diagram.

e. Draw the phasor diagram of the currents $\mathbf{I}_s$, $\mathbf{I}_R$, and $\mathbf{I}_L$, and the voltage $\mathbf{E}$.

f. Find the current $\mathbf{I}_C$ for each capacitor using only Kirchhoff's current law.

g. Find the series circuit of one resistive and reactive element that will have the same impedance as the original circuit.

***42.** Repeat Problem 41 if the inductance is changed to 1 H.

43. Find the element or elements that must be in the closed container of Fig. 15.149 to satisfy the following conditions. (Find the simplest parallel circuit that will satisfy the indicated conditions.)

a. Average power to the circuit = 3000 W.

b. Circuit has a lagging power factor.

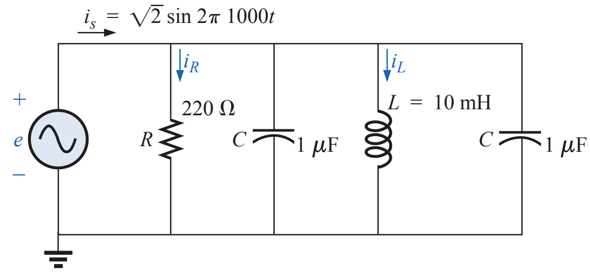


FIG. 15.148
Problems 41 and 42.

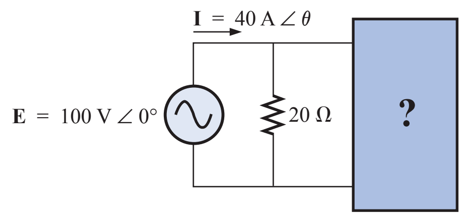


FIG. 15.149
Problem 43.

SECTION 15.13 Phase Measurements (Dual-Trace Oscilloscope)

44. For the circuit of Fig. 15.150, determine the phase relationship between the following using a dual-trace oscilloscope. The circuit can be reconstructed differently for each part, but do not use sensing resistors. Show all connections on a redrawn diagram.

a. e and v_C

b. e and i_s

c. e and v_L

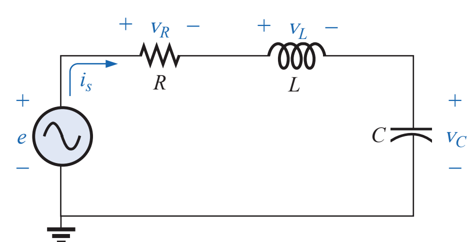


FIG. 15.150
Problem 44.

45. For the network of Fig. 15.151, determine the phase relationship between the following using a dual-trace oscilloscope. The network must remain as constructed in Fig. 15.151, but sensing resistors can be introduced. Show all connections on a redrawn diagram.

a. e and v_{R_2}

b. e and i_s

c. i_L and i_C

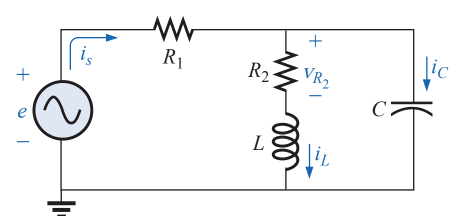


FIG. 15.151
Problem 45.

46. For the oscilloscope traces of Fig. 15.152:
   a. Determine the phase relationship between the waveforms, and indicate which one leads or lags.
   b. Determine the peak-to-peak and rms values of each waveform.
   c. Find the frequency of each waveform.

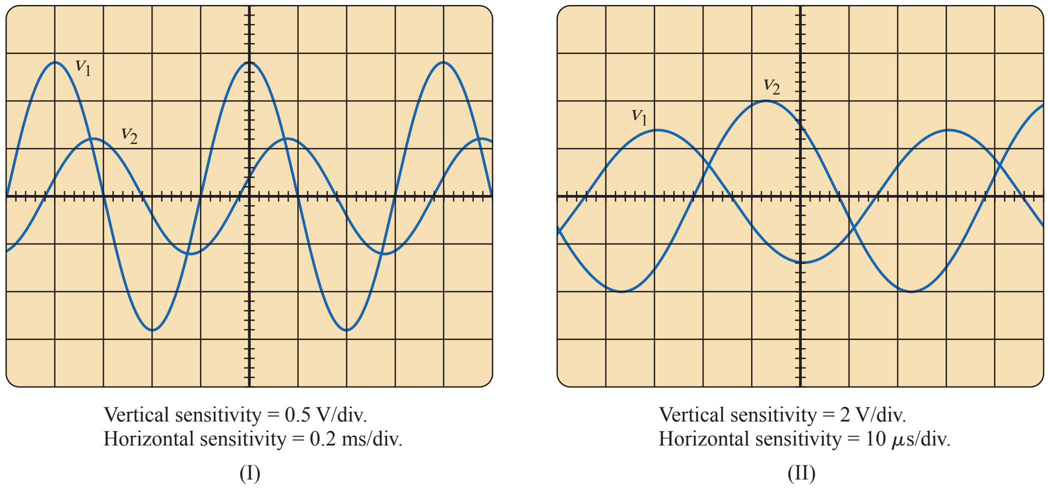


FIG. 15.152
Problem 46.

SECTION 15.15 Computer Analysis

PSpice or Electronics Workbench

47. For the network of Fig. 15.119 (use $f = 1$ kHz):
   a. Determine the rms values of the voltages $\mathbf{V}_R$ and $\mathbf{V}_L$ and the current $\mathbf{I}$.
   b. Plot v_R, v_L, and i versus time on separate plots.
   c. Place e, v_R, v_L, and i on the same plot, and label accordingly.

48. For the network of Fig. 15.138:
   a. Determine the rms values of the currents $\mathbf{I}_s$, $\mathbf{I}_R$, and $\mathbf{I}_L$.
   b. Plot i_s, i_R, and i_L versus time on separate plots.
   c. Place e, i_s, i_R, and i_L on the same plot, and label accordingly.

49. For the network of Fig. 15.121:
   a. Plot the impedance of the network versus frequency from 0 to 10 kHz.
   b. Plot the current i versus frequency for the frequency range zero to 10 kHz.

***50.** For the network of Fig. 15.129:
   a. Find the rms values of the voltages v_R and v_C at a frequency of 1 kHz.
   b. Plot v_C versus frequency for the frequency range zero to 10 kHz.
   c. Plot the phase angle between e and i for the frequency range zero to 10 kHz.

Programming Language (C++, QBASIC, Pascal, etc.)

51. Write a program to generate the sinusoidal expression for the current of a resistor, inductor, or capacitor given the value of R, L, or C and the applied voltage in sinusoidal form.

52. Given the impedance of each element in rectangular form, write a program to determine the total impedance in rectangular form of any number of series elements.

53. Given two phasors in polar form in the first quadrant, write a program to generate the sum of the two phasors in polar form.

GLOSSARY

Admittance A measure of how easily a network will "admit" the passage of current through that system. It is measured in siemens, abbreviated S, and is represented by the capital letter *Y*.

Admittance diagram A vector display that clearly depicts the magnitude of the admittance of the conductance, capacitive susceptance, and inductive susceptance, and the magnitude and angle of the total admittance of the system.

Current divider rule A method by which the current through either of two parallel branches can be determined in an ac network without first finding the voltage across the parallel branches.

Equivalent circuits For every series ac network there is a parallel ac network (and vice versa) that will be "equivalent" in the sense that the input current and impedance are the same.

Impedance diagram A vector display that clearly depicts the magnitude of the impedance of the resistive, reactive, and capacitive components of a network, and the magnitude and angle of the total impedance of the system.

Parallel ac circuits A connection of elements in an ac network in which all the elements have two points in common. The voltage is the same across each element.

Phasor diagram A vector display that provides at a glance the magnitude and phase relationships among the various voltages and currents of a network.

Series ac configuration A connection of elements in an ac network in which no two impedances have more than one terminal in common and the current is the same through each element.

Susceptance A measure of how "susceptible" an element is to the passage of current through it. It is measured in siemens, abbreviated S, and is represented by the capital letter *B*.

Voltage divider rule A method through which the voltage across one element of a series of elements in an ac network can be determined without first having to find the current through the elements.

16

Series-Parallel ac Networks

16.1 INTRODUCTION

In this chapter, we shall utilize the fundamental concepts of the previous chapter to develop a technique for solving **series-parallel ac networks.** A brief review of Chapter 7 may be helpful before considering these networks since the approach here will be quite similar to that undertaken earlier. The circuits to be discussed will have only one source of energy, either potential or current. Networks with two or more sources will be considered in Chapters 17 and 18, using methods previously described for dc circuits.

In general, when working with series-parallel ac networks, consider the following approach:

1. ***Redraw the network, employing block impedances to combine obvious series and parallel elements, which will reduce the network to one that clearly reveals the fundamental structure of the system.***
2. ***Study the problem and make a brief mental sketch of the overall approach you plan to use. Doing this may result in time- and energy-saving shortcuts. In some cases a lengthy, drawn-out analysis may not be necessary. A single application of a fundamental law of circuit analysis may result in the desired solution.***
3. ***After the overall approach has been determined, it is usually best to consider each branch involved in your method independently before tying them together in series-parallel combinations. In most cases, work back from the obvious series and parallel combinations to the source to determine the total impedance of the network. The source current can then be determined, and the path back to specific unknowns can be defined. As you progress back to the source, continually define those unknowns that have not been lost in the reduction process. It will save time when you have to work back through the network to find specific quantities.***

4. ***When you have arrived at a solution, check to see that it is reasonable by considering the magnitudes of the energy source and the elements in the circuit. If not, either solve the network using another approach, or check over your work very carefully. At this point a computer solution can be an invaluable asset in the validation process.***

16.2 ILLUSTRATIVE EXAMPLES

EXAMPLE 16.1 For the network of Fig. 16.1:

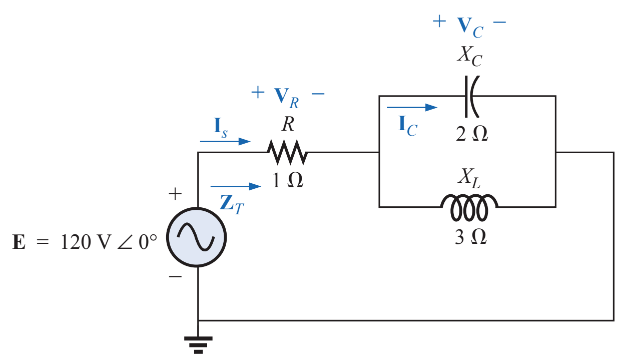


FIG. 16.1
Example 16.1.

a. Calculate $\mathbf{Z}_T$.
b. Determine $\mathbf{I}_s$.
c. Calculate $\mathbf{V}_R$ and $\mathbf{V}_C$.
d. Find $\mathbf{I}_C$.
e. Compute the power delivered.
f. Find F_p of the network.

Solutions:

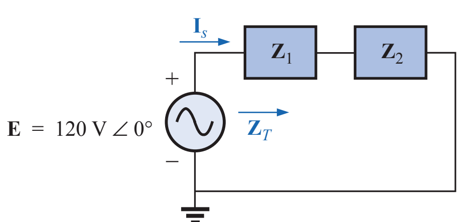


FIG. 16.2
Network of Fig. 16.1 after assigning the block impedances.

a. As suggested in the introduction, the network has been redrawn with block impedances, as shown in Fig. 16.2. The impedance $\mathbf{Z}_1$ is simply the resistor R of 1 Ω, and $\mathbf{Z}_2$ is the parallel combination of X_C and X_L. The network now clearly reveals that it is fundamentally a series circuit, suggesting a direct path toward the total impedance and the source current. As noted in the introduction, for many such problems you must work back to the source to find first the total impedance and then the source current. When the unknown quantities are found in terms of these subscripted impedances, the numerical values can then be substituted to find the magnitude and phase angle of the unknown. In other words, try to find the desired solution solely in terms of the subscripted impedances before substituting numbers. This approach will usually enhance the clarity of the chosen path toward a solution while saving time and preventing careless calculation errors. Note also in Fig. 16.2 that all the unknown quantities except $\mathbf{I}_C$ have been preserved, meaning that we can use Fig. 16.2 to determine these quantities rather than having to return to the more complex network of Fig. 16.1.

The total impedance is defined by

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2$$

with

$$\mathbf{Z}_1 = R\angle 0° = 1\,\Omega\angle 0°$$

$$\mathbf{Z}_2 = \mathbf{Z}_C \parallel \mathbf{Z}_L = \frac{(X_C\angle -90°)(X_L\angle 90°)}{-j\,X_C + j\,X_L} = \frac{(2\,\Omega\angle -90°)(3\,\Omega\angle 90°)}{-j\,2\,\Omega + j\,3\,\Omega}$$

$$= \frac{6\,\Omega\angle 0°}{j\,1} = \frac{6\,\Omega\angle 0°}{1\angle 90°} = 6\,\Omega\angle -90°$$

and

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 1\,\Omega - j\,6\,\Omega = \mathbf{6.08\,\Omega\angle -80.54°}$$

b. $\mathbf{I}_s = \dfrac{\mathbf{E}}{\mathbf{Z}_T} = \dfrac{120\text{ V}\angle 0°}{6.08\,\Omega\angle -80.54°} = \mathbf{19.74\text{ A}\angle 80.54°}$

c. Referring to Fig. 16.2, we find that $\mathbf{V}_R$ and $\mathbf{V}_C$ can be found by a direct application of Ohm's law:

$$\mathbf{V}_R = \mathbf{I}_s\mathbf{Z}_1 = (19.74\text{ A}\angle 80.54°)(1\,\Omega\angle 0°) = \mathbf{19.74\text{ V}\angle 80.54°}$$

$$\mathbf{V}_C = \mathbf{I}_s\mathbf{Z}_2 = (19.74\text{ A}\angle 80.54°)(6\,\Omega\angle -90°)$$

$$= \mathbf{118.44\text{ V}\angle -9.46°}$$

d. Now that $\mathbf{V}_C$ is known, the current $\mathbf{I}_C$ can also be found using Ohm's law.

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{\mathbf{Z}_C} = \frac{118.44\text{ V}\angle -9.46°}{2\,\Omega\angle -90°} = \mathbf{59.22\text{ A}\angle 80.54°}$$

e. $P_{\text{del}} = I_s^2R = (19.74\text{ A})^2(1\,\Omega) = \mathbf{389.67\text{ W}}$

f. $F_p = \cos\theta = \cos 80.54° = \mathbf{0.164\ leading}$

The fact that the total impedance has a negative phase angle (revealing that $\mathbf{I}_s$ leads $\mathbf{E}$) is a clear indication that the network is capacitive in nature and therefore has a leading power factor. The fact that the network is capacitive can be determined from the original network by first realizing that, for the parallel *L-C* elements, the smaller impedance predominates and results in an *R-C* network.

EXAMPLE 16.2 For the network of Fig. 16.3:

a. If $\mathbf{I}$ is 50 A $\angle 30°$, calculate $\mathbf{I}_1$ using the current divider rule.
b. Repeat part (a) for $\mathbf{I}_2$.
c. Verify Kirchhoff's current law at one node.

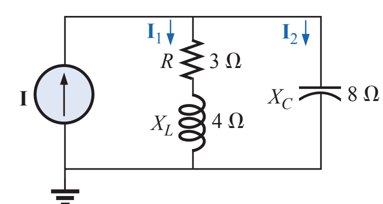


FIG. 16.3
Example 16.2.

Solutions:

a. Redrawing the circuit as in Fig. 16.4, we have

$$\mathbf{Z}_1 = R + j\,X_L = 3\,\Omega + j\,4\,\Omega = 5\,\Omega\angle 53.13°$$

$$\mathbf{Z}_2 = -j\,X_C = -j\,8\,\Omega = 8\,\Omega\angle -90°$$

Using the current divider rule yields

$$\mathbf{I}_1 = \frac{\mathbf{Z}_2\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \frac{(8\,\Omega\angle -90°)(50\text{ A}\angle 30°)}{(-j\,8\,\Omega) + (3\,\Omega + j\,4\,\Omega)} = \frac{400\angle -60°}{3 - j\,4}$$

$$= \frac{400\angle -60°}{5\angle -53.13°} = \mathbf{80\text{ A}\angle -6.87°}$$

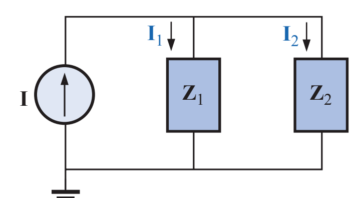


FIG. 16.4
Network of Fig. 16.3 after assigning the block impedances.

b. $$\mathbf{I}_2 = \frac{\mathbf{Z}_1\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \frac{(5\ \Omega \angle 53.13°)(50\ \text{A} \angle 30°)}{5\ \Omega \angle -53.13°} = \frac{250 \angle 83.13°}{5 \angle -53.13°} = \mathbf{50\ A \angle 136.26°}$$

c. $$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$
$$\begin{aligned} 50\ \text{A} \angle 30° &= 80\ \text{A} \angle -6.87° + 50\ \text{A} \angle 136.26° \\ &= (79.43 - j\,9.57) + (-36.12 + j\,34.57) \\ &= 43.31 + j\,25.0 \\ 50\ \text{A} \angle 30° &= 50\ \text{A} \angle 30° \quad \text{(checks)} \end{aligned}$$

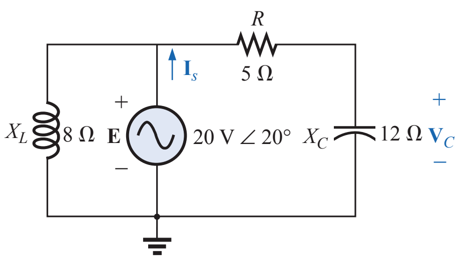


FIG. 16.5
Example 16.3.

EXAMPLE 16.3 For the network of Fig. 16.5:
a. Calculate the voltage $\mathbf{V}_C$ using the voltage divider rule.
b. Calculate the current $\mathbf{I}_s$.

Solutions:
a. The network is redrawn as shown in Fig. 16.6, with

$$\begin{aligned} \mathbf{Z}_1 &= 5\ \Omega = 5\ \Omega \angle 0° \\ \mathbf{Z}_2 &= -j\,12\ \Omega = 12\ \Omega \angle -90° \\ \mathbf{Z}_3 &= +j\,8\ \Omega = 8\ \Omega \angle 90° \end{aligned}$$

Since $\mathbf{V}_C$ is desired, we will not combine R and X_C into a single block impedance. Note also how Fig. 16.6 clearly reveals that **E** is the total voltage across the series combination of $\mathbf{Z}_1$ and $\mathbf{Z}_2$, permitting the use of the voltage divider rule to calculate $\mathbf{V}_C$. In addition, note that all the currents necessary to determine $\mathbf{I}_s$ have been preserved in Fig. 16.6, revealing that there is no need to ever return to the network of Fig. 16.5—everything is defined by Fig. 16.6.

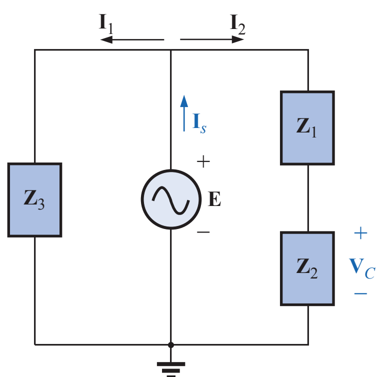


FIG. 16.6
Network of Fig. 16.5 after assigning the block impedances.

$$\mathbf{V}_C = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(12\ \Omega \angle -90°)(20\ \text{V} \angle 20°)}{5\ \Omega - j\,12\ \Omega} = \frac{240\ \text{V} \angle -70°}{13 \angle -67.38°} = \mathbf{18.46\ V \angle -2.62°}$$

b. $$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_3} = \frac{20\ \text{V} \angle 20°}{8\ \Omega \angle 90°} = 2.5\ \text{A} \angle -70°$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{20\ \text{V} \angle 20°}{13\ \Omega \angle -67.38°} = 1.54\ \text{A} \angle 87.38°$$

and

$$\begin{aligned} \mathbf{I}_s &= \mathbf{I}_1 + \mathbf{I}_2 \\ &= 2.5\ \text{A} \angle -70° + 1.54\ \text{A} \angle 87.38° \\ &= (0.86 - j\,2.35) + (0.07 + j\,1.54) \\ \mathbf{I}_s &= 0.93 - j\,0.81 = \mathbf{1.23\ A \angle -41.05°} \end{aligned}$$

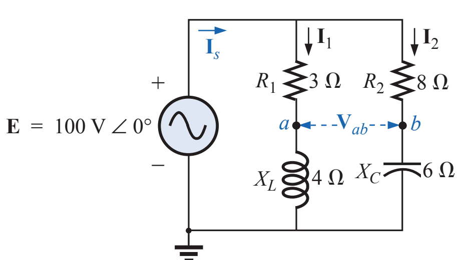


FIG. 16.7
Example 16.4.

EXAMPLE 16.4 For Fig. 16.7:
a. Calculate the current $\mathbf{I}_s$.
b. Find the voltage $\mathbf{V}_{ab}$.

Solutions:
a. Redrawing the circuit as in Fig. 16.8, we obtain

$$\begin{aligned} \mathbf{Z}_1 &= R_1 + j\,X_L = 3\ \Omega + j\,4\ \Omega = 5\ \Omega \angle 53.13° \\ \mathbf{Z}_2 &= R_2 - j\,X_C = 8\ \Omega - j\,6\ \Omega = 10\ \Omega \angle -36.87° \end{aligned}$$

In this case the voltage $\mathbf{V}_{ab}$ is lost in the redrawn network, but the currents $\mathbf{I}_1$ and $\mathbf{I}_2$ remain defined for future calculations necessary to determine $\mathbf{V}_{ab}$. Figure 16.8 clearly reveals that the total impedance can be found using the equation for two parallel impedances:

$$\mathbf{Z}_T = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(5\ \Omega\ \angle 53.13^\circ)(10\ \Omega\ \angle -36.87^\circ)}{(3\ \Omega + j\,4\ \Omega) + (8\ \Omega - j\,6\ \Omega)}$$

$$= \frac{50\ \Omega\ \angle 16.26^\circ}{11 - j\,2} = \frac{50\ \Omega\ \angle 16.26^\circ}{11.18\ \angle -10.30^\circ}$$

$$= \mathbf{4.472\ \Omega\ \angle 26.56^\circ}$$

and $$\mathbf{I}_s = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\ \text{V}\ \angle 0^\circ}{4.472\ \Omega\ \angle 26.56^\circ} = \mathbf{22.36\ A\ \angle -26.56^\circ}$$

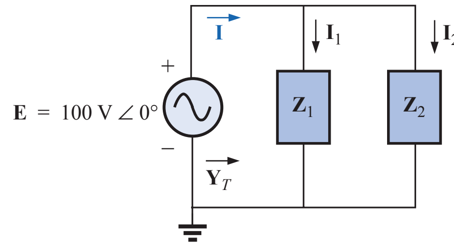


FIG. 16.8
Network of Fig. 16.7 after assigning the block impedances.

b. By Ohm's law,

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{100\ \text{V}\ \angle 0^\circ}{5\ \Omega\ \angle 53.13^\circ} = \mathbf{20\ A\ \angle -53.13^\circ}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \frac{100\ \text{V}\ \angle 0^\circ}{10\ \Omega\ \angle -36.87^\circ} = \mathbf{10\ A\ \angle 36.87^\circ}$$

Returning to Fig. 16.7, we have

$$\mathbf{V}_{R_1} = \mathbf{I}_1\mathbf{Z}_{R_1} = (20\ \text{A}\ \angle -53.13^\circ)(3\ \Omega\ \angle 0^\circ) = \mathbf{60\ V\ \angle -53.13^\circ}$$
$$\mathbf{V}_{R_2} = \mathbf{I}_1\mathbf{Z}_{R_2} = (10\ \text{A}\ \angle +36.87^\circ)(8\ \Omega\ \angle 0^\circ) = \mathbf{80\ V\ \angle +36.87^\circ}$$

Instead of using the two steps just shown, we could have determined $\mathbf{V}_{R_1}$ or $\mathbf{V}_{R_2}$ in one step using the voltage divider rule:

$$\mathbf{V}_{R_1} = \frac{(3\ \Omega\ \angle 0^\circ)(100\ \text{V}\ \angle 0^\circ)}{3\ \Omega\ \angle 0^\circ + 4\ \Omega\ \angle 90^\circ} = \frac{300\ \text{V}\ \angle 0^\circ}{5\ \angle 53.13^\circ} = \mathbf{60\ V\ \angle -53.13^\circ}$$

To find $\mathbf{V}_{ab}$, Kirchhoff's voltage law must be applied around the loop (Fig.16.9) consisting of the 3-Ω and 8-Ω resistors. By Kirchhoff's voltage law,

$$\mathbf{V}_{ab} + \mathbf{V}_{R_1} - \mathbf{V}_{R_2} = 0$$

or
$$\mathbf{V}_{ab} = \mathbf{V}_{R_2} - \mathbf{V}_{R_1}$$
$$= 80\ \text{V}\ \angle 36.87^\circ - 60\ \text{V}\ \angle -53.13^\circ$$
$$= (64 + j\,48) - (36 - j\,48)$$
$$= 28 + j\,96$$
$$\mathbf{V}_{ab} = \mathbf{100\ V\ \angle 73.74^\circ}$$

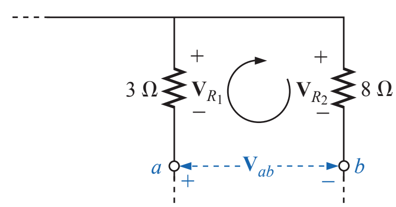


FIG. 16.9
Determining the voltage $\mathbf{V}_{ab}$ *for the network of Fig. 16.7.*

EXAMPLE 16.5 The network of Fig. 16.10 is frequently encountered in the analysis of transistor networks. The transistor equivalent circuit includes a current source **I** and an output impedance R_o. The resistor R_C is a biasing resistor to establish specific dc conditions, and the resistor R_i represents the loading of the next stage. The coupling capacitor is designed to be an open circuit for dc and to have as low an impedance as possible for the frequencies of interest to ensure that $\mathbf{V}_L$ is a maximum value. The frequency range of the example includes the entire audio (hearing) spectrum from 100 Hz to 20 kHz. The purpose of the example is to demonstrate that, for the full audio range, the effect of the capacitor can be ignored. It performs its function as a dc blocking agent but permits the ac to pass through with little disturbance.

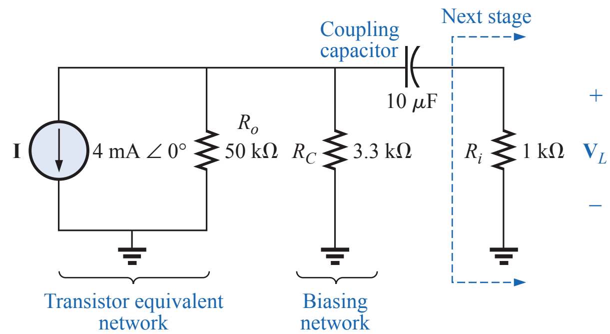


FIG. 16.10
Basic transistor amplifier.

a. Determine $\mathbf{V}_L$ for the network of Fig. 16.10 at a frequency of 100 Hz.
b. Repeat part (a) at a frequency of 20 kHz.
c. Compare the results of parts (a) and (b).

Solutions:

a. The network is redrawn with subscripted impedances in Fig. 16.11.

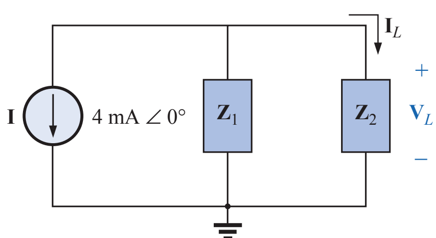


FIG. 16.11
Network of Fig. 16.10 following the assignment of the block impedances.

$$\mathbf{Z}_1 = 50\text{ k}\Omega\ \angle 0^\circ \parallel 3.3\text{ k}\Omega\ \angle 0^\circ = 3.096\text{ k}\Omega\ \angle 0^\circ$$
$$\mathbf{Z}_2 = R_i - j\,X_C$$

$$\text{At } f = 100\text{ Hz: } X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(100\text{ Hz})(10\ \mu\text{F})} = 159.16\ \Omega$$

and $$\mathbf{Z}_2 = 1\text{ k}\Omega - j\,159.16\ \Omega$$

Current divider rule:

$$\begin{aligned}\mathbf{I}_L &= \frac{\mathbf{Z}_1\mathbf{I}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(3.096\text{ k}\Omega\ \angle 0^\circ)(4\text{ mA}\ \angle 0^\circ)}{3.096\text{ k}\Omega + 1\text{ k}\Omega - j\,159.16\ \Omega}\\ &= \frac{12.384\text{ A}\ \angle 0^\circ}{4096 - j\,159.16} = \frac{12.384\text{ A}\ \angle 0^\circ}{4099\ \angle -2.225^\circ}\\ &= 3.021\text{ mA}\ \angle 2.225^\circ\end{aligned}$$

and $$\begin{aligned}\mathbf{V}_L &= \mathbf{I}_L\mathbf{Z}_R\\ &= (3.021\text{ mA}\ \angle 2.225^\circ)(1\text{ k}\Omega\ \angle 0^\circ)\\ &= \mathbf{3.021\ V\ \angle 2.225^\circ}\end{aligned}$$

b. $$\text{At } f = 20\text{ kHz: } X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(20\text{ kHz})(10\ \mu\text{F})} = 0.796\ \Omega$$

Note the dramatic change in X_C with frequency. Obviously, the higher the frequency, the better the short-circuit approximation for X_C for ac conditions.

$$\mathbf{Z}_2 = 1\text{ k}\Omega - j\,0.796\ \Omega$$

Current divider rule:

$$\begin{aligned}\mathbf{I}_L &= \frac{\mathbf{Z}_1\mathbf{I}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(3.096\text{ k}\Omega\ \angle 0^\circ)(4\text{ mA}\ \angle 0^\circ)}{3.096\text{ k}\Omega + 1\text{ k}\Omega - j\,0.796\ \Omega}\\ &= \frac{12.384\text{ A}\ \angle 0^\circ}{4096 - j\,0.796\ \Omega} = \frac{12.384\text{ A}\ \angle 0^\circ}{4096\ \angle -0.011^\circ}\\ &= 3.023\text{ mA}\ \angle 0.011^\circ\end{aligned}$$

and
$$\begin{aligned}\mathbf{V}_L &= \mathbf{I}_L\mathbf{Z}_R\\ &= (3.023\text{ mA }\angle 0.011^\circ)(1\text{ k}\Omega\ \angle 0^\circ)\\ &= \mathbf{3.023\ V\ \angle 0.011^\circ}\end{aligned}$$

c. The results clearly indicate that the capacitor had little effect on the frequencies of interest. In addition, note that most of the supply current reached the load for the typical parameters employed.

EXAMPLE 16.6 For the network of Fig. 16.12:

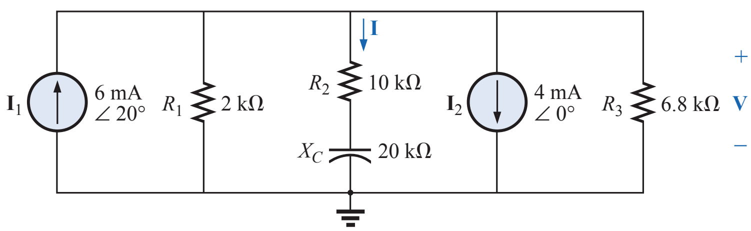


FIG. 16.12
Example 16.6.

a. Determine the current **I**.
b. Find the voltage **V**.

Solutions:

a. The rules for parallel current sources are the same for dc and ac networks. That is, the equivalent current source is their sum or difference (as phasors). Therefore,

$$\begin{aligned}\mathbf{I}_T &= 6\text{ mA }\angle 20^\circ - 4\text{ mA }\angle 0^\circ\\ &= 5.638\text{ mA} + j\,2.052\text{ mA} - 4\text{ mA}\\ &= 1.638\text{ mA} + j\,2.052\text{ mA}\\ &= 2.626\text{ mA }\angle 51.402^\circ\end{aligned}$$

Redrawing the network using block impedances will result in the network of Fig. 16.13 where

$$\mathbf{Z}_1 = 2\text{ k}\Omega\ \angle 0^\circ \parallel 6.8\text{ k}\Omega\ \angle 0^\circ = 1.545\text{ k}\Omega\ \angle 0^\circ$$

and
$$\mathbf{Z}_2 = 10\text{ k}\Omega - j\,20\text{ k}\Omega = 22.361\text{ k}\Omega\ \angle -63.435^\circ$$

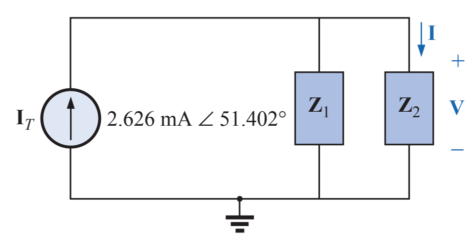


FIG. 16.13
Network of Fig. 16.12 following the assignment of the subscripted impedances.

Note that **I** and **V** are still defined in Fig. 16.13.

Current divider rule:

$$\begin{aligned}\mathbf{I} &= \frac{\mathbf{Z}_1\mathbf{I}_T}{\mathbf{Z}_1+\mathbf{Z}_2} = \frac{(1.545\text{ k}\Omega\ \angle 0^\circ)(2.626\text{ mA }\angle 51.402^\circ)}{1.545\text{ k}\Omega + 10\text{ k}\Omega - j\,20\text{ k}\Omega}\\ &= \frac{4.057\text{ A }\angle 51.402^\circ}{11.545\times 10^3 - j\,20\times 10^3} = \frac{4.057\text{ A }\angle 51.402^\circ}{23.093\times 10^3\angle -60.004^\circ}\\ &= \mathbf{0.176\ mA\ \angle 111.406^\circ}\end{aligned}$$

b.
$$\begin{aligned}\mathbf{V} &= \mathbf{I}\mathbf{Z}_2\\ &= (0.176\text{ mA }\angle 111.406^\circ)(22.36\text{ k}\Omega\ \angle -63.435^\circ)\\ &= \mathbf{3.936\ V\ \angle 47.971^\circ}\end{aligned}$$

EXAMPLE 16.7 For the network of Fig. 16.14:

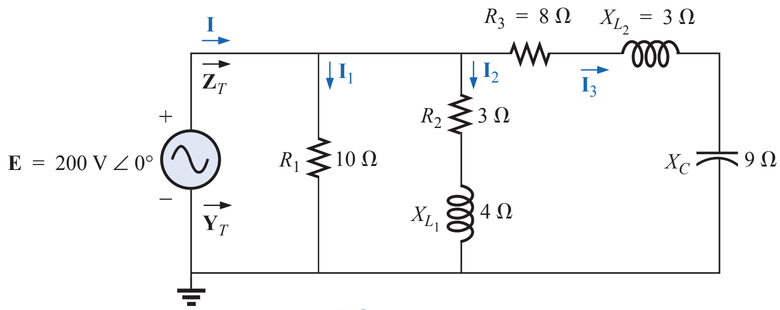


FIG. 16.14
Example 16.7.

a. Compute **I**.
b. Find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.
c. Verify Kirchhoff's current law by showing that

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

d. Find the total impedance of the circuit.

Solutions:

a. Redrawing the circuit as in Fig. 16.15 reveals a strictly parallel network where

$$\mathbf{Z}_1 = R_1 = 10\ \Omega\ \angle 0°$$
$$\mathbf{Z}_2 = R_2 + j\,X_{L_1} = 3\ \Omega + j\,4\ \Omega$$
$$\mathbf{Z}_3 = R_3 + j\,X_{L_2} - j\,X_C = 8\ \Omega + j\,3\ \Omega - j\,9\ \Omega = 8\ \Omega - j\,6\ \Omega$$

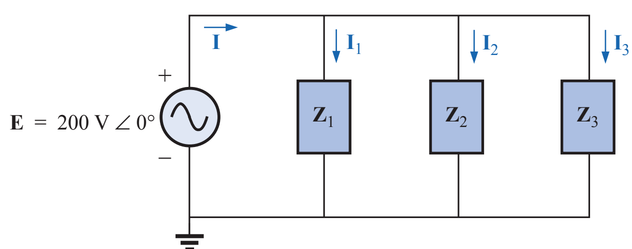


FIG. 16.15
Network of Fig. 16.14 following the assignment of the subscripted impedances.

The total admittance is

$$\begin{aligned}
\mathbf{Y}_T &= \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 \\
&= \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} = \frac{1}{10\ \Omega} + \frac{1}{3\ \Omega + j\,4\ \Omega} + \frac{1}{8\ \Omega - j\,6\ \Omega} \\
&= 0.1\ \text{S} + \frac{1}{5\ \Omega\ \angle 53.13°} + \frac{1}{10\ \Omega\ \angle -36.87°} \\
&= 0.1\ \text{S} + 0.2\ \text{S}\ \angle -53.13° + 0.1\ \text{S}\ \angle 36.87° \\
&= 0.1\ \text{S} + 0.12\ \text{S} - j\,0.16\ \text{S} + 0.08\ \text{S} + j\,0.06\ \text{S} \\
&= 0.3\ \text{S} - j\,0.1\ \text{S} = 0.316\ \text{S}\ \angle -18.435°
\end{aligned}$$

Calculator The above mathematical exercise presents an excellent opportunity to demonstrate the power of some of today's calculators. Using the TI-86, the above operation would appear as follows on the display:

```
1/(10,0)+1/(3,4)+1/(8,−6)
```

with the result:

```
(300.000E−3,−100.000E−3)
```

Converting to polar form:

```
Ans ▶ Pol
(316.228E−3∠−18.435E0)
```

The current **I**:

$$\mathbf{I} = \mathbf{E}\mathbf{Y}_T = (200\text{ V}\angle 0°)(0.316\text{ S}\angle -18.435°)$$
$$= \mathbf{63.2\text{ A}\angle -18.435°}$$

b. Since the voltage is the same across parallel branches,

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{200\text{ V}\angle 0°}{10\ \Omega\angle 0°} = \mathbf{20\text{ A}\angle 0°}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \frac{200\text{ V}\angle 0°}{5\ \Omega\angle 53.13°} = \mathbf{40\text{ A}\angle -53.13°}$$

$$\mathbf{I}_3 = \frac{\mathbf{E}}{\mathbf{Z}_3} = \frac{200\text{ V}\angle 0°}{10\ \Omega\angle -36.87°} = \mathbf{20\text{ A}\angle +36.87°}$$

c. $$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

$$60 - j\,20 = 20\angle 0° + 40\angle -53.13° + 20\angle +36.87°$$
$$= (20 + j\,0) + (24 - j\,32) + (16 + j\,12)$$
$$60 - j\,20 = 60 - j\,20 \quad \text{(checks)}$$

d. $$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.316\text{ S}\angle -18.435°}$$
$$= \mathbf{3.165\ \Omega\angle 18.435°}$$

EXAMPLE 16.8 For the network of Fig. 16.16:

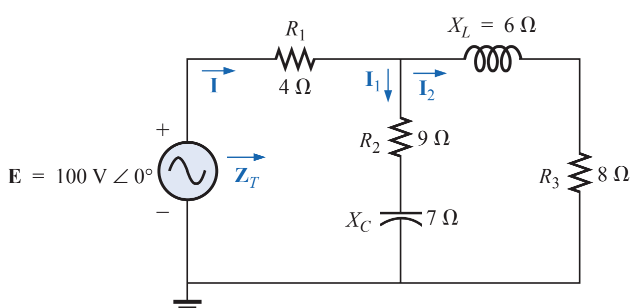


FIG. 16.16
Example 16.8.

a. Calculate the total impedance $\mathbf{Z}_T$.
b. Compute **I**.
c. Find the total power factor.
d. Calculate $\mathbf{I}_1$ and $\mathbf{I}_2$.
e. Find the average power delivered to the circuit.

Solutions:

a. Redrawing the circuit as in Fig. 16.17, we have

$$\mathbf{Z}_1 = R_1 = 4\ \Omega\ \angle 0°$$
$$\mathbf{Z}_2 = R_2 - j\,X_C = 9\ \Omega - j\,7\ \Omega = 11.40\ \Omega\ \angle -37.87°$$
$$\mathbf{Z}_3 = R_3 + j\,X_L = 8\ \Omega + j\,6\ \Omega = 10\ \Omega\ \angle +36.87°$$

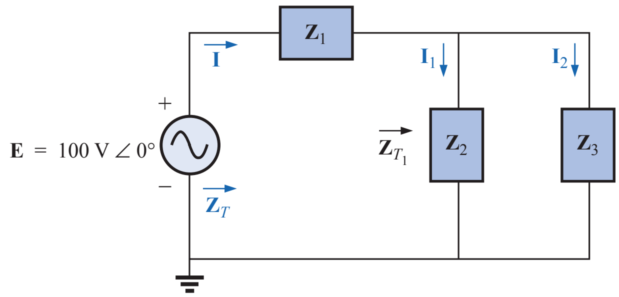


FIG. 16.17
Network of Fig. 16.16 following the assignment of the subscripted impedances.

Notice that all the desired quantities were conserved in the redrawn network. The total impedance:

$$\begin{aligned}
\mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_{T_1} \\
&= \mathbf{Z}_1 + \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3} \\
&= \frac{4\ \Omega + (11.4\ \Omega\ \angle -37.87°)(10\ \Omega\ \angle 36.87°)}{(9\ \Omega - j\,7\ \Omega) + (8\ \Omega + j\,6\ \Omega)} \\
&= 4\ \Omega + \frac{114\ \Omega\ \angle -1.00°}{17.03\ \angle -3.37°} = 4\ \Omega + 6.69\ \Omega\ \angle 2.37° \\
&= 4\ \Omega + 6.68\ \Omega + j\,0.28\ \Omega = 10.68\ \Omega + j\,0.28\ \Omega \\
\mathbf{Z}_T &= \mathbf{10.684\ \Omega\ \angle 1.5°}
\end{aligned}$$

Mathcad Solution: The complex algebra just presented in detail provides an excellent opportunity to practice our Mathcad skills with complex numbers. Remember that the *j* must follow the numerical value of the imaginary part and **is not** multiplied by the numerical value. Simply type in the numerical value and then *j*. Also recall that unless you make a global change in the format, an *i* will appear with the imaginary part of the solution. As shown in Fig. 16.18, each impedance is first defined with **Shift:.** Then each impedance is entered in sequence on the same line or succeeding lines. Next, the equation for the total impedance is defined using the brackets to ensure that the bottom summation is carried out before the division and also to provide the same format to the equation as appearing above. Then enter **ZT,** select the equal sign key, and the rectangular form for the total impedance will appear as shown.

The polar form can be obtained by first going to the **Calculator** toolbar to obtain the magnitude operation and inserting **ZT** as shown in Fig. 16.18. Then selecting the equal sign will result in the magnitude of 10.693 Ω. The angle is obtained by first going to the **Greek** toolbar and picking up theta, entering **T,** and defining the variable. The **π** comes from the **Calculator** toolbar, and the **arg( )** from **Insert-*f(x)*-**

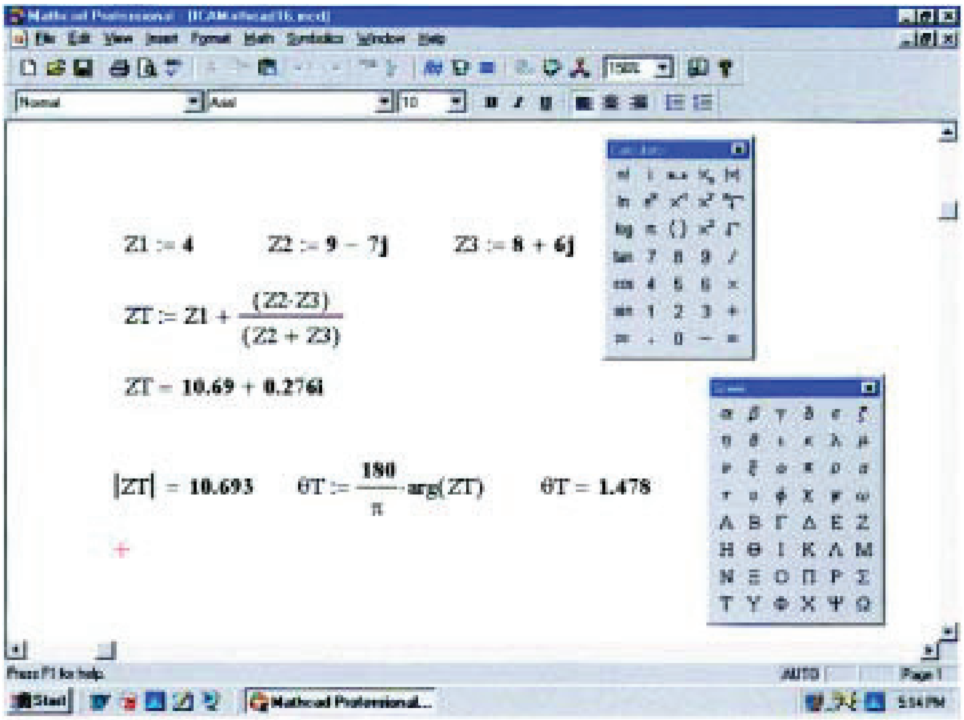


FIG. 16.18
Using Mathcad to determine the total impedance for the network of Fig.16.16.

Function Name-arg. Finally the variable is written again and the equal sign selected to obtain an angle of 1.478°. The computer solution of 10.693 Ω ∠1.478° is an excellent verification of the theoretical solution of 10.684 Ω ∠1.5°.

Calculator Another opportunity to demonstrate the versatility of the calculator! For the above operation, however, one must be aware of the priority of the mathematical operations, as demonstrated in the calculator display below. In most cases, the operations are performed in the same order they would be performed longhand.

(4,0)+((9,−7)+(8,6))$^{-1}$* (11.4∠−37.87)(10∠36.87) ENTER
(10.689E0,276.413E−3)
Ans ▶ Pol ENTER
(10.692E0∠1.481E0)

b. $\mathbf{I} = \dfrac{\mathbf{E}}{\mathbf{Z}_T} = \dfrac{100\text{ V }\angle 0^\circ}{10.684\ \Omega\ \angle 1.5^\circ} = \mathbf{9.36\ A\ \angle -1.5^\circ}$

c. $F_p = \cos\theta_T = \dfrac{R}{Z_T} = \dfrac{10.68\ \Omega}{10.684\ \Omega} \cong \mathbf{1}$

(essentially resistive, which is interesting, considering the complexity of the network)

d. Current divider rule:

$$\mathbf{I}_2 = \frac{\mathbf{Z}_2\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(11.40\ \Omega\ \angle -37.87^\circ)(9.36\ \text{A}\ \angle -1.5^\circ)}{(9\ \Omega - j\,7\ \Omega) + (8\ \Omega + j\,6\ \Omega)}$$
$$= \frac{106.7\ \text{A}\ \angle -39.37^\circ}{17 - j\,1} = \frac{106.7\ \text{A}\ \angle -39.37^\circ}{17.03\ \angle -3.37^\circ}$$
$$\mathbf{I}_2 = \mathbf{6.27\ A\ \angle -36^\circ}$$

Applying Kirchhoff's current law (rather than another application of the current divider rule) yields

$$\mathbf{I}_1 = \mathbf{I} - \mathbf{I}_2$$

or
$$\mathbf{I} = \mathbf{I}_1 - \mathbf{I}_2$$
$$= (9.36\ \text{A}\ \angle -1.5^\circ) - (6.27\ \text{A}\ \angle -36^\circ)$$
$$= (9.36\ \text{A} - j\,0.25\ \text{A}) - (5.07\ \text{A} - j\,3.69\ \text{A})$$
$$\mathbf{I}_1 = 4.29\ \text{A} + j\,3.44\ \text{A} = \mathbf{5.5\ A\ \angle 38.72^\circ}$$

e. $P_T = EI\cos\theta_T$
$$= (100\ \text{V})(9.36\ \text{A})\cos 1.5^\circ$$
$$= (936)(0.99966)$$
$$P_T = \mathbf{935.68\ W}$$

16.3 LADDER NETWORKS

Ladder networks were discussed in some detail in Chapter 7. This section will simply apply the first method described in Section 7.3 to the general sinusoidal ac ladder network of Fig. 16.19. The current $\mathbf{I}_6$ is desired.

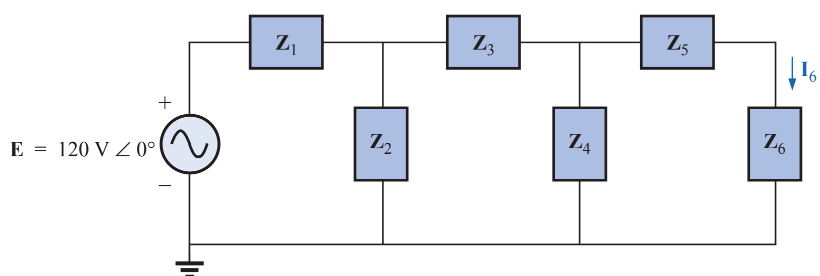


FIG. 16.19
Ladder network.

Impedances $\mathbf{Z}_T$, $\mathbf{Z}'_T$, and $\mathbf{Z}''_T$ and currents $\mathbf{I}_1$ and $\mathbf{I}_3$ are defined in Fig. 16.20:

$$\mathbf{Z}''_T = \mathbf{Z}_5 + \mathbf{Z}_6$$

and
$$\mathbf{Z}'_T = \mathbf{Z}_3 + \mathbf{Z}_4 \parallel \mathbf{Z}''_T$$

with
$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}'_T$$

Then
$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T}$$

and
$$\mathbf{I}_3 = \frac{\mathbf{Z}_2\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}'_T}$$

with
$$\mathbf{I}_6 = \frac{\mathbf{Z}_4\mathbf{I}_3}{\mathbf{Z}_4 + \mathbf{Z}''_T}$$

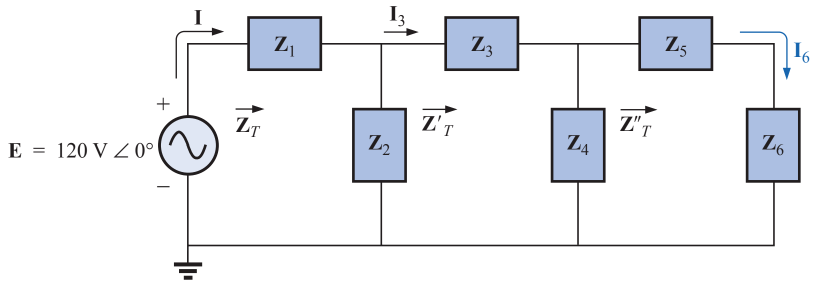


FIG. 16.20
Defining an approach to the analysis of ladder networks.

16.4 APPLICATIONS

The vast majority of the applications appearing throughout the text have been of the series-parallel variety. The following are simply two more that include series-parallel combinations of elements and systems to perform important everyday tasks. The ground fault interrupter (GFI) outlet employs series protective switches and sensing coils and a parallel control system, while the ideal equivalent circuit for the coax cable employs a series-parallel combination of inductors and capacitors.

GFI (Ground Fault Interrupter)

The National Electric Code, the "bible" for all electrical contractors, now requires that ground fault interrupter (GFI) outlets be used in any area where water and dampness could result in serious injury, such as in bathrooms, pools, marinas, and so on. The outlet looks like any other except that it has a reset button and a test button in the center of the unit as shown in Fig. 16.21(a). Its primary difference between an ordinary outlet is that it will shut the power off much more quickly than the breaker all the way down in the basement could. You may still feel a shock with a GFI outlet, but the current will cut off so quickly (in a few milliseconds) that a person in normal health should not receive a serious electrical injury. Whenever in doubt about its use, remember that the cost is such that it should be installed. It works just as a regular outlet does, but it provides an increased measure of safety.

The basic operation is best described by the simple network of Fig. 16.21(b). The protection circuit separates the power source from the outlet itself. Note in Fig. 16.21(b) the importance of grounding the protection circuit to the central ground of the establishment (a water pipe, ground bar, and so on, connected to the main panel). In general, the outlet will be grounded to the same connection. Basically, the network shown in Fig. 16.21(b) senses both the current entering (I_i) and the current leaving (I_o) and provides a direct connection to the outlet when they are equal. If a fault should develop such as caused by someone touching the hot leg while standing on a wet floor, the return current will be less than the feed current (just a few milliamperes is enough). The protection circuitry will sense this difference, establish an open circuit in the line, and cut off the power to the outlet.

In Fig. 16.22(a) you can see the feed and return lines passing through the sensing coils. The two sensing coils are separately connected to the printed circuit board. There are two pulse control switches in the line and a return to establish an open circuit under errant condi-

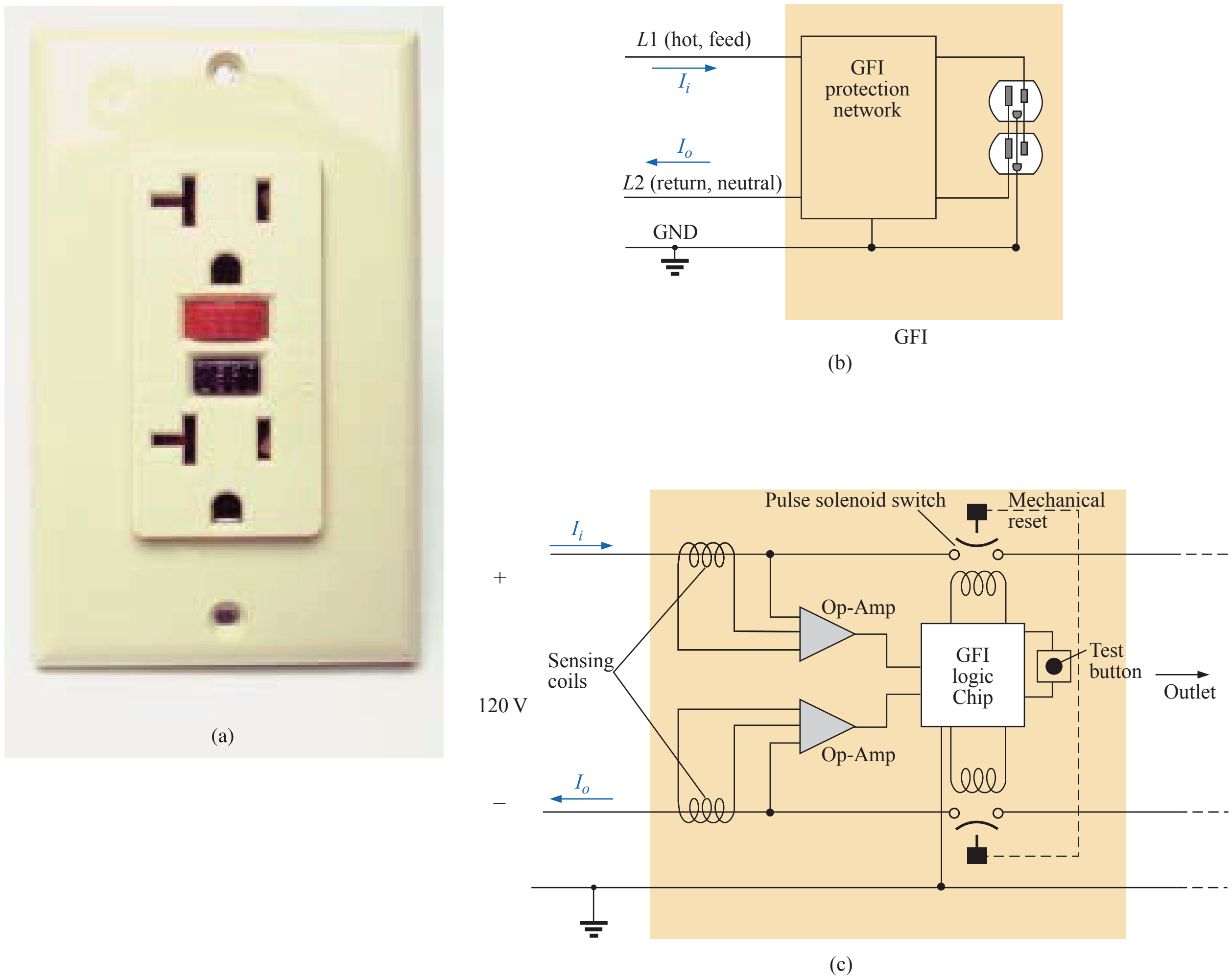


FIG. 16.21
GFI outlet: (a) wall-mounted appearance; (b) basic operation; (c) schematic.

tions. The two contacts in Fig. 16.22(a) are the contacts that provide conduction to the outlet. When a fault develops, another set of similar contacts in the housing will slide away, providing the desired open-circuit condition. The separation is created by the solenoid appearing in Fig. 16.22(b). When the solenoid is energized due to a fault condition, it will pull the plunger toward the solenoid, compressing the spring. At the same time, the slots in the lower plastic piece (connected directly to the plunger) will shift down, causing a disconnect by moving the structure inserted in the slots. The test button is connected to the brass bar across the unit in Fig. 16.22(c) below the reset button. When pressed, it will place a large resistor between the line and ground to "unbalance" the line and cause a fault condition. When the button is released, the resistor will be separated from the line, and the unbalance condition will be removed. The resistor is actually connected directly to one end of the bar and moves down with pressure on the bar as shown in Fig. 16.22(d). Note in Fig. 16.22(c) how the metal ground connection passes right through the entire unit and how it is connected to the ground terminal of an applied plug. Also note how it is separated from the rest of the network with the plastic housing. Although this unit

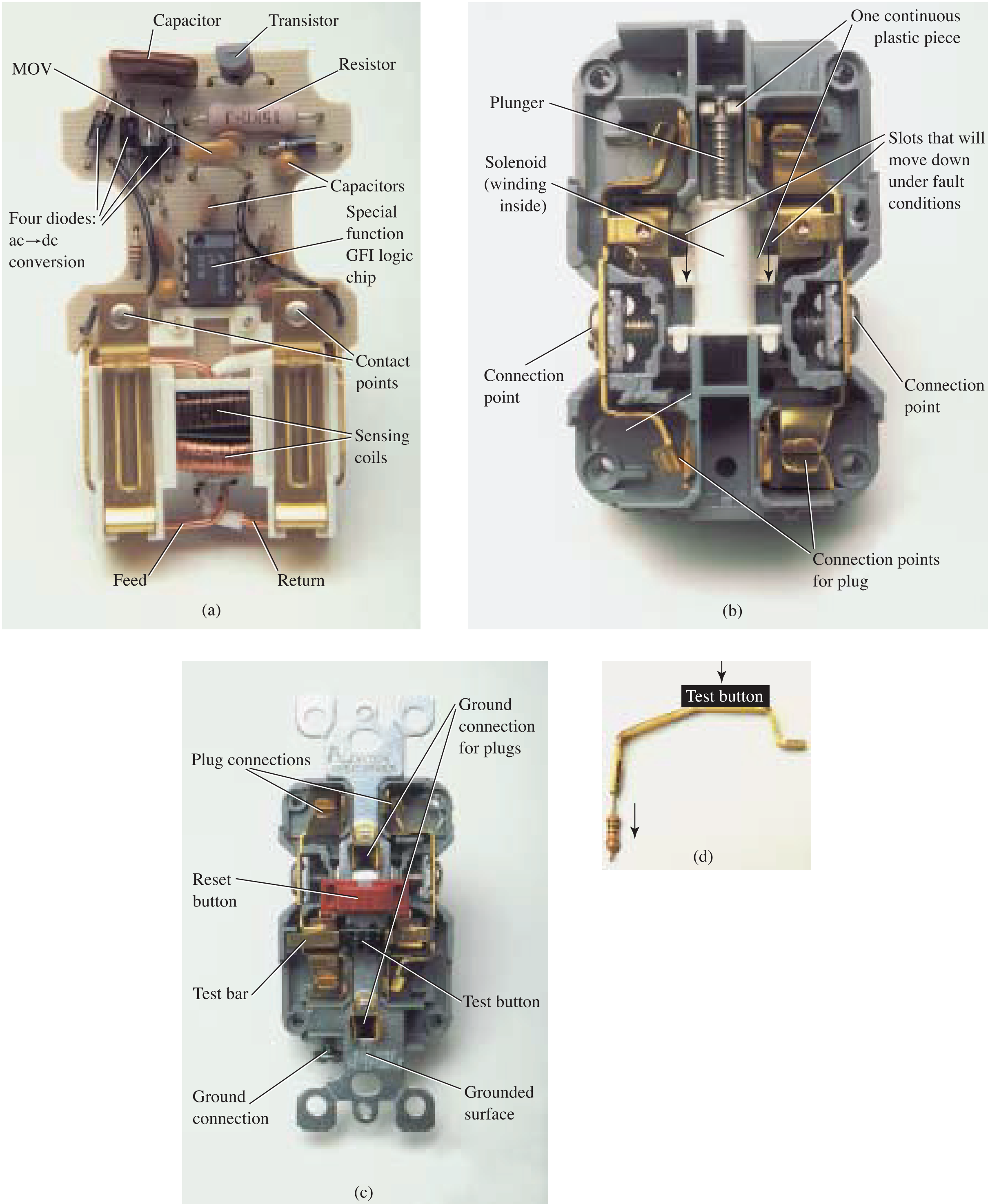


FIG. 16.22

GFI construction: (a) sensing coils; (b) solenoid control (bottom view); (c) grounding (top view); (d) test bar.

appears simple on the outside and is relatively small in size, it is beautifully designed and contains a great deal of technology and innovation.

Before leaving the subject, note the logic chip in the center of Fig. 16.22(a) and the various sizes of capacitors and resistors. Note also the four diodes in the upper left region of the circuit board used as a bridge rectifier for the ac-to-dc conversion process. The transistor is the black element with the half-circle appearance. It is part of the driver circuit for the controlling solenoid. Because of the size of the unit, there wasn't a lot of room to provide the power to quickly open the circuit. The result is the use of a pulse circuit to control the motion of the controlling solenoid. In other words, the solenoid is pulsed for a short period of time to cause the required release. If the design used a system that would hold the circuit open on a continuing basis, the power requirement would be greater and the size of the coil larger. A small coil can handle the required power pulse for a short period of time without any long-term damage.

As mentioned earlier, if unsure, then install a GFI. It provides a measure of safety—at a very reasonable cost—that should not be ignored.

Coax Cable

In recent years it appears that coax cable is everywhere, from TV connections to medical equipment, from stereos to computer connections. What makes this type of connection so special? What are its advantages over the standard two-wire connection?

The primary purpose of coax cable is to provide a channel for communication between two points without picking up noise from the surrounding medium—a direct link in its purest form. You may wonder whether noise pollution is really that bad and whether this concern is overkill, but simply think of all the signals passing through the air that we cannot see, for example, for cellular phones, pagers, and radio and TV stations. Then you start to realize that there is a lot going on out there that we can't see. None of us would like our EKG signal from our heart to be disturbed by extraneous noise or to have our stereo pick up channels other than those of interest. It is a real problem that must be solved, and it appears that the best solution is to use coax cable. Compared to standard conductors, coax displays a lower loss of signal in transmission and has much improved high-frequency transmission characteristics.

It is the construction that offers the protection we desire. The basic construction of a 75-Ω coax cable as typically used in the home appears in Fig. 16.23(a) with its terminal connection in Fig. 16.23(b). It has a

FIG. 16.23
75-Ω coax cable: (a) construction; (b) terminal connection.

solid inner conductor surrounded by a polyethylene dielectric (insulator). Copper or aluminum braid woven over the dielectric forms the outer conductor. Finally, a waterproof jacket placed over the braided wire provides protection against moisture. Since the entire outer surface of the braided wire is at the same potential, it completely isolates the solid conductor in the center of the coax cable from the outside signals—an isolation referred to as *shielding.* The question is sometimes asked, Why is the outside wire braided rather than just a flat sheet of conducting material? It is braided to reduce the effects of the fields established by any currents that pass through the outside conductor. In Fig. 16.24(a), a current in the outer conductor has established circular magnetic fields that can be additive and can create transmission problems. However, as shown in Fig. 16.24(b), if the wire is braided, the magnetic field established by one wire in the braid may be canceled by a neighboring conductor crossing the conductor on an angle. Note the opposite direction of the fields in the region between the two braided wires. Of course, the total magnetic flux may not be canceled, but the situation is certainly improved compared to that with a solid flat conductor. For added protection, a duofoil covering is sometimes added as shown in Fig. 16.23(a) to ensure 100% shielding.

Because a coax cable is most commonly referred to as an *RF* (radio-frequency) *transmission line,* most people associate the use of coax cables with high frequencies. However, this is certainly not the case, as evidenced by medical technology that deals with static dc levels and low-voltage (in microvolts or millivolts), "slow" (less than 5 Hz) ac. In general, coax cables should be used wherever there is a need to ensure that the transmitted signal is undisturbed by any surrounding noise. Coax cables are acceptable for the full range of frequencies from 0 Hz to a few hundred gigahertz, with sound frequencies extending from about 15 Hz to 20 kHz, radio frequencies from 20 kHz to 300 MHz, and microwave frequencies from 300 MHz to 300 GHz. Our discussion thus far has centered on protecting the transmitted signal from external noise. It is important to realize also that when a coax cable is used, it will not act as a transmitter for the signal that it is carrying. This fact is very important as we hook up electronic appliances such as VCRs to our TVs. If we simply used a twin lead wire between the VCR and TV, not only would the wire pick up signals by acting like an antenna, but it would also transmit channel 3 (or 4) to the surrounding medium

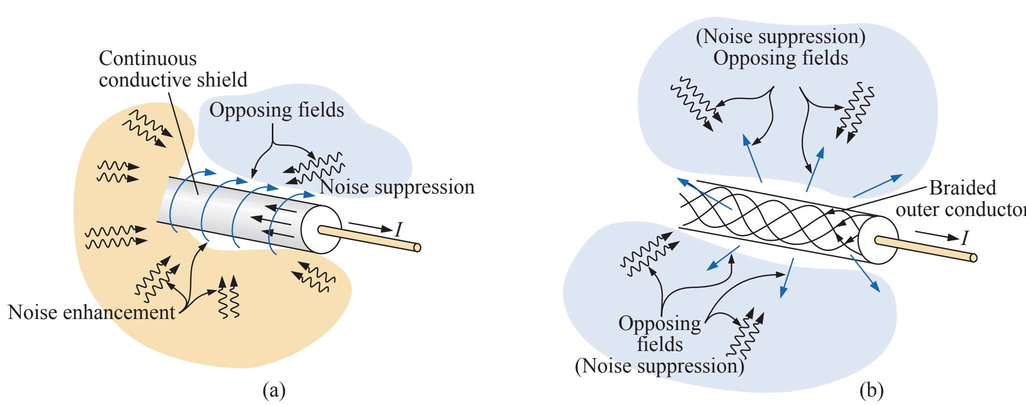


FIG. 16.24
Shielding: (a) solid outside inductor; (b) braided outside conductor.

which would affect not only your TV's response but also that of any other TV or receiver in the area.

For the coupling between the systems in which coax cable is typically used, it is not the level of voltage or current that is the primary concern but whether there is a good "match" between components and the cable. Every transmission line composed of two parallel conductors will have capacitance between the conductors, and every conductor that is carrying current has a certain level of inductance. For a transmission line an equivalent model can be composed of the lumped series-parallel combination of Fig. 16.25(a), where each capacitor or inductor is for a short length of the wire. For an infinitely long chain of the elements of Fig. 16.25(a), the combination has an input impedance called the *characteristic impedance* that is proportional to $\sqrt{L/C}$ where L and C are the inductance and capacitance of a unit length of the transmission line. Although Figure 16.25(a) suggests that a transmission line is purely reactive, there is resistance in the line because of the resistance of the wire, and this resistance will absorb power. It is therefore important to realize when hooking up coax cable that the TV farthest from the source will receive the least amount of signal power, and if it is very distant, the resulting loss may be sufficient to affect the picture quality. Rearranging the equations for v_L and I_C and substituting as follows will reveal that the characteristic impedance is purely resistive and is measured in ohms:

$$v_L = L\frac{di_L}{dt} \Rightarrow L = v_L\frac{dt}{di_L}$$

$$i_C = C\frac{dv_C}{dt} \Rightarrow C = i_C\frac{dt}{dv_C}$$

so that

$$\sqrt{\frac{L}{C}} = \sqrt{\frac{v_L\dfrac{dt}{di_L}}{i_C\dfrac{dt}{dv_C}}} = \sqrt{\frac{v_L}{i_C}\cdot\frac{dv_C}{di_L}} = \sqrt{\Omega\cdot\Omega} = \sqrt{\Omega^2} = \Omega$$

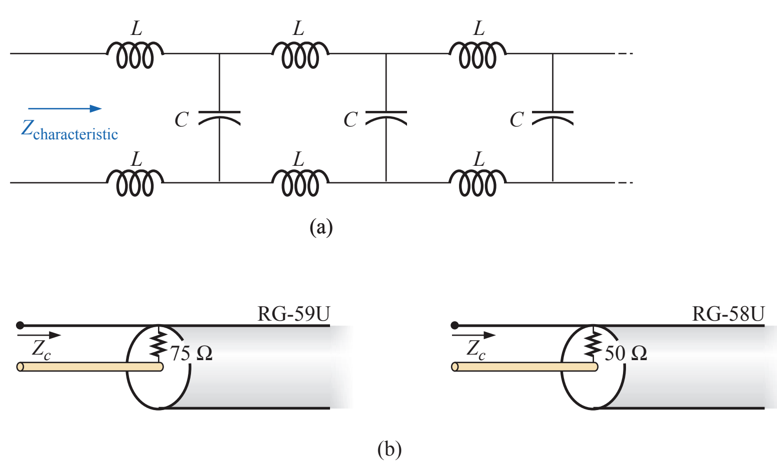


FIG. 16.25
Coax cable: (a) electrical equivalent (lossless line); (b) characteristic impedance.

The most common coax cables have characteristic impedances of either 50 Ω or 75 Ω, as shown in Fig. 16.25(b). In actuality they may be 53.5-Ω and 73.5-Ω lines, respectively, but they are usually grouped in the category of 50- or 75-Ω lines. The 75-Ω line is typically used for applications such as cable TV and RF equipment, while the 50-Ω line is typically used for test equipment, ham radio stations, and medical equipment. Two of the most common coax cables are listed in Table 16.1 with specific information about their characteristics.

TABLE 16.1

Characteristics of two frequently used coax cables.

	RG-59U 75 Ω (actually 73.5 Ω)	**RG-58U 50 Ω (actually 53.5 Ω)**
Core wire:	20 AWG, 40% aluminum	20 AWG, 95% tinned
Resistance:	44.5 Ω/1000 ft	10 Ω/1000 ft
Coating:	Duofoil, 100% shield coverage	Polyethylene
PVC jacket:	0.237-in. outside diameter	0.193-in. outside diameter
Capacitance:	16.2 pF/ft	28.5 pF/ft
Losses:	1 MHz, 0.8 dB/100 ft	1 MHz, 0.3 dB/100 ft
	10 MHz, 1 dB/100 ft	10 MHz, 1.1 dB/100 ft
	50 MHz, 1.8 dB/100 ft	50 MHz, 2.5 dB/100 ft
	100 MHz, 2.5 dB/100 ft	100 MHz, 3.8 dB/100 ft
	1 GHz, 7.9 dB/100 ft	1 GHz, 14.5 dB/100 ft

In reality, a transmission line will not be infinite in length as required for the definition of characteristic impedance. The result is that a 20-ft length of 75-Ω cable will not have an input impedance of 20 Ω but rather one that is determined by the load applied to the cable. However, if the transmission line is terminated by a resistance of 75 Ω, the characteristic impedance of 75 Ω will appear at the source. In other words, terminating a coax cable by its characteristic impedance will make it appear as an infinite line to the source. When the applied load equals the characteristic impedance of the line, the line is said to be *matched.* An applied load equal to the characteristic impedance also results in maximum power transfer to the load as established by the maximum power theorem. Any loading other than the characteristic impedance will result in a "reflection of power" back to the source. Matching the load to the line is therefore a major concern when using coax cables. For instance, take the folded-dipole antenna referred to as a *yagi* that was a common sight on roof tops before cable came along. The twin line cable running from the antenna to the TV had a characteristic impedance of 300 Ω. Today, most TVs have an input impedance of 75 Ω, and thus such antennas would have to be connected to the TV with a *matching transformer* (called a *Balun transformer*) that would make the 75-Ω load look like 300 Ω to the antenna for maximum power transfer, as shown in Fig. 16.26. In today's world, TVs are referred to as *cable ready* if they have a coax connection and an input impedance of 75 Ω to match the cable system.

FIG. 16.26

Balun matching transformer.

One of the mistakes frequently made when installing a coax system is to hook up a splitter and fail to terminate all the output terminals. In Fig. 16.27(a), a three-way splitter is connected to two TVs with the third terminal left open for any possible future additions. The open third terminal will cause a mismatch on the incoming line, and less power

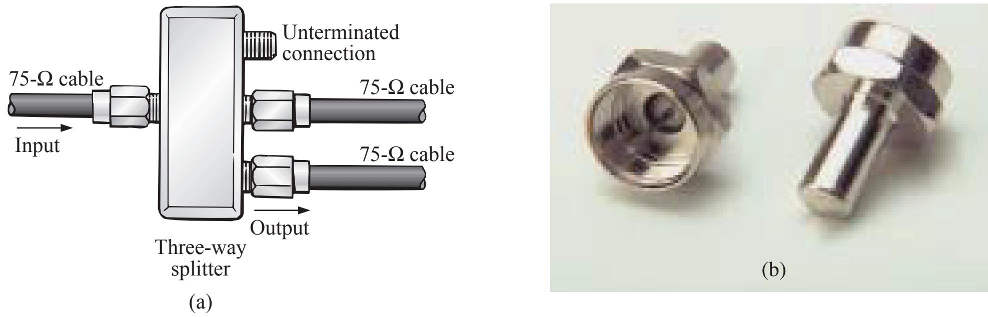


FIG. 16.27
Signal splitting: (a) three-way splitter; (b) F-type 75-Ω coax terminator.

will get to the connected TVs. This situation is corrected by terminating the unused terminal with a commercially available connector as shown in Fig. 16.27(b), which simply has a 75-Ω resistor inside. It is also important to realize that each time you split the signal, you lose power to each of the TVs connected to the system. In fact, you lose 3 dB for each split as shown in Fig. 16.28(a). Splitting the signal in two will result in a loss of 3 dB for each TV, while splitting it three ways will result in a 6-dB loss for each TV. The concept of decibels will be covered in Chapter 24, but be aware for the moment that a 3-dB drop represents a drop in power of one-half—certainly a significant amount. A TV can still respond pretty well with a drop of 3 dB or 6 dB, but anything approaching a 12-dB drop will probably result in a poor image and should be avoided. Whenever using a splitter, it is always best to connect an amplifier before the splitter as shown in Fig. 16.28(b). In essence, the amplifier compensates for the loss introduced by splitters and also (if well designed) will permit leaving a terminal open without disturbing the resulting signal power flow. In other words, a good

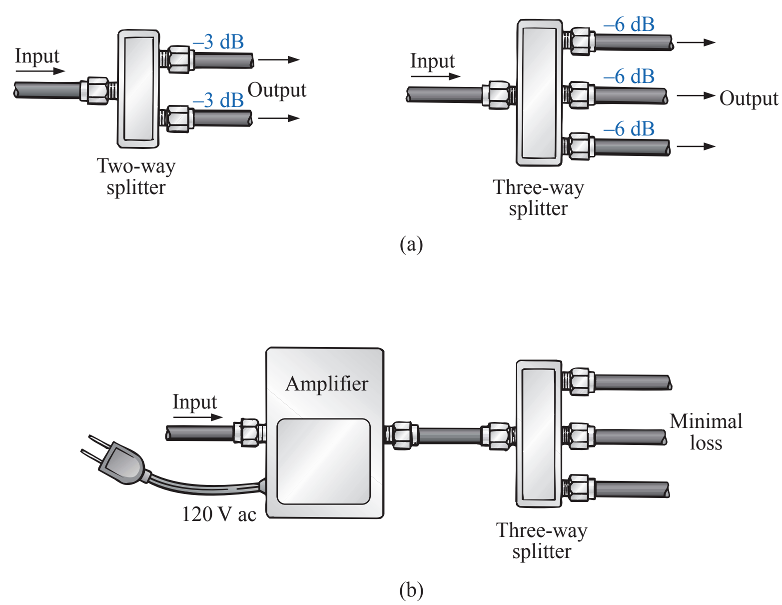


FIG. 16.28
Coax splitting losses: (a) dB losses introduced by two-way and three-way splitters; (b) using an amplifier.

amplifier knows how to compensate for a terminal that is improperly terminated.

Table 16.1 reveals that there is a measurable loss in power (dB) for every 100 ft of cable. For each cable, about 3 dB are lost for every 100 ft at 100 MHz, primarily because of the resistance of the center conductor (44.5 Ω/1000 ft for the 75-Ω line and 10 Ω/1000 ft for the 50-Ω line). This is one reason why it is not recommended to first split the signal and apply the amplifier at the location of the TV. In Fig. 16.29(a), the signal-to-noise (unwanted signals) ratio is quite high, and the reception will be quite good. However, as shown in Fig. 16.29(b), if the signal is sent down a 100-ft cable to a room distant from the source, there will be a drop in signal, and even if the noise component does not increase, the signal-to-noise ratio at the TV will be much higher. If an amplifier is connected at this point, it will amplify both the signal and the unwanted noise, and the poorer signal-to-noise ratio will be fed to the TV, resulting in a poorer reception. In general, therefore, amplifiers should be applied where the signal-to-noise ratio is the highest.

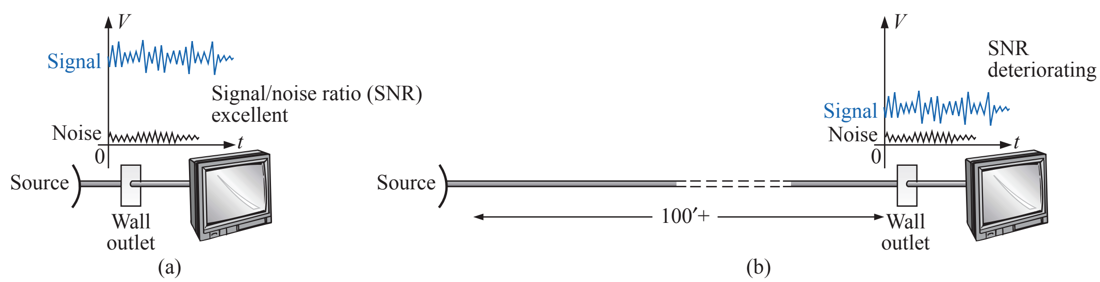


FIG. 16.29
Signal-to-noise ratios: (a) negligible line loss; (b) measurable line loss.

The discussion of coax cables and their proper use could go on for a number of pages. Priorities, however, require that any further investigation be left to the reader. Simply be aware that the matching process is an important one and that coax cables are not ideal systems and do have an internal resistance that can affect transmission—especially over long distances.

16.5 COMPUTER ANALYSIS

PSpice

ac Bridge Network We will be using Example 16.4 to demonstrate the power of the **VPRINT** option in the **SPECIAL** library. It permits a direct determination of the magnitude and angle of any voltage in an ac network. Similarly the **IPRINT** option does the same for ac currents. In Example 16.4, the ac voltages across R_1 and R_2 were first determined, and then Kirchhoff's voltage law was applied to determine the voltage between the two known points. Since PSpice is designed primarily to determine the voltage at a point with respect to ground, the network of Fig. 16.7 is entered as shown in Fig. 16.30 to permit a direct calculation of the voltages across R_1 and R_2.

The source and network elements are entered using a procedure that has been demonstrated several times in previous chapters, although for

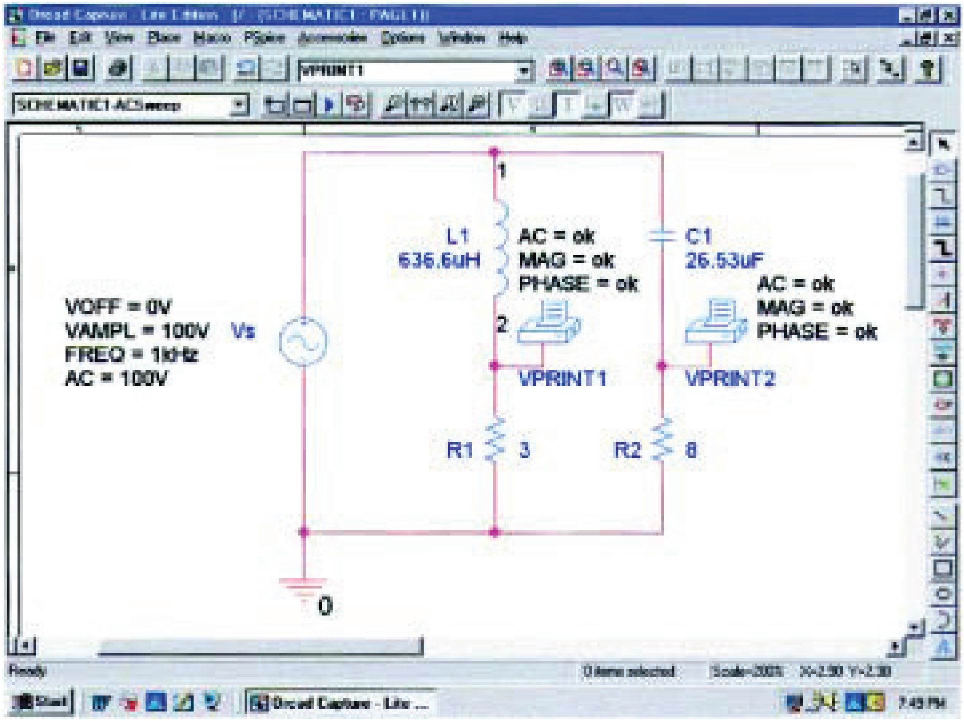


FIG. 16.30

*Determining the voltage across R_1 and R_2 using the **VPRINT** option of a PSpice analysis.*

the **AC Sweep** analysis to be performed in this example, the source must carry an **AC** level also. Fortunately, it is the same as **VAMPL** as shown in Fig. 16.30. It is introduced into the source description by double-clicking on the source symbol to obtain the **Property Editor** dialog box. The **AC** column is selected and the 100 V entered in the box below. Then **Display** is selected and **Name and Value** chosen. Click **OK** followed by **Apply,** and you can exit the dialog box. The result is **AC** = 100 V added to the source description on the diagram and in the system. Using the reactance values of Fig. 16.7, the values for L and C were determined using a frequency of 1 kHz. The voltage across R_1 and R_2 can be determined using the **Trace** command in the same manner as described in the previous chapter or by using the **VPRINT** option. Both approaches will be discussed in this section because they have application to any ac network.

The **VPRINT** option is under the **SPECIAL** library in the **Place Part** dialog box. Once selected, the printer symbol will appear on the screen next to the cursor, and it can be placed near the point of interest. Once the printer symbol is in place, a double-click on it will result in the **Property Editor** dialog box. Scrolling from left to right, type the word **ok** under **AC, MAG,** and **PHASE.** When each is active, the **Display** key should be selected and the option **Name and Value** chosen followed by **OK.** When all the entries have been made, choose **Apply** and exit the dialog box. The result appears Fig. 16.30 for the two applications of the **VPRINT** option. If you prefer, **VPRINT1** and **VPRINT2** can be added to distinguish between the two when you review the output data. This is accomplished by returning to the **Property Editor** dialog box for each by double-clicking on the printer symbol of each and selecting **Value** and then **Display** followed by **Value Only.** We are now ready for the simulation.

The simulation is initiated by selecting the **New Simulation Profile** icon and entering **ACSweep** as the **Name.** Then select **Create** to bring up the **Simulation Settings** dialog box. This time, we want to analyze the network at 1 kHz but are not interested in plots against time. Thus, the **AC Sweep/Noise** option will be selected under **Analysis type** in the **Analysis** section. An **AC Sweep Type** region will then appear in the dialog asking for the **Start Frequency.** Since we are interested in the response at only one frequency, the **Start** and **End Frequency** will be the same: 1 kHz. Since we need only one point of analysis, the **Points/Decade** will be 1. Click **OK,** and the **Run PSpice** icon can be selected. The **SCHEMATIC1** screen will appear, and the voltage across R_1 can be determined by selecting **Trace** followed by **Add Trace** and then **V(R1:1).** The result is the bottom display of Fig. 16.31 with only one plot point at 1 kHz. Since we fixed the frequency of interest at 1 kHz, this is the only frequency with a response. The magnitude of the voltage across R_1 is 60 V to match the longhand solution of Example 16.4. The phase angle associated with the voltage can be determined by the sequence **Plot-Add Plot to Window-Trace-Add Trace-P( )** from the **Functions or Macros** list and then **V(R1:1)** to obtain **P(V(R1:1))** in the **Trace Expression** box. Click **OK,** and the resulting plot shows that the phase angle is near just less than $-50°$ which is certainly a close match with the $-53.13°$ obtained in Example 16.4.

The above process made no use of the new **VPRINT** option just introduced. We will now see what this option provides. When the **SCHEMATIC1** window appears after the simulation, the window should be exited using the **X**, and **PSpice** should be selected on the top

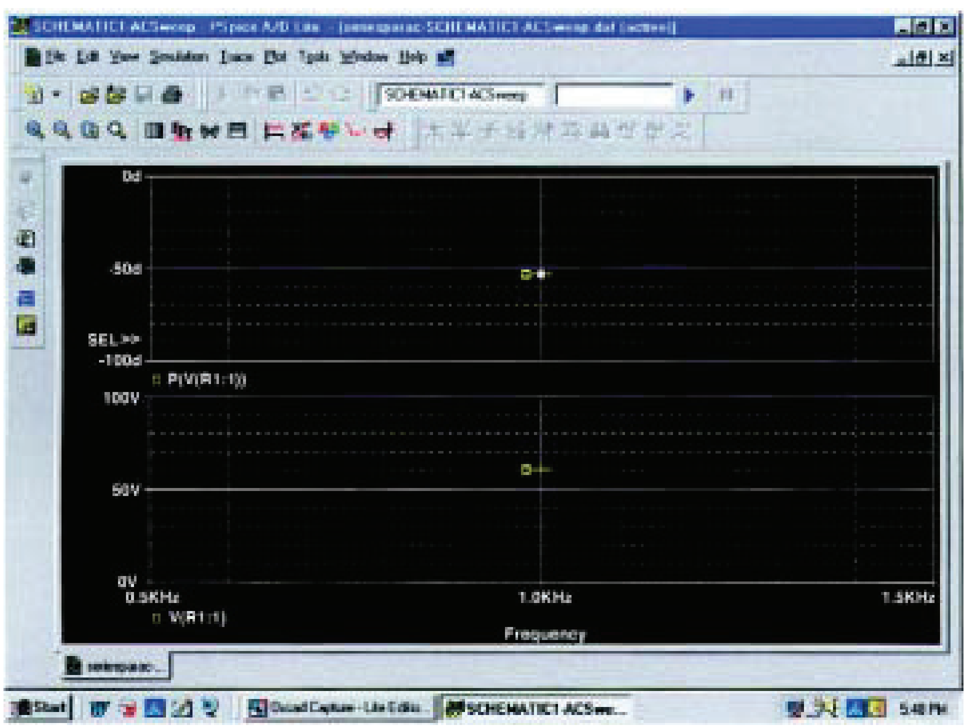


FIG. 16.31
The resulting magnitude and phase angle for the voltage V_{R_1} of Fig. 16.30.

menu bar of the resulting screen. A list will appear of which **View Output File** is an option. Selecting this option will result in a long list of data about the construction of the network and the results obtained from the simulation. In Fig. 16.32 the portion of the output file listing the resulting magnitude and phase angle for the voltages defined by **VPRINT1** and **VPRINT2** is provided. Note that the voltage across R_1 defined by **VPRINT1** is 60 V at an angle of 53.13°. The voltage across R_2 as defined by **VPRINT2** is 80 V at an angle of 36.87°. Both are exact matches of the solutions of Example 16.4. In the future, therefore, if the **VPRINT** option is used, the results will appear in the output file.

```
 ** Profile: "SCHEMATIC1-ACSweep"  [ C:\PSpice\seriesparac-SCHEMATIC1-ACSweep.s
im ]

 ****     AC ANALYSIS                        TEMPERATURE =   27.000 DEG C

******************************************************************************

  FREQ        VM(N00809)  VP(N00809)

   1.000E+03   6.000E+01  -5.313E+01

**** 06/30/01 17:45:20 ************** PSpice Lite (Mar 2000) *****************

 ** Profile: "SCHEMATIC1-ACSweep"  [ C:\PSpice\seriesparac-SCHEMATIC1-ACSweep.s
im ]

 ****     AC ANALYSIS                        TEMPERATURE =   27.000 DEG C

******************************************************************************

  FREQ        VM(N00717)  VP(N00717)

   1.000E+03   8.000E+01   3.687E+01
```

FIG. 16.32

*The **VPRINT1** (V_{R_1}) and **VPRINT2** (V_{R_2}) response for the network of Fig. 16.30.*

Now we will determine the voltage across the two branches from point *a* to point *b*. Return to **SCHEMATIC1,** and select **Trace** followed by **Add Trace** to obtain the list of **Simulation Output Variables.** Then, by applying Kirchhoff's voltage law around the closed loop, we find that the desired voltage is **V(R1:1)-V(R2:1)** which when followed by **OK** will result in the plot point in the screen in the bottom of Fig. 16.33. Note that it is exactly 100 V as obtained in the longhand solution. The phase angle can then be determined through **Plot-Add Plot to Window-Trace-Add Trace** and creating the expression **P(V(R1:1)-V(R2:1)).** Remember that the expression can be generated using the lists of **Output variables** and **Functions**, but it can

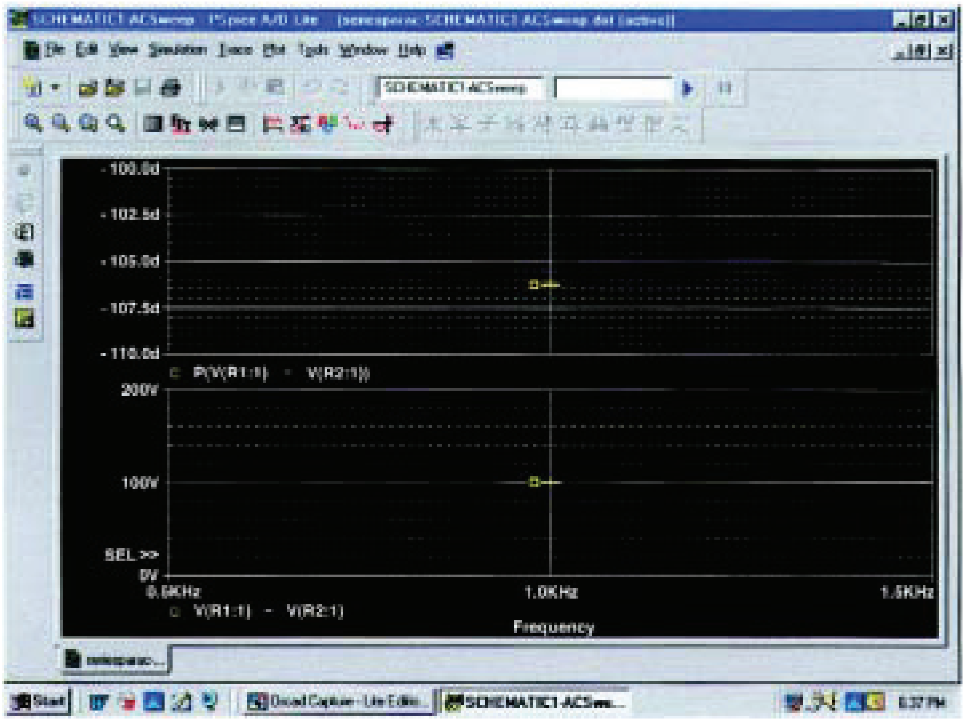


FIG. 16.33

The PSpice reponse for the voltage between the two points above resistors R_1 and R_2.

also be simply typed in from the keyboard. However, always be sure that there are as many left parentheses as there are right. Click **OK,** and a solution near $-105°$ appears. A better reading can be obtained by using **Plot-Axis Settings-Y Axis-User Defined** and changing the scale to $-100°$ to $-110°$. The result is the top screen of Fig. 16.33 with an angle closer to $-106.5°$ or $73.5°$ which is very close to the theoretical solution of $73.74°$.

Finally, the last way to find the desired bridge voltage is to remove the **VPRINT2** option and place the ground at that point as shown in Fig. 16.34. Now the voltage generated from a point above R_1 to ground will be the desired voltage. Repeating a full simulation will then result in the plot of Fig. 16.35 with the the same results as Fig. 16.33. Note, however, that even though the two figures look the same, the quantities listed in the bottom left of each plot are different.

Electronics Workbench

Electronics Workbench will now be used to determine the voltage across the last element of the ladder network of Fig. 16.36. The mathematical content of this chapter would certainly suggest that this analysis would be a lengthy exercise in complex algebra, with one mistake (a single sign or an incorrect angle) enough to invalidate the results. However, it will take only a few minutes to "draw" the network on the screen and only a few seconds to generate the results—results you can usually assume are correct if all the parameters were entered correctly. The results are certainly an excellent check against a longhand solution.

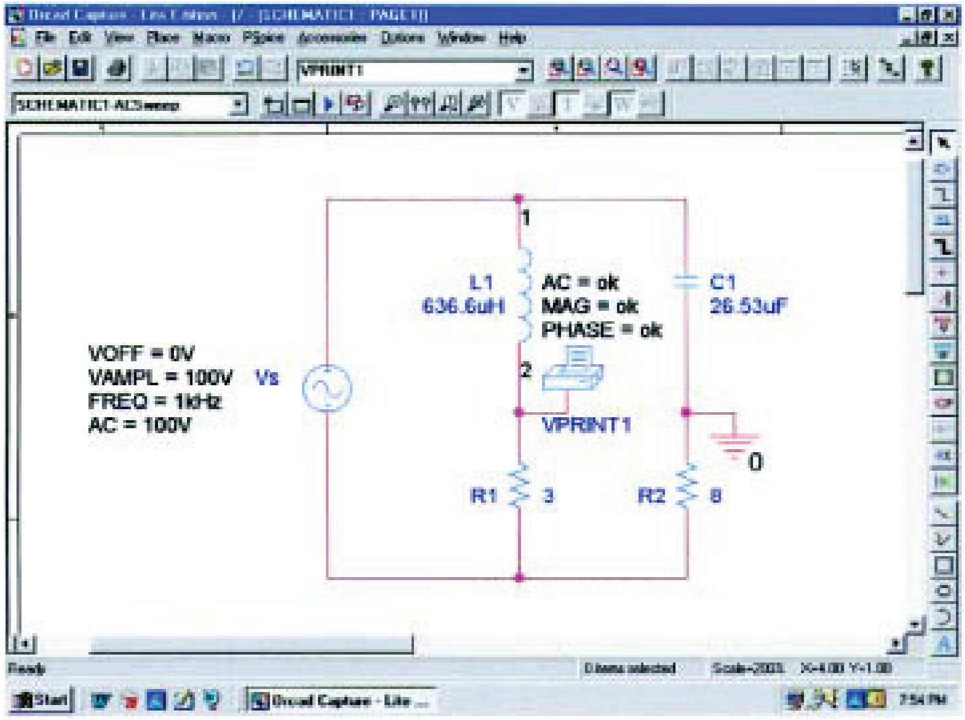


FIG. 16.34

*Determining the voltage between the two points above resistors R_1 and R_2 by moving the ground connection of Fig. 16.30 to the position of **VPRINT2**.*

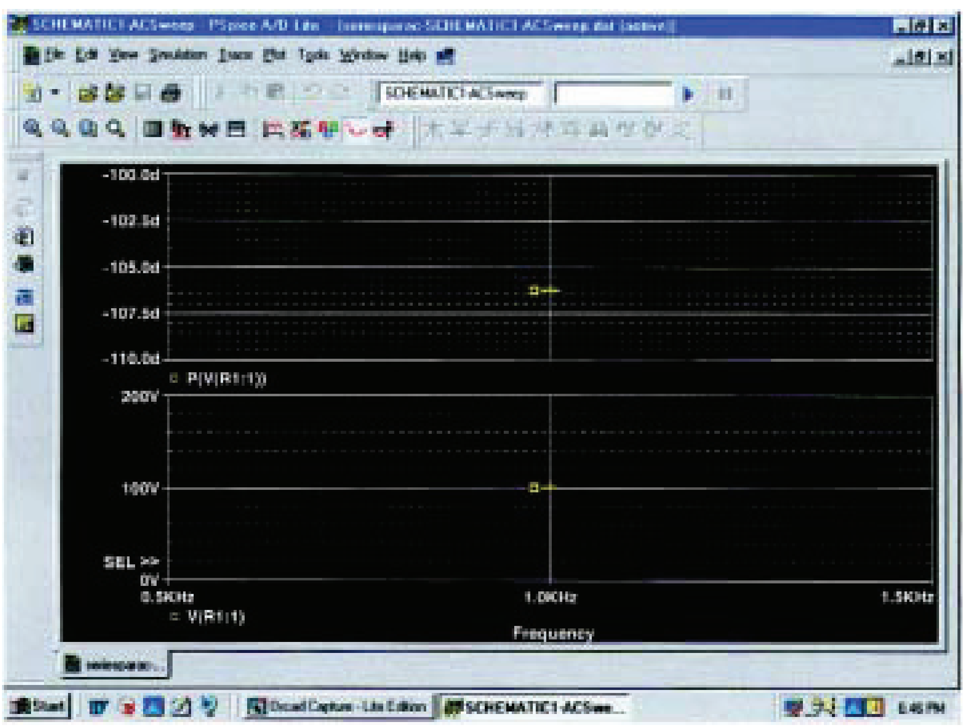


FIG. 16.35

PSpice response to the simulation of the network of Fig. 16.34.

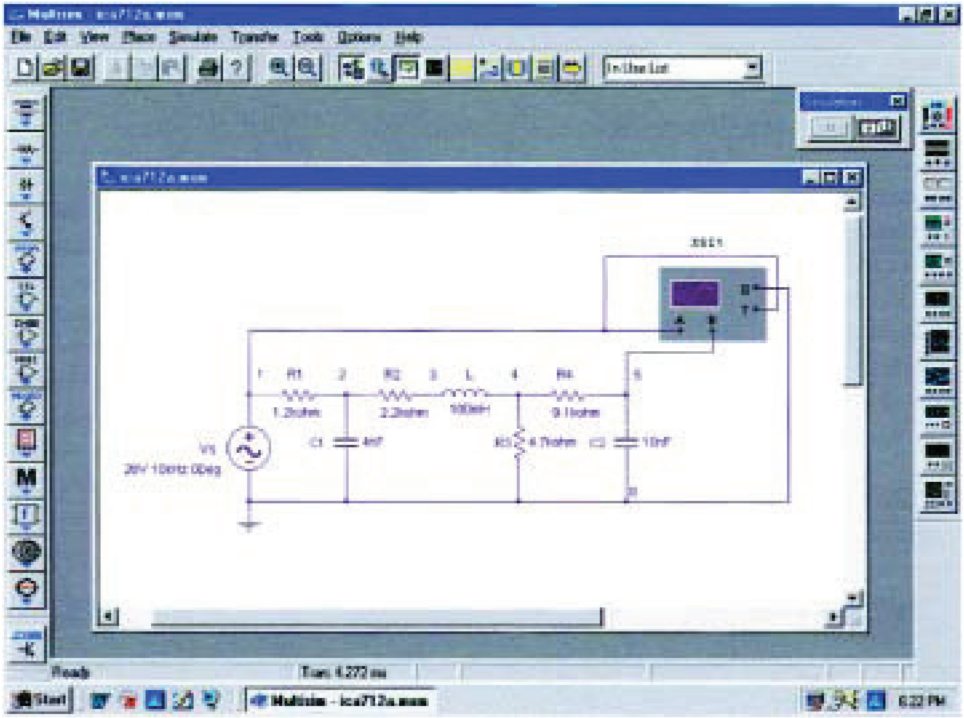


FIG. 16.36

Using the oscilloscope of Electronics Workbench to determine the voltage across the capacitor C_2.

Our first approach will be to use an oscilloscope to measure the amplitude and phase angle of the output voltage as shown in Fig. 16.36. Note that five nodes are defined, with node 5 the desired voltage. The oscilloscope settings include a **Time base** of 20 μs/div. since the period of the 10-kHz signal is 100 μs. Channel **A** was set on 10 V/div. so that the full 20 V of the applied signal will have a peak value encompassing two divisions. Note that **Channel A** in Fig. 16.36 is connected directly to the source **Vs** and to the **Trigger** input for synchronization. Expecting the output voltage to have a smaller amplitude resulted in a vertical sensitivity of 1 V/div. for **Channel B.** The analysis was initiated by placing the **Simulation** switch in the **1** position. It is important to realize that

when simulation is initiated, it will take time for networks with reactive elements to settle down and for the response to reach its steady-state condition. It is therefore wise to let a system run for a while after simulation before selecting **Single** ***on the oscilloscope to obtain a steady waveform for analysis.***

The resulting plots of Fig. 16.37 clearly show that the applied voltage has an amplitude of 20 V and a period of 100 μs (5 div. at 20 μs/div). The cursors sit ready for use at the left and right edges of the screen. Clicking on the small red arrow (with number 1) at the top of the oscilloscope screen will permit you to drag it to any location on the horizontal axis. As you move the cursor, the magnitude of each waveform will appear in the **T1** box below. By comparing positive slopes through the origin, you should see that the applied voltage is leading the output voltage by an angle that is more than 90°. Setting the cursor at the

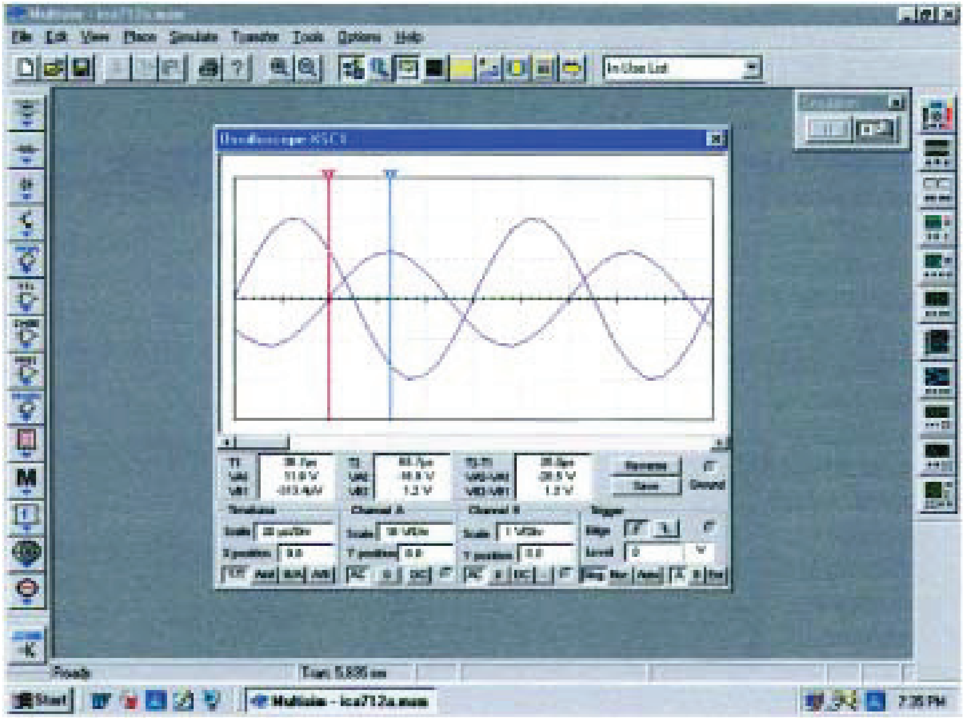


FIG. 16.37

Using Electronics Workbench to display the applied voltage and voltage across the capacitor C_2 for the network of Fig. 16.36.

point where the output voltage on channel B passes through the origin with a positive slope, we find that we cannot achieve exactly 0 V; but $-313.4\ \mu V = -0.313$ mV **(VB1)** is certainly very close at 39.7 μs **(T1).**

Knowing that the applied voltage passed through the origin at 0 μs permits the following claculation for the phase angle:

$$\frac{39.7\ \mu s}{100\ \mu s} = \frac{\theta}{360°}$$

$$\theta = 142.92°$$

with the result that the output voltage has an angle of $-142.92°$ associated with it. The second cursor is found at the right edge of the screen and has a blue color. Selecting it and moving it to the peak value of the output voltage results in **VB2** = 1.2 V at 65.7 μs **(T2).** The result of all the above is

$$\mathbf{V}_{C_2} = 1.2\text{ V} \angle -142.92°.$$

Our second approach will be to use the **AC Analysis** option under the **Simulate** heading. First, realize that when we were using the oscilloscope as we did above, there was no need to pass through the sequence of dialog boxes to choose the desired analysis. All that was necessary was to simulate using either the switch or the **PSpice-Run** sequence—the oscilloscope was there to measure the output voltage. Remember that the source defined the magnitude of the applied voltage, the frequency, and the phase shift. This time we will use the sequence

Simulate-Analyses-AC Analysis to obtain the **AC Analysis** dialog box in which the **Start** and **Stop frequencies** will be 10 kHz and the **Selected variable for analysis** will be node 5 only. Selecting **Simulate** will then result in a magnitude-phase plot with no apparent indicators at 10 kHz. However, this is easily corrected by first selecting one of the plots by clicking on the **Voltage** label at the left of the plot. Then select the **Show/Hide Grid, Show/Hide Legend,** and **Show/Hide Cursors** keys to obtain the cursors, legend, and **AC Analysis** dialog box. Hook on the red cursor and move it to 10 kHz. At that location, and that location only, **x1** will appear as 10 kHz in the dialog box, and **y1** will be 1.1946 as shown in Fig. 16.38. In other words, the cursor has defined the magnitude of the voltage across the output capacitor as 1.1946 V or approximately 1.2 V as obtained above. If you then select the **Phase** curve and repeat the procedure, you will find that at 10 kHz **(x1)** the angle is −142.15° **(y1)** which is very close to the −142.92° obtained above.

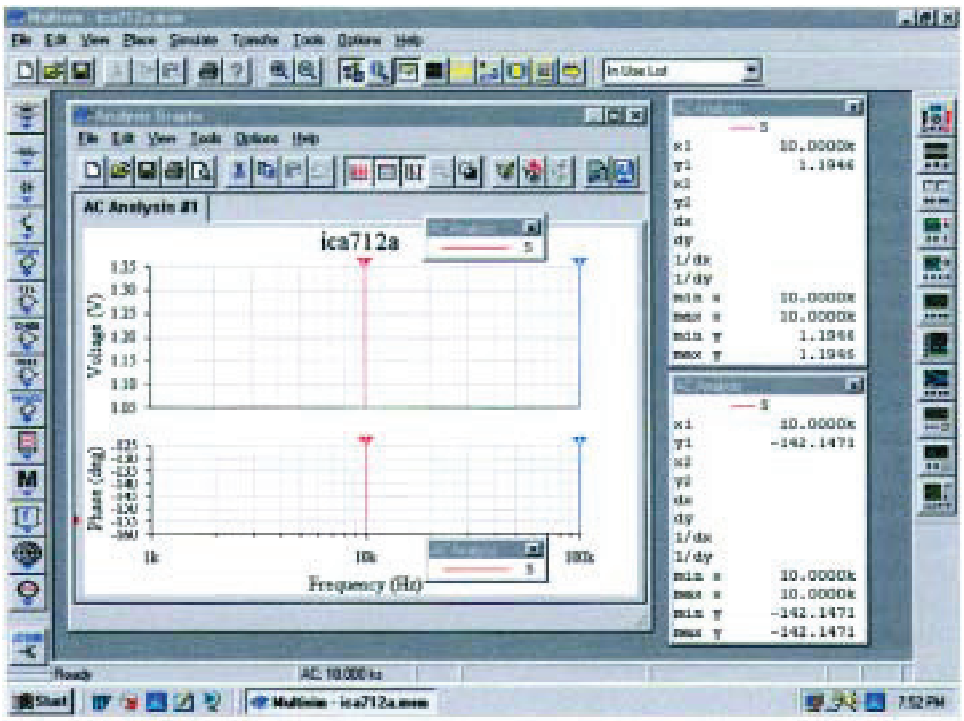


FIG. 16.38
*Using the **AC Analysis** option under Electronics Workbench to determine the magnitude and phase angle for the voltage V_{C_2} for the network of Fig. 16.36.*

In total, therefore, we have two methods to obtain an ac voltage in a network—one by instrumentation and the other through the computer methods. Both are valid, although, as expected, the computer approach has a higher level of accuracy.

PROBLEMS

SECTION 16.2 Illustrative Examples

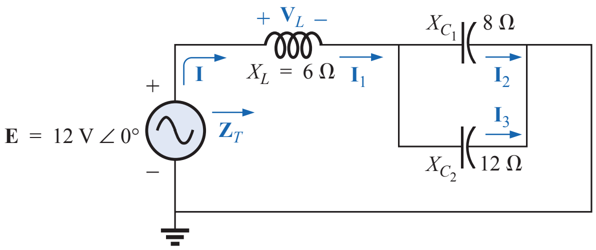


FIG. 16.39
Problems 1 and 19.

1. For the series-parallel network of Fig. 16.39:
   - **a.** Calculate $\mathbf{Z}_T$.
   - **b.** Determine $\mathbf{I}$.
   - **c.** Determine $\mathbf{I}_1$.
   - **d.** Find $\mathbf{I}_2$ and $\mathbf{I}_3$.
   - **e.** Find $\mathbf{V}_L$.

2. For the network of Fig. 16.40:
   - **a.** Find the total impedance $\mathbf{Z}_T$.
   - **b.** Determine the current $\mathbf{I}_s$.
   - **c.** Calculate $\mathbf{I}_C$ using the current divider rule.
   - **d.** Calculate $\mathbf{V}_L$ using the voltage divider rule.

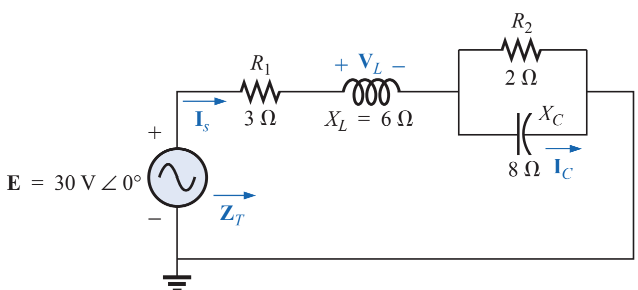


FIG. 16.40
Problems 2 and 15.

3. For the network of Fig. 16.41:
   - **a.** Find the total impedance $\mathbf{Z}_T$ and the total admittance $\mathbf{Y}_T$.
   - **b.** Find the current $\mathbf{I}_s$.
   - **c.** Calculate $\mathbf{I}_2$ using the current divider rule.
   - **d.** Calculate $\mathbf{V}_C$.
   - **e.** Calculate the average power delivered to the network.

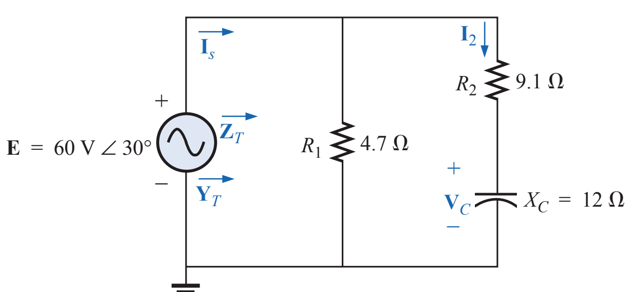


FIG. 16.41
Problems 3 and 20.

4. For the network of Fig. 16.42:
   a. Find the total impedance $\mathbf{Z}_T$.
   b. Calculate the voltage $\mathbf{V}_2$ and the current $\mathbf{I}_L$.
   c. Find the power factor of the network.

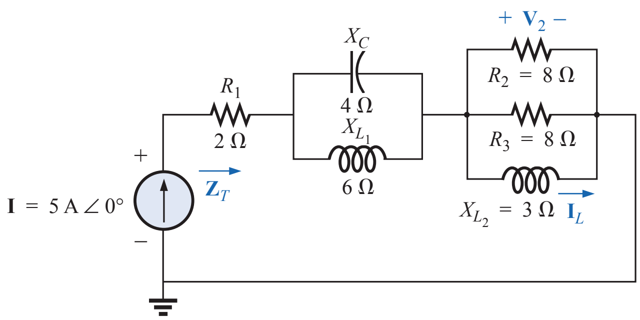


FIG. 16.42
Problem 4.

5. For the network of Fig. 16.43:
   a. Find the current **I.**
   b. Find the voltage $\mathbf{V}_C$.
   c. Find the average power delivered to the network.

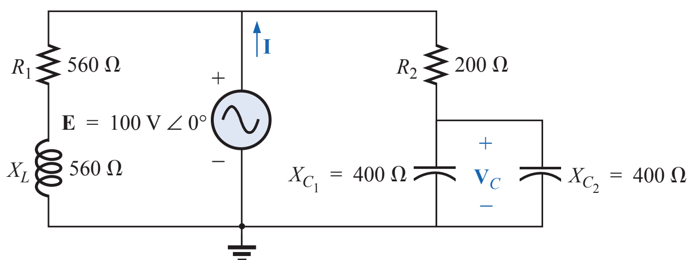


FIG. 16.43
Problems 5 and 21.

*6. For the network of Fig. 16.44:
   a. Find the current $\mathbf{I}_1$.
   b. Calculate the voltage $\mathbf{V}_C$ using the voltage divider rule.
   c. Find the voltage $\mathbf{V}_{ab}$.

*7. For the network of Fig. 16.45:
   a. Find the current $\mathbf{I}_1$.
   b. Find the voltage $\mathbf{V}_1$.
   c. Calculate the average power delivered to the network.

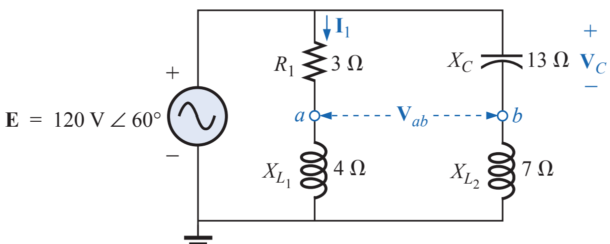


FIG. 16.44
Problem 6.

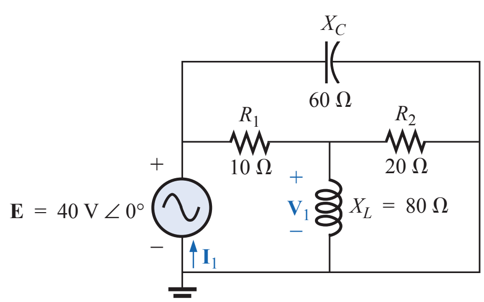


FIG. 16.45
Problems 7 and 16.

8. For the network of Fig. 16.46:

a. Find the total impedance $\mathbf{Z}_T$ and the admittance $\mathbf{Y}_T$.

b. Find the currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.

c. Verify Kirchhoff's current law by showing that $\mathbf{I}_s = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$.

d. Find the power factor of the network, and indicate whether it is leading or lagging.

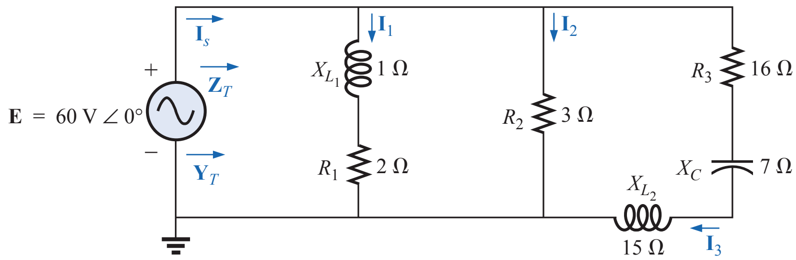


FIG. 16.46
Problem 8.

***9.** For the network of Fig. 16.47:

a. Find the total admittance $\mathbf{Y}_T$.

b. Find the voltages $\mathbf{V}_1$ and $\mathbf{V}_2$.

c. Find the current $\mathbf{I}_3$.

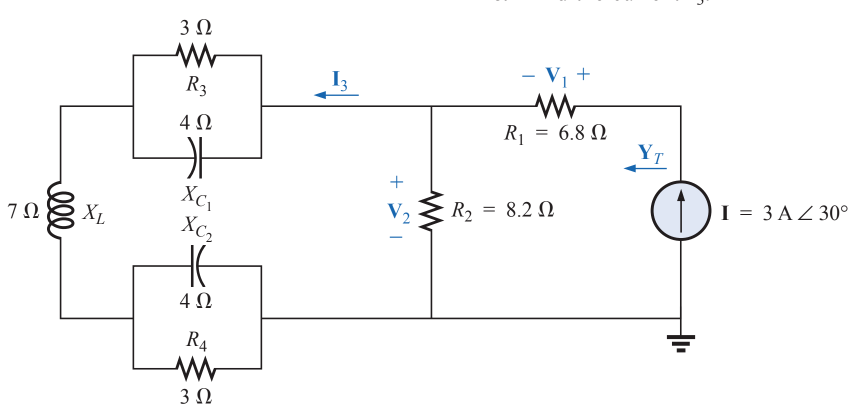


FIG. 16.47
Problem 9.

***10.** For the network of Fig. 16.48:

a. Find the total impedance $\mathbf{Z}_T$ and the admittance $\mathbf{Y}_T$.

b. Find the source current $\mathbf{I}_s$ in phasor form.

c. Find the currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in phasor form.

d. Find the voltages $\mathbf{V}_1$ and $\mathbf{V}_{ab}$ in phasor form.

e. Find the average power delivered to the network.

f. Find the power factor of the network, and indicate whether it is leading or lagging.

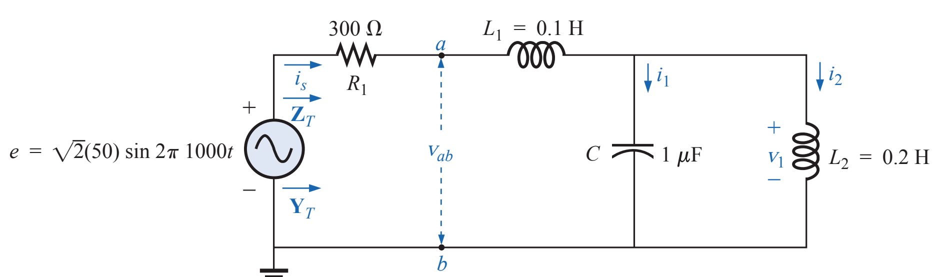


FIG. 16.48
Problem 10.

***11.** Find the current **I** for the network of Fig. 16.49.

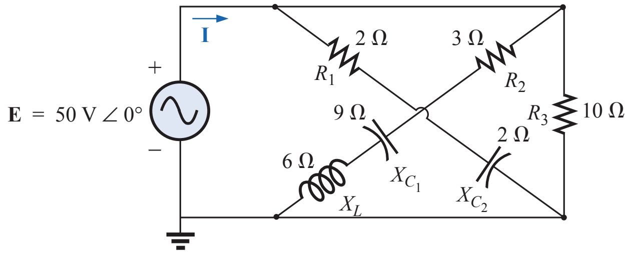


FIG. 16.49
Problems 11 and 17.

SECTION 16.3 Ladder Networks

12. Find the current $\mathbf{I}_5$ for the network of Fig. 16.50. Note the effect of one reactive element on the resulting calculations.

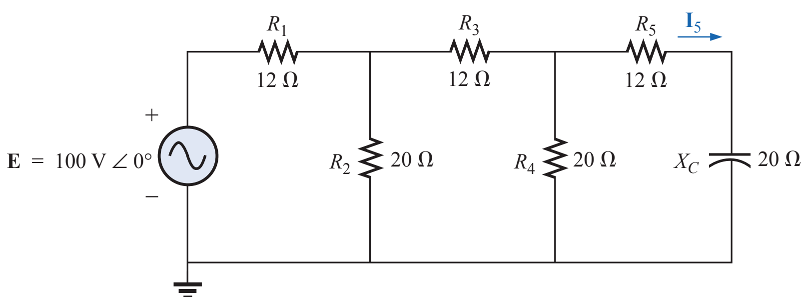


FIG. 16.50
Problem 12.

13. Find the average power delivered to R_4 in Fig. 16.51.

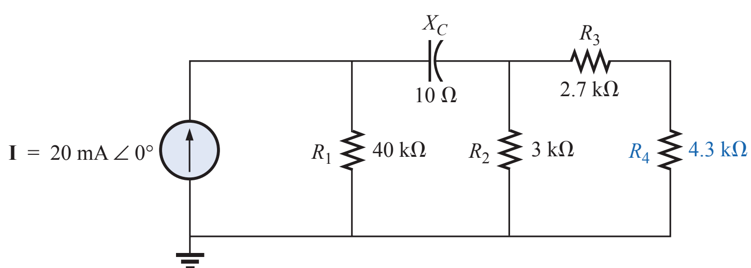


FIG. 16.51
Problem 13.

14. Find the current $\mathbf{I}_1$ for the network of Fig. 16.52.

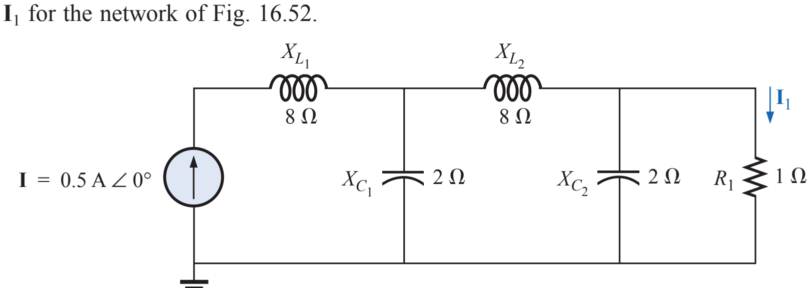


FIG. 16.52
Problems 14 and 18.

SECTION 16.5 Computer Analysis

PSpice or Electronics Workbench

For Problems 15 through 18, use a frequency of 1 kHz to determine the inductive and capacitive levels required for the input files. In each case write the required input file.

***15.** Repeat Problem 2 using PSpice or EWB.

***16.** Repeat Problem 7, parts (a) and (b), using PSpice or EWB.

***17.** Repeat Problem 11 using PSpice or EWB.

***18.** Repeat Problem 14 using PSpice or EWB.

Programming Language (C++, QBASIC, Pascal, etc.)

19. Write a program to provide a general solution to Problem 1; that is, given the reactance of each element, generate a solution for parts (a) through (e).

20. Given the network of Fig. 16.41, write a program to generate a solution for parts (a) and (b) of Problem 2. Use the values given.

21. Generate a program to obtain a general solution for the network of Fig. 16.43 for the questions asked in parts (a) through (c) of Problem 2. That is, given the resistance and reactance of the elements, determine the requested current, voltage, and power.

GLOSSARY

Ladder network A repetitive combination of series and parallel branches that has the appearance of a ladder.

Series-parallel ac network A combination of series and parallel branches in the same network configuration. Each branch may contain any number of elements whose impedance is dependent on the applied frequency.

17

Methods of Analysis and Selected Topics (ac)

17.1 INTRODUCTION

For networks with two or more sources that are not in series or parallel, the methods described in the last two chapters cannot be applied. Rather, methods such as mesh analysis or nodal analysis must be employed. Since these methods were discussed in detail for dc circuits in Chapter 8, this chapter will consider the variations required to apply these methods to ac circuits. Dependent sources will also be introduced for both mesh and nodal analysis.

The branch-current method will not be discussed again because it falls within the framework of mesh analysis. In addition to the methods mentioned above, the bridge network and Δ-Y, Y-Δ conversions will also be discussed for ac circuits.

Before we examine these topics, however, we must consider the subject of independent and controlled sources.

17.2 INDEPENDENT VERSUS DEPENDENT (CONTROLLED) SOURCES

In the previous chapters, each source appearing in the analysis of dc or ac networks was an **independent source,** such as E and I (or $\mathbf{E}$ and $\mathbf{I}$) in Fig. 17.1.

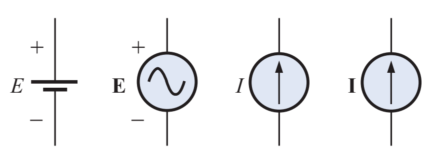


FIG. 17.1
Independent sources.

The term* independent *specifies that the magnitude of the source is independent of the network to which it is applied and that the source displays its terminal characteristics even if completely isolated.

A* dependent *or* controlled source *is one whose magnitude is determined (or controlled) by a current or voltage of the system in which it appears.

Currently two symbols are used for controlled sources. One simply uses the independent symbol with an indication of the controlling element, as shown in Fig. 17.2. In Fig. 17.2(a), the magnitude and phase of the voltage are controlled by a voltage **V** elsewhere in the system, with the magnitude further controlled by the constant k_1. In Fig.

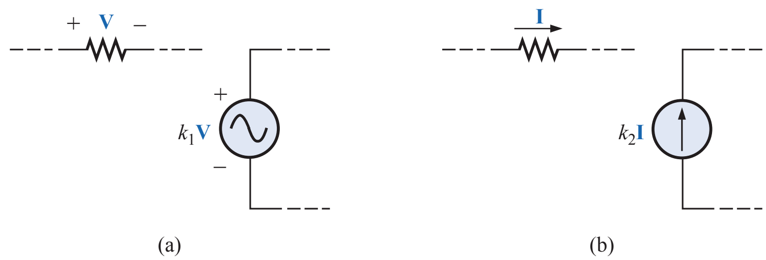


FIG. 17.2
Controlled or dependent sources.

17.2(b), the magnitude and phase of the current source are controlled by a current **I** elsewhere in the system, with the magnitude further controlled by the constant k_2. To distinguish between the dependent and independent sources, the notation of Fig. 17.3 was introduced. In recent years many respected publications on circuit analysis have accepted the notation of Fig. 17.3, although a number of excellent publications in the area of electronics continue to use the symbol of Fig. 17.2, especially in the circuit modeling for a variety of electronic devices such as the transistor and FET. This text will employ the symbols of Fig. 17.3.

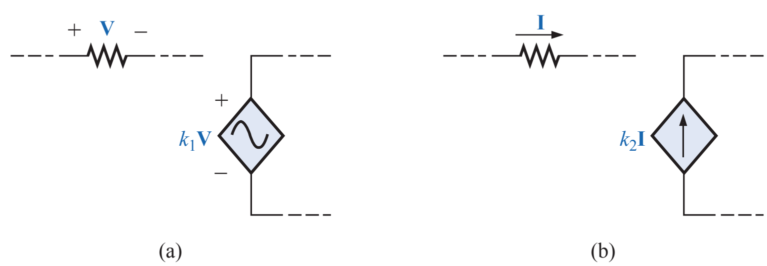


FIG. 17.3
Special notation for controlled or dependent sources.

Possible combinations for controlled sources are indicated in Fig. 17.4. Note that the magnitude of current sources or voltage sources can be controlled by a voltage and a current, respectively. Unlike with the independent source, isolation such that **V** or **I** = 0 in Fig. 17.4(a) will result in the short-circuit or open-circuit equivalent as indicated in Fig. 17.4(b). Note that the type of representation under these conditions is controlled by whether it is a current source or a voltage source, not by the controlling agent (**V** or **I**).

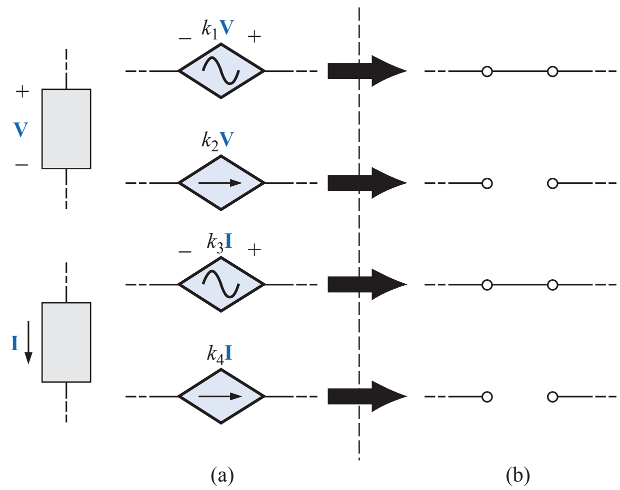


FIG. 17.4
Conditions of V = 0 V and I = 0 A for a controlled source.

17.3 SOURCE CONVERSIONS

When applying the methods to be discussed, it may be necessary to convert a current source to a voltage source, or a voltage source to a current source. This **source conversion** can be accomplished in much the same manner as for dc circuits, except now we shall be dealing with phasors and impedances instead of just real numbers and resistors.

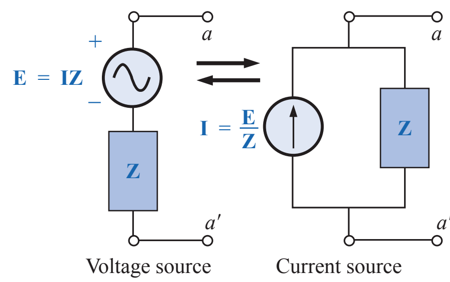


FIG. 17.5
Source conversion.

Independent Sources

In general, the format for converting one type of independent source to another is as shown in Fig. 17.5.

EXAMPLE 17.1 Convert the voltage source of Fig. 17.6(a) to a current source.

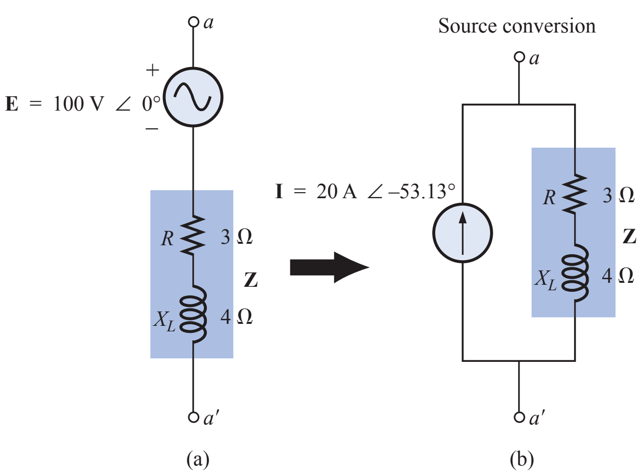


FIG. 17.6
Example 17.1.

Solution:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{100\text{ V}\angle 0^\circ}{5\ \Omega \angle 53.13^\circ}$$
$$= \mathbf{20\ A \angle -53.13^\circ} \qquad \text{[Fig. 17.6(b)]}$$

EXAMPLE 17.2 Convert the current source of Fig. 17.7(a) to a voltage source.

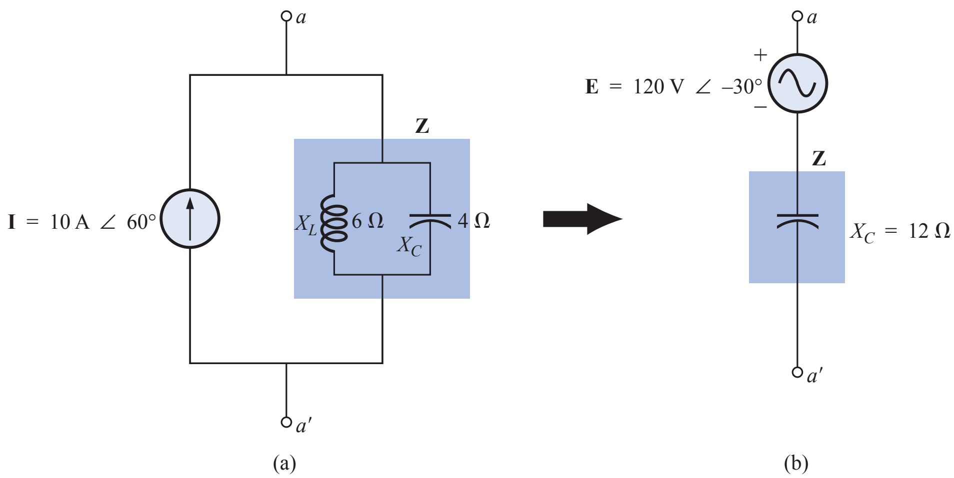


FIG. 17.7
Example 17.2.

Solution:

$$\mathbf{Z} = \frac{\mathbf{Z}_C\mathbf{Z}_L}{\mathbf{Z}_C + \mathbf{Z}_L} = \frac{(X_C \angle -90^\circ)(X_L \angle 90^\circ)}{-j\,X_C + j\,X_L}$$
$$= \frac{(4\ \Omega \angle -90^\circ)(6\ \Omega \angle 90^\circ)}{-j\,4\ \Omega + j\,6\ \Omega} = \frac{24\ \Omega \angle 0^\circ}{2 \angle 90^\circ}$$
$$= \mathbf{12\ \Omega \angle -90^\circ} \qquad \text{[Fig. 17.7(b)]}$$
$$\mathbf{E} = \mathbf{IZ} = (10\text{ A} \angle 60^\circ)(12\ \Omega \angle -90^\circ)$$
$$= \mathbf{120\ V \angle -30^\circ} \qquad \text{[Fig. 17.7(b)]}$$

Dependent Sources

For dependent sources, the direct conversion of Fig. 17.5 can be applied if the controlling variable (**V** or **I** in Fig. 17.4) is not determined by a portion of the network to which the conversion is to be applied. For example, in Figs. 17.8 and 17.9, **V** and **I**, respectively, are controlled by an external portion of the network. Conversions of the other kind, where **V** and **I** are controlled by a portion of the network to be converted, will be considered in Sections 18.3 and 18.4.

EXAMPLE 17.3 Convert the voltage source of Fig. 17.8(a) to a current source.

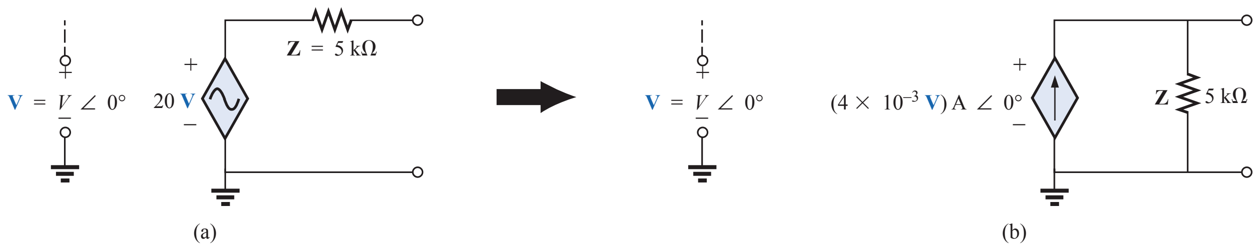


FIG. 17.8
Source conversion with a voltage-controlled voltage source.

Solution:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{(20\mathbf{V})\ \text{V} \angle 0°}{5\ \text{k}\Omega \angle 0°}$$
$$= \mathbf{(4 \times 10^{-3}\ V)\ A \angle 0°} \qquad \text{[Fig. 17.8(b)]}$$

EXAMPLE 17.4 Convert the current source of Fig. 17.9(a) to a voltage source.

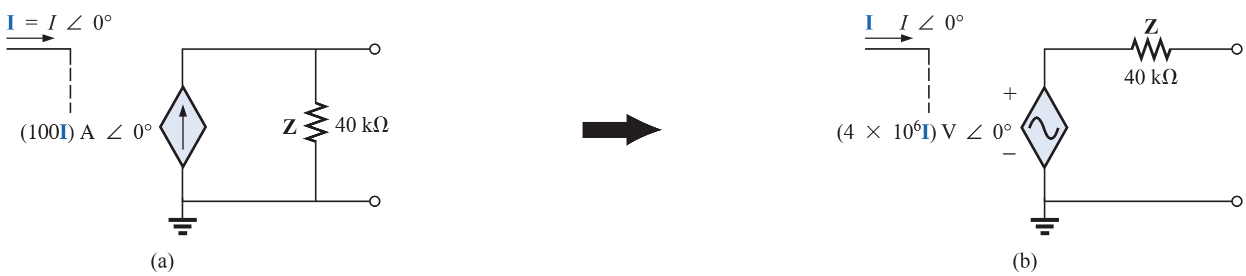


FIG. 17.9
Source conversion with a current-controlled current source.

Solution:

$$\mathbf{E} = \mathbf{IZ} = [(100\mathbf{I})\ \text{A} \angle 0°][40\ \text{k}\Omega \angle 0°]$$
$$= \mathbf{(4 \times 10^{6} I)\ V \angle 0°} \qquad \text{[Fig. 17.9(b)]}$$

17.4 MESH ANALYSIS

General Approach

Independent Voltage Sources Before examining the application of the method to ac networks, the student should first review the appropriate sections on **mesh analysis** in Chapter 8 since the content of this section will be limited to the general conclusions of Chapter 8.

The general approach to mesh analysis for independent sources includes the same sequence of steps appearing in Chapter 8. In fact, throughout this section the only change from the dc coverage will be to substitute impedance for resistance and admittance for conductance in the general procedure.

1. Assign a distinct current in the clockwise direction to each independent closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current. However, it eliminates the need to have to choose a direction for each application. Any direction can be chosen for each loop current with no loss in accuracy as long as the remaining steps are followed properly.
2. Indicate the polarities within each loop for each impedance as determined by the assumed direction of loop current for that loop.
3. Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. Again, the clockwise direction was chosen to establish uniformity and to prepare us for the format approach to follow.
   a. If an impedance has two or more assumed currents through it, the total current through the impedance is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents passing through in the opposite direction.
   b. The polarity of a voltage source is unaffected by the direction of the assigned loop currents.
4. Solve the resulting simultaneous linear equations for the assumed loop currents.

The technique is applied as above for all networks with independent sources or for networks with *dependent sources where the controlling variable is not a part of the network under investigation.* If the controlling variable is part of the network being examined, a method to be described shortly must be applied.

EXAMPLE 17.5 Using the general approach to mesh analysis, find the current $\mathbf{I}_1$ in Fig. 17.10.

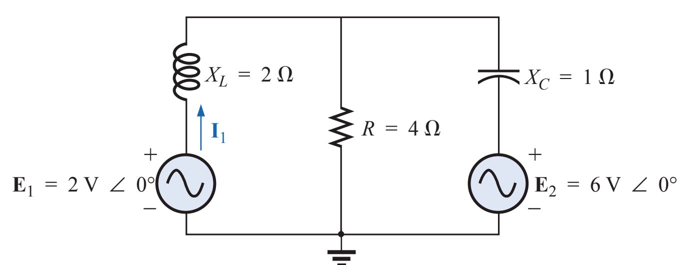


FIG. 17.10
Example 17.5.

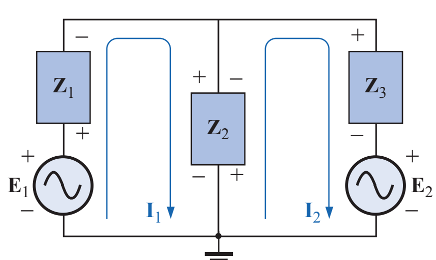


FIG. 17.11
Assigning the mesh currents and subscripted impedances for the network of Fig. 17.10.

Solution: When applying these methods to ac circuits, it is good practice to represent the resistors and reactances (or combinations thereof) by subscripted impedances. When the total solution is found in terms of these subscripted impedances, the numerical values can be substituted to find the unknown quantities.

The network is redrawn in Fig. 17.11 with subscripted impedances:

$$\mathbf{Z}_1 = +j\,X_L = +j\,2\ \Omega \qquad \mathbf{E}_1 = 2\ \text{V}\ \angle 0°$$
$$\mathbf{Z}_2 = R = 4\ \Omega \qquad \mathbf{E}_2 = 6\ \text{V}\ \angle 0°$$
$$\mathbf{Z}_3 = -j\,X_C = -j\,1\ \Omega$$

Steps 1 and 2 are as indicated in Fig. 17.11.

Step 3:

$$+\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{Z}_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$
$$-\mathbf{Z}_2(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{E}_2 = 0$$

or

$$\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_1\mathbf{Z}_2 + \mathbf{I}_2\mathbf{Z}_2 = 0$$
$$-\mathbf{I}_2\mathbf{Z}_2 + \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{E}_2 = 0$$

so that

$$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1$$
$$\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 = -\mathbf{E}_2$$

which are rewritten as

$$\begin{aligned}\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 \qquad\qquad &= \mathbf{E}_1\\ -\mathbf{I}_1\mathbf{Z}_2 \qquad + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) &= -\mathbf{E}_2\end{aligned}$$

Step 4: Using determinants, we obtain

$$\mathbf{I}_1 = \frac{\begin{vmatrix} \mathbf{E}_1 & -\mathbf{Z}_2 \\ -\mathbf{E}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & -\mathbf{Z}_2 \\ -\mathbf{Z}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}$$
$$= \frac{\mathbf{E}_1(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{E}_2(\mathbf{Z}_2)}{(\mathbf{Z}_1 + \mathbf{Z}_2)(\mathbf{Z}_2 + \mathbf{Z}_3) - (\mathbf{Z}_2)^2}$$
$$= \frac{(\mathbf{E}_1 - \mathbf{E}_2)\mathbf{Z}_2 + \mathbf{E}_1\mathbf{Z}_3}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}$$

Substituting numerical values yields

$$\mathbf{I}_1 = \frac{(2\text{ V} - 6\text{ V})(4\ \Omega) + (2\text{ V})(-j\,1\ \Omega)}{(+j\,2\ \Omega)(4\ \Omega) + (+j\,2\ \Omega)(-j\,2\ \Omega) + (4\ \Omega)(-j\,2\ \Omega)}$$
$$= \frac{-16 - j\,2}{j\,8 - j^2 2 - j\,4} = \frac{-16 - j\,2}{2 + j\,4} = \frac{16.12\text{ A}\angle -172.87^\circ}{4.47\angle 63.43^\circ}$$
$$= \mathbf{3.61\ A\angle -236.30^\circ} \quad \text{or} \quad \mathbf{3.61\ A\angle 123.70^\circ}$$

Dependent Voltage Sources For dependent voltage sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent voltage sources.
2. Step 3 is modified as follows: Treat each dependent source like an independent source when Kirchhoff's voltage law is applied to each independent loop. However, once the equation is written, substitute the equation for the controlling quantity to ensure that the unknowns are limited solely to the chosen mesh currents.
3. Step 4 is as before.

EXAMPLE 17.6 Write the mesh currents for the network of Fig. 17.12 having a dependent voltage source.

Solution:

Steps 1 and 2 are defined on Fig. 17.12.

Step 3:
$$\mathbf{E}_1 - \mathbf{I}_1 R_1 - R_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$
$$R_2(\mathbf{I}_2 - \mathbf{I}_1) + \mu\mathbf{V}_x - \mathbf{I}_2 R_3 = 0$$

Then substitute $\mathbf{V}_x = (\mathbf{I}_1 - \mathbf{I}_2)R_2$

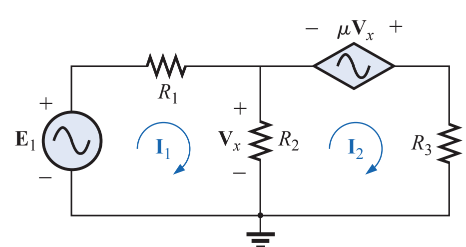


FIG. 17.12
Applying mesh analysis to a network with a voltage-controlled voltage source.

The result is two equations and two unknowns.

$$\mathbf{E}_1 - \mathbf{I}_1 R_1 - R_2(\mathbf{I} - \mathbf{I}_2) = 0$$
$$R_2(\mathbf{I}_2 - \mathbf{I}_1) + \mu R_2(\mathbf{I}_1 - \mathbf{I}_2) - \mathbf{I}_2 R_3 = 0$$

Independent Current Sources For independent current sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.
2. Step 3 is modified as follows: Treat each current source as an open circuit (recall the *supermesh* designation of Chapter 8), and write the mesh equations for each remaining independent path. Then relate the chosen mesh currents to the dependent sources to ensure that the unknowns of the final equations are limited simply to the mesh currents.
3. Step 4 is as before.

EXAMPLE 17.7 Write the mesh currents for the network of Fig. 17.13 having an independent current source.

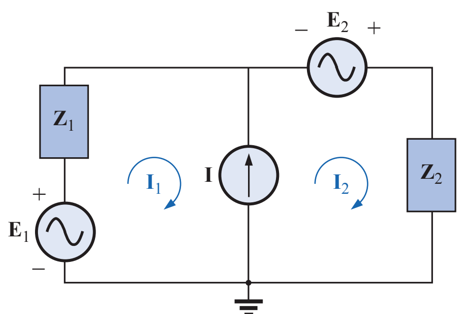


FIG. 17.13
Applying mesh analysis to a network with an independent current source.

Solution:

Steps 1 and 2 are defined on Fig. 17.13.

Step 3: $\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 + \mathbf{E}_2 - \mathbf{I}_2\mathbf{Z}_2 = 0$ (only remaining independent path)

with $\mathbf{I}_1 + \mathbf{I} = \mathbf{I}_2$

The result is two equations and two unknowns.

Dependent Current Sources For dependent current sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.
2. Step 3 is modified as follows: The procedure is essentially the same as that applied for independent current sources, except now the dependent sources have to be defined in terms of the chosen mesh currents to ensure that the final equations have only mesh currents as the unknown quantities.
3. Step 4 is as before.

EXAMPLE 17.8 Write the mesh currents for the network of Fig. 17.14 having a dependent current source.

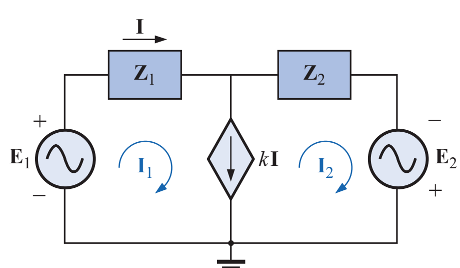


FIG. 17.14
Applying mesh analysis to a network with a current-controlled current source.

Solution:

Steps 1 and 2 are defined on Fig. 17.14.

Step 3: $\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_2 + \mathbf{E}_2 = 0$

and $k\mathbf{I} = \mathbf{I}_1 - \mathbf{I}_2$

Now $\mathbf{I} = \mathbf{I}_1$ so that $k\mathbf{I}_1 = \mathbf{I}_1 - \mathbf{I}_2$ or $\mathbf{I}_2 = \mathbf{I}_1(1 - k)$

The result is two equations and two unknowns.

Format Approach

The format approach was introduced in Section 8.9. The steps for applying this method are repeated here with changes for its use in ac circuits:

1. ***Assign a loop current to each independent closed loop (as in the previous section) in a clockwise direction.***
2. ***The number of required equations is equal to the number of chosen independent closed loops. Column 1 of each equation is formed by simply summing the impedance values of those impedances through which the loop current of interest passes and multiplying the result by that loop current.***
3. ***We must now consider the mutual terms that are always subtracted from the terms in the first column. It is possible to have more than one mutual term if the loop current of interest has an element in common with more than one other loop current. Each mutual term is the product of the mutual impedance and the other loop current passing through the same element.***
4. ***The column to the right of the equality sign is the algebraic sum of the voltage sources through which the loop current of interest passes. Positive signs are assigned to those sources of voltage having a polarity such that the loop current passes from the negative to the positive terminal. Negative signs are assigned to those potentials for which the reverse is true.***
5. ***Solve the resulting simultaneous equations for the desired loop currents.***

The technique is applied as above for all networks with independent sources or for networks with dependent sources where the controlling variable is not a part of the network under investigation. If the controlling variable is part of the network being examined, additional care must be taken when applying the above steps.

EXAMPLE 17.9 Using the format approach to mesh analysis, find the current $\mathbf{I}_2$ in Fig. 17.15.

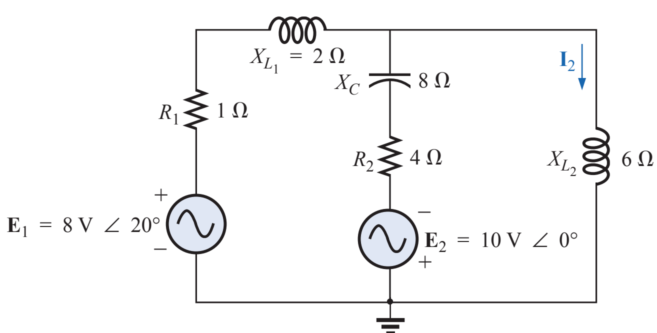


FIG. 17.15
Example 17.9.

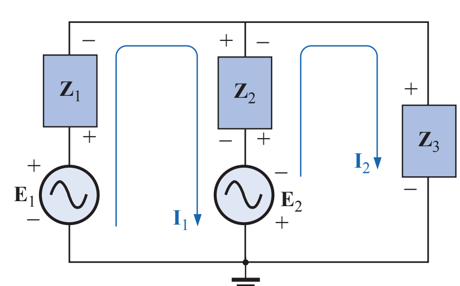


FIG. 17.16
Assigning the mesh currents and subscripted impedances for the network of Fig. 17.15.

Solution 1: The network is redrawn in Fig. 17.16:

$$\mathbf{Z}_1 = R_1 + j\,X_{L_1} = 1\ \Omega + j\,2\ \Omega \qquad \mathbf{E}_1 = 8\text{ V} \angle 20°$$
$$\mathbf{Z}_2 = R_2 - j\,X_C = 4\ \Omega - j\,8\ \Omega \qquad \mathbf{E}_2 = 10\text{ V} \angle 0°$$
$$\mathbf{Z}_3 = +j\,X_{L_2} = +j\,6\ \Omega$$

Note the reduction in complexity of the problem with the substitution of the subscripted impedances.

Step 1 is as indicated in Fig. 17.16.

Steps 2 to 4:

$$\begin{aligned} \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 &= \mathbf{E}_1 + \mathbf{E}_2 \\ \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 &= -\mathbf{E}_2 \end{aligned}$$

which are rewritten as

$$\begin{aligned} \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 \qquad\qquad &= \mathbf{E}_1 + \mathbf{E}_2 \\ -\mathbf{I}_1\mathbf{Z}_2 \qquad + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) &= -\mathbf{E}_2 \end{aligned}$$

Step 5: Using determinants, we have

$$\mathbf{I}_2 = \frac{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & \mathbf{E}_1 + \mathbf{E}_2 \\ -\mathbf{Z}_2 & -\mathbf{E}_2 \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & -\mathbf{Z}_2 \\ -\mathbf{Z}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}$$

$$= \frac{-(\mathbf{Z}_1 + \mathbf{Z}_2)\mathbf{E}_2 + \mathbf{Z}_2(\mathbf{E}_1 + \mathbf{E}_2)}{(\mathbf{Z}_1 + \mathbf{Z}_2)(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{Z}_2^2}$$

$$= \frac{\mathbf{Z}_2\mathbf{E}_1 - \mathbf{Z}_1\mathbf{E}_2}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}$$

Substituting numerical values yields

$$\mathbf{I}_2 = \frac{(4\ \Omega - j\,8\ \Omega)(8\ \text{V} \angle 20^\circ) - (1\ \Omega + j\,2\ \Omega)(10\ \text{V} \angle 0^\circ)}{(1\ \Omega + j\,2\ \Omega)(4\ \Omega - j\,8\ \Omega) + (1\ \Omega + j\,2\ \Omega)(+j\,6\ \Omega) + (4\ \Omega - j\,8\ \Omega)(+j\,6\ \Omega)}$$

$$= \frac{(4 - j\,8)(7.52 + j\,2.74) - (10 + j\,20)}{20 + (j\,6 - 12) + (j\,24 + 48)}$$

$$= \frac{(52.0 - j\,49.20) - (10 + j\,20)}{56 + j\,30} = \frac{42.0 - j\,69.20}{56 + j\,30} = \frac{80.95\ \text{A} \angle -58.74^\circ}{63.53 \angle 28.18^\circ}$$

$$= \mathbf{1.27\ A \angle -86.92^\circ}$$

Calculator The calculator (TI-86 or equivalent) can be an effective tool in performing the long, laborious calculations involved with the final equation appearing above. However, you must be very careful to use the correct number of brackets and to define by brackets the order of the arithmetic operations.

```
((4,−8)*8(∠20)−(1,2)*(10∠0))/((1,2)*(4,−8)+(1,2)*(0,6)+(4,−8)*(0,6)) ENTER
(67.854E−3,−1.272E0)
Ans ▶ Pol
(1.274E0∠−86.956E0)
```

CALC. 17.1

Mathcad Solution: This example provides an excellent opportunity to demonstrate the power of Mathcad. First the impedances and parameters are defined for the equations to follow as shown in Fig. 17.17. Then the **guess** values of the mesh currents $\mathbf{I}_1$ and $\mathbf{I}_2$ are entered. The label **Given** must then be entered followed by the equations for the net-

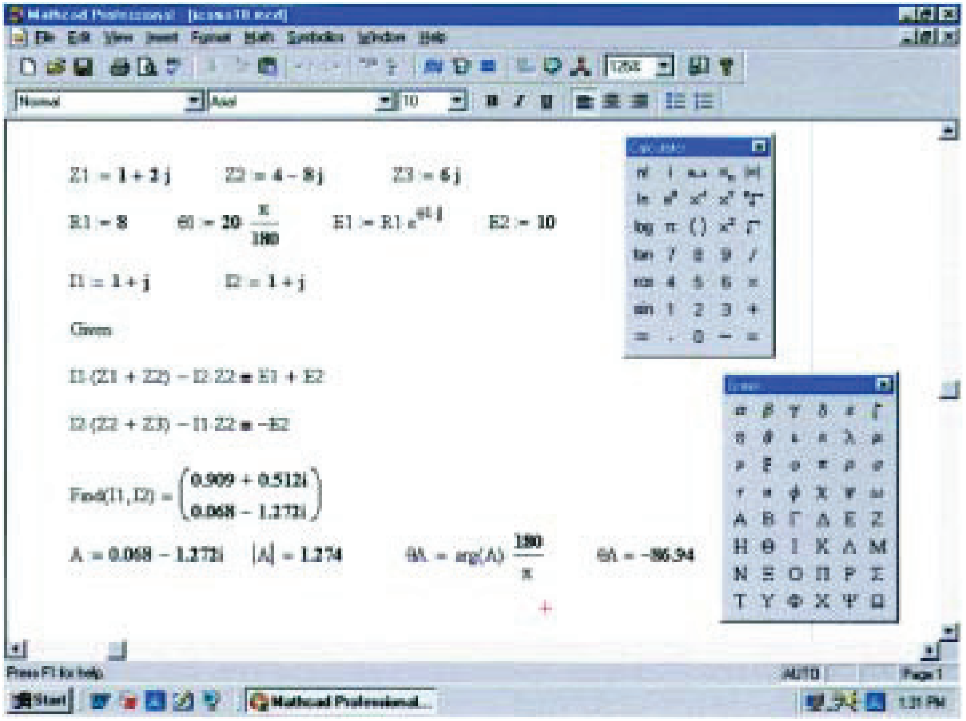


FIG. 17.17
Using Mathcad to verify the results of Example 17.9.

work. Note that in this example, we are not continuing with the analysis until the matrix is defined—we are working directly from the network equations. Once the equations have been properly entered, **Find(I1,I2)** is entered. Then selecting the equal sign will result in the single-column matrix with the results in rectangular form. Conversion to polar form requires defining a variable **A** and then calling for the magnitude and angle using the definitions entered earlier in the listing and both the **Calculator** and **Greek** toolbars. The result for $\mathbf{I}_2$ is 1.274 A $\angle -86.94°$ which is an excellent match with the theoretical solution.

EXAMPLE 17.10 Write the mesh equations for the network of Fig. 17.18. Do not solve.

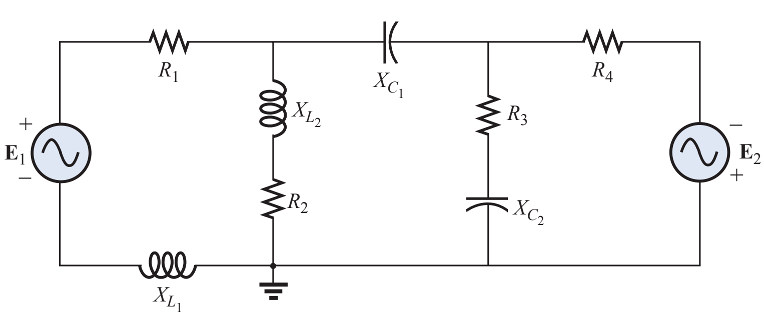


FIG. 17.18
Example 17.10.

Solution: The network is redrawn in Fig. 17.19. Again note the reduced complexity and increased clarity provided by the use of subscripted impedances:

$$\begin{aligned} \mathbf{Z}_1 &= R_1 + j X_{L_1} \qquad & \mathbf{Z}_4 &= R_3 - j X_{C_2} \\ \mathbf{Z}_2 &= R_2 + j X_{L_2} & \mathbf{Z}_5 &= R_4 \\ \mathbf{Z}_3 &= j X_{C_1} \end{aligned}$$

and

$$\begin{aligned} \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 &= \mathbf{E}_1 \\ \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4) - \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_3\mathbf{Z}_4 &= 0 \\ \mathbf{I}_3(\mathbf{Z}_4 + \mathbf{Z}_5) - \mathbf{I}_2\mathbf{Z}_4 &= \mathbf{E}_2 \end{aligned}$$

or

$$\begin{array}{llll} \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) & - \mathbf{I}_2(\mathbf{Z}_2) & + 0 & = \mathbf{E}_1 \\ \mathbf{I}_1\mathbf{Z}_2 & - \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4) & + \mathbf{I}_3(\mathbf{Z}_4) & = 0 \\ 0 & - \mathbf{I}_2(\mathbf{Z}_4) & + \mathbf{I}_3(\mathbf{Z}_4 + \mathbf{Z}_5) & = \mathbf{E}_2 \end{array}$$

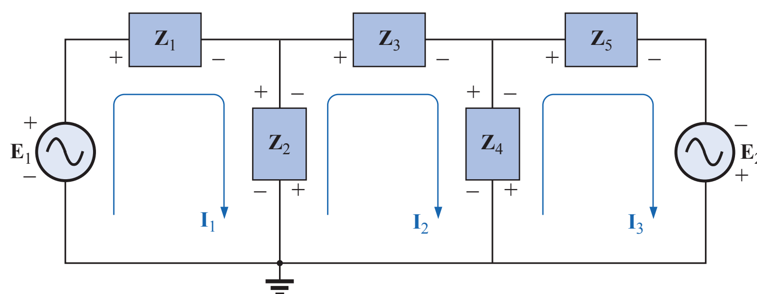


FIG. 17.19
Assigning the mesh currents and subscripted impedances for the network of Fig. 17.18.

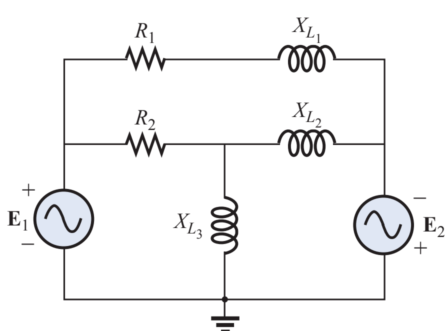


FIG. 17.20
Example 17.11

EXAMPLE 17.11 Using the format approach, write the mesh equations for the network of Fig. 17.20.

Solution: The network is redrawn as shown in Fig. 17.21, where

$$\begin{aligned} \mathbf{Z}_1 &= R_1 + j X_{L_1} \qquad & \mathbf{Z}_3 &= j X_{L_2} \\ \mathbf{Z}_2 &= R_2 & \mathbf{Z}_4 &= j X_{L_3} \end{aligned}$$

and

$$\begin{aligned} \mathbf{I}_1(\mathbf{Z}_2 + \mathbf{Z}_4) - \mathbf{I}_2\mathbf{Z}_2 - \mathbf{I}_3\mathbf{Z}_4 &= \mathbf{E}_1 \\ \mathbf{I}_2(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_3\mathbf{Z}_3 &= 0 \\ \mathbf{I}_3(\mathbf{Z}_3 + \mathbf{Z}_4) - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{I}_1\mathbf{Z}_4 &= \mathbf{E}_2 \end{aligned}$$

or

$$\begin{array}{llll} \mathbf{I}_1(\mathbf{Z}_2 + \mathbf{Z}_4) - \mathbf{I}_2\mathbf{Z}_2 & & - \mathbf{I}_3\mathbf{Z}_4 & = \mathbf{E}_1 \\ -\mathbf{I}_1\mathbf{Z}_2 & + \mathbf{I}_2(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3) & - \mathbf{I}_3\mathbf{Z}_3 & = 0 \\ -\mathbf{I}_1\mathbf{Z}_4 & - \mathbf{I}_2\mathbf{Z}_3 & + \mathbf{I}_3(\mathbf{Z}_3 + \mathbf{Z}_4) & = \mathbf{E}_2 \end{array}$$

Note the symmetry *about* the diagonal axis; that is, note the location of $-\mathbf{Z}_2$, $-\mathbf{Z}_4$, and $-\mathbf{Z}_3$ off the diagonal.

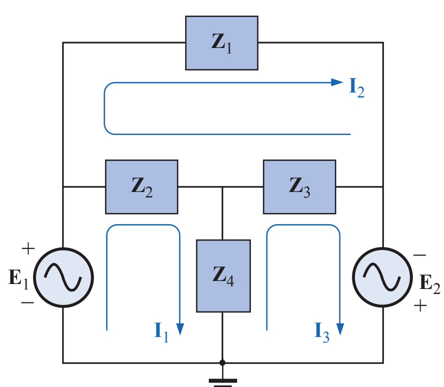


FIG. 17.21
Assigning the mesh currents and subscripted impedances for the network of Fig. 17.20.

17.5 NODAL ANALYSIS

General Approach

Independent Sources Before examining the application of the method to ac networks, a review of the appropriate sections on **nodal**

analysis in Chapter 8 is suggested since the content of this section will be limited to the general conclusions of Chapter 8.

The fundamental steps are the following:

1. ***Determine the number of nodes within the network.***
2. ***Pick a reference node and label each remaining node with a subscripted value of voltage:*** $\mathbf{V}_1$***,*** $\mathbf{V}_2$***, and so on.***
3. ***Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law.***
4. ***Solve the resulting equations for the nodal voltages.***

A few examples will refresh your memory about the content of Chapter 8 and the general approach to a nodal-analysis solution.

EXAMPLE 17.12 Determine the voltage across the inductor for the network of Fig. 17.22.

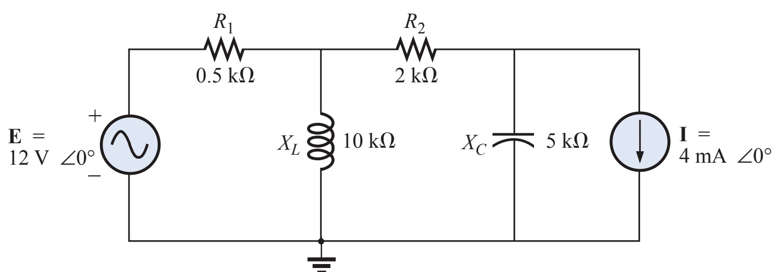


FIG. 17.22
Example 17.12.

Solution 1:

Steps 1 and 2 are as indicated in Fig. 17.23.

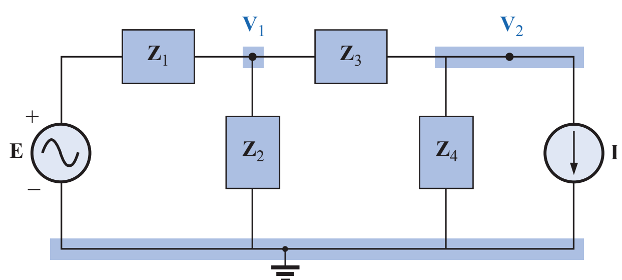


FIG. 17.23
Assigning the nodal voltages and subscripted impedances to the network of Fig. 17.22.

Step 3: Note Fig. 17.24 for the application of Kirchhoff's current law to node $\mathbf{V}_1$:

$$\Sigma\,\mathbf{I}_i = \Sigma\,\mathbf{I}_o$$

$$0 = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

$$\frac{\mathbf{V}_1 - \mathbf{E}}{\mathbf{Z}_1} + \frac{\mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_3} = 0$$

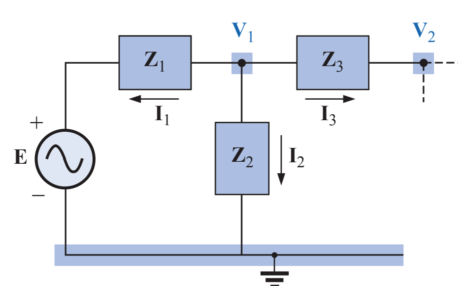


FIG. 17.24
Applying Kirchhoff's current law to the node $\mathbf{V}_1$ *of Fig. 17.23.*

Rearranging terms:

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3}\right]-\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}\right]=\frac{\mathbf{E}_1}{\mathbf{Z}_1} \qquad \textbf{(17.1)}$$

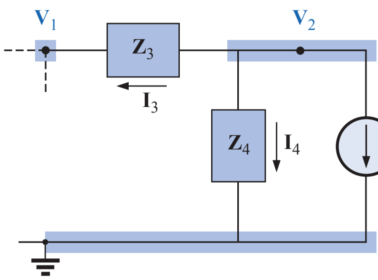


FIG. 17.25
Applying Kirchhoff's current law to the node $\mathbf{V}_2$ *of Fig. 17.23.*

Note Fig. 17.25 for the application of Kirchhoff's current law to node $\mathbf{V}_2$.

$$0 = \mathbf{I}_3 + \mathbf{I}_4 + \mathbf{I}$$

$$\frac{\mathbf{V}_2-\mathbf{V}_1}{\mathbf{Z}_3}+\frac{\mathbf{V}_2}{\mathbf{Z}_4}+\mathbf{I}=0$$

Rearranging terms:

$$\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_4}\right]-\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_3}\right]=-\mathbf{I} \qquad \textbf{(17.2)}$$

Grouping equations:

$$\begin{array}{l}
\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3}\right]-\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}\right]=\frac{\mathbf{E}}{\mathbf{Z}_1}\\
\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_3}\right]-\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_4}\right]=\mathbf{I}
\end{array}$$

$$\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3}=\frac{1}{0.5\text{ k}\Omega}+\frac{1}{j\,10\text{ k}\Omega}+\frac{1}{2\text{ k}\Omega}=2.5\text{ mS}\angle-2.29^\circ$$

$$\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_4}=\frac{1}{2\text{ k}\Omega}+\frac{1}{-j\,5\text{ k}\Omega}=0.539\text{ mS}\angle 21.80^\circ$$

and

$$\begin{array}{l}
\mathbf{V}_1[2.5\text{ mS}\angle-2.29^\circ]-\mathbf{V}_2[0.5\text{ mS}\angle 0^\circ]=24\text{ mA}\angle 0^\circ\\
\mathbf{V}_1[0.5\text{ mS}\angle 0^\circ]-\mathbf{V}_2[0.539\text{ mS}\angle 21.80^\circ]=4\text{ mA}\angle 0^\circ
\end{array}$$

with

$$\mathbf{V}_1=\frac{\begin{vmatrix}24\text{ mA}\angle 0^\circ & -0.5\text{ mS}\angle 0^\circ\\ 4\text{ mA}\angle 0^\circ & -0.539\text{ mS}\angle 21.80^\circ\end{vmatrix}}{\begin{vmatrix}2.5\text{ mS}\angle-2.29^\circ & -0.5\text{ mS}\angle 0^\circ\\ 0.5\text{ mS}\angle 0^\circ & -0.539\text{ mS}\angle 21.80^\circ\end{vmatrix}}$$

$$=\frac{(24\text{ mA}\angle 0^\circ)(-0.539\text{ mS}\angle 21.80^\circ)+(0.5\text{ mS}\angle 0^\circ)(4\text{ mA}\angle 0^\circ)}{(2.5\text{ mS}\angle-2.29^\circ)(-0.539\text{ mS}\angle 21.80^\circ)+(0.5\text{ mS}\angle 0^\circ)(0.5\text{ mS}\angle 0^\circ)}$$

$$=\frac{-12.94\times10^{-6}\text{ V}\angle 21.80^\circ+2\times10^{-6}\text{ V}\angle 0^\circ}{-1.348\times10^{-6}\angle 19.51^\circ+0.25\times10^{-6}\angle 0^\circ}$$

$$=\frac{-(12.01+j\,4.81)\times10^{-6}\text{ V}+2\times10^{-6}\text{ V}}{-(1.271+j\,0.45)\times10^{-6}+0.25\times10^{-6}}$$

$$=\frac{-10.01\text{ V}-j\,4.81\text{ V}}{-1.021-j\,0.45}=\frac{11.106\text{ V}\angle-154.33^\circ}{1.116\angle-156.21^\circ}$$

$$\mathbf{V}_1=\mathbf{9.95\text{ V}\angle 1.88^\circ}$$

Mathcad Solution: The length and the complexity of the above mathematical development strongly suggest the use of an alternative approach such as Mathcad. The printout of Fig. 17.26 first defines the letters **k** and **m** to specific numerical values so that the power-of-ten format did not have to be included in the equations. Thus, the results are cleaner and easier to review. When entering the equations, remember that the j is entered as $1j$ **without** the multiplication sign between the

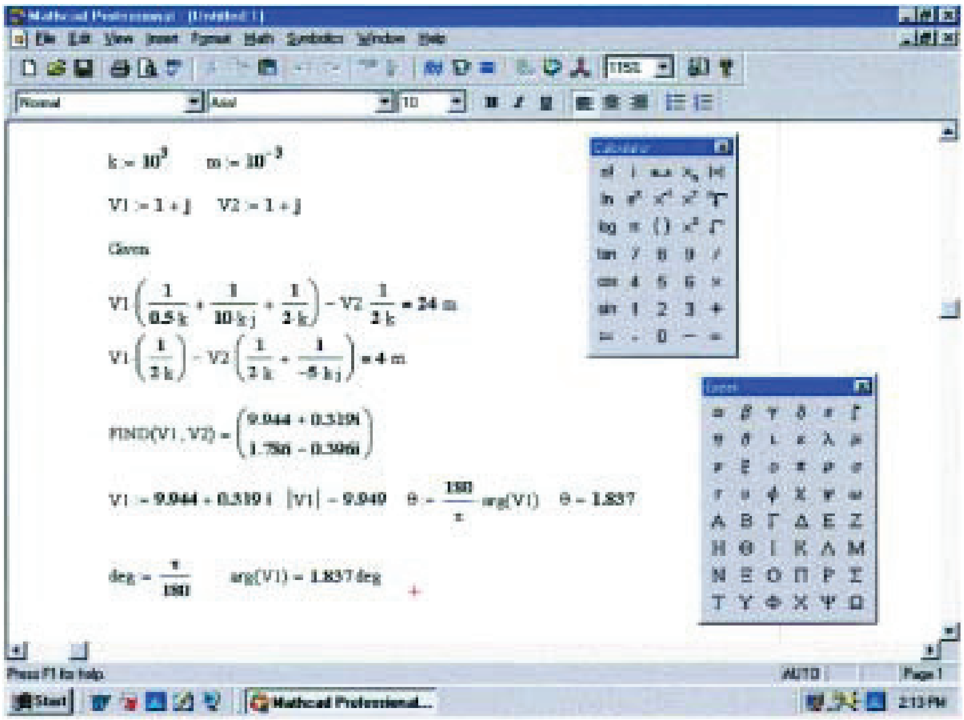


FIG. 17.26
Using Mathcad to verify the results of Example 17.12.

1 and the *j*. A multiplication sign between the two will define the *j* as another variable. Also be sure that the multiplication process is inserted between the nodal variables and the brackets. If an error signal continues to surface, it is often best to simply reenter the entire listing—errors are often not easy to spot simply by looking at the resulting equations.

Finally the results are obtained and converted to polar form for comparison with the theoretical solution. The solution of 9.949 A $\angle 1.837°$ is a very close confirmation of the longhand solution.

Before leaving this example, let's look at another method for obtaining the polar form of the solution. The method appears in the bottom of Fig. 17.26. First **deg** is defined as shown, and then **arg** is picked up from the **Insert-*f(x)*-Insert Function-arg** sequence. Next **V1** is entered; the result will be in radian form but with a small black rectangle in the place where the units normally appear. Click on that black rectangle, and the bracket will appear and **deg** can be typed. When the equal sign is selected, the angle in degrees will appear.

Dependent Current Sources For dependent current sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.
2. Step 3 is modified as follows: Treat each dependent current source like an independent source when Kirchhoff's current law is applied to each defined node. However, once the equations are established, substitute the equation for the controlling quantity to ensure that the unknowns are limited solely to the chosen nodal voltages.
3. Step 4 is as before.

EXAMPLE 17.13 Write the nodal equations for the network of Fig. 17.27 having a dependent current source.

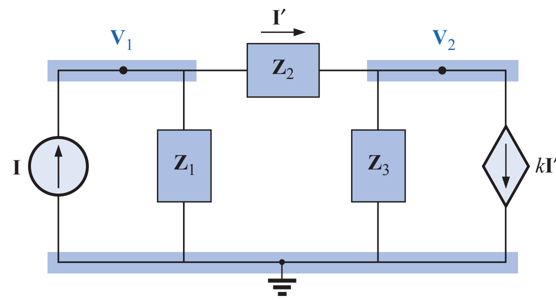


FIG. 17.27
Applying nodal analysis to a network with a current-controlled current source.

Solution:

Steps 1 and 2 are as defined in Fig. 17.27.

Step 3: At node $\mathbf{V}_1$,

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$

$$\frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_2} - \mathbf{I} = 0$$

and
$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}\right] - \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_2}\right] = \mathbf{I}$$

At node $\mathbf{V}_2$,

$$\mathbf{I}_2 + \mathbf{I}_3 + k\mathbf{I}' = 0$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_2}{\mathbf{Z}_3} + k\left[\frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_2}\right] = 0$$

and
$$\mathbf{V}_1\left[\frac{1-k}{\mathbf{Z}_2}\right] - \mathbf{V}_2\left[\frac{1-k}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}\right] = 0$$

resulting in two equations and two unknowns.

Independent Voltage Sources between Assigned Nodes For independent voltage sources between assigned nodes, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.
2. Step 3 is modified as follows: Treat each source between defined nodes as a short circuit (recall the *supernode* classification of Chapter 8), and write the nodal equations for each remaining independent node. Then relate the chosen nodal voltages to the independent voltage source to ensure that the unknowns of the final equations are limited solely to the nodal voltages.
3. Step 4 is as before.

EXAMPLE 17.14 Write the nodal equations for the network of Fig. 17.28 having an independent source between two assigned nodes.

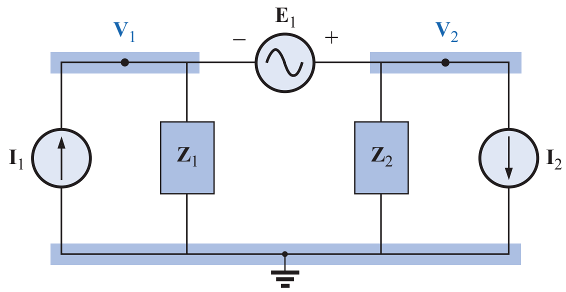


FIG. 17.28
Applying nodal analysis to a network with an independent voltage source between defined nodes.

Solution:

Steps 1 and 2 are defined in Fig. 17.28.

Step 3: Replacing the independent source **E** with a short-circuit equivalent results in a supernode that will generate the following equation when Kirchhoff's current law is applied to node $\mathbf{V}_1$:

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{\mathbf{V}_2}{\mathbf{Z}_2} + \mathbf{I}_2$$

with $$\mathbf{V}_2 - \mathbf{V}_1 = \mathbf{E}$$

and we have two equations and two unknowns.

Dependent Voltage Sources between Defined Nodes For dependent voltage sources between defined nodes, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent voltage sources.
2. Step 3 is modified as follows: The procedure is essentially the same as that applied for independent voltage sources, except now the dependent sources have to be defined in terms of the chosen nodal voltages to ensure that the final equations have only nodal voltages as their unknown quantities.
3. Step 4 is as before.

EXAMPLE 17.15 Write the nodal equations for the network of Fig. 17.29 having a dependent voltage source between two defined nodes.

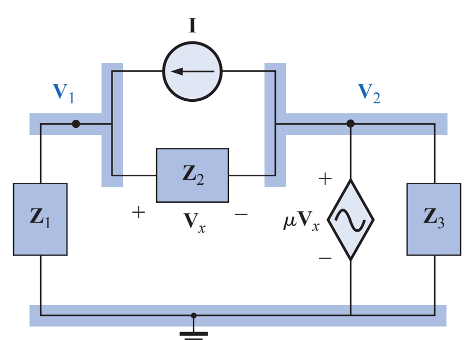


FIG. 17.29
Applying nodal analysis to a network with a voltage-controlled voltage source.

Solution:

Steps 1 and 2 are defined in Fig. 17.29.

Step 3: Replacing the dependent source $\mu\mathbf{V}_x$ with a short-circuit equivalent will result in the following equation when Kirchhoff's current law is applied at node $\mathbf{V}_1$:

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$

$$\frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{\mathbf{Z}_2} - \mathbf{I} = 0$$

and $$\mathbf{V}_2 = \mu\mathbf{V}_x = \mu[\mathbf{V}_1 - \mathbf{V}_2]$$

or $$\mathbf{V}_2 = \frac{\mu}{1+\mu}\mathbf{V}_1$$

resulting in two equations and two unknowns. Note that because the impedance $\mathbf{Z}_3$ is in parallel with a voltage source, it does not appear in the analysis. It will, however, affect the current through the dependent voltage source.

Format Approach

A close examination of Eqs. (17.1) and (17.2) in Example 17.12 will reveal that they are the same equations that would have been obtained using the format approach introduced in Chapter 8. Recall that the approach required that the voltage source first be converted to a current source, but the writing of the equations was quite direct and minimized any chances of an error due to a lost sign or missing term.

The sequence of steps required to apply the format approach is the following:

1. ***Choose a reference node and assign a subscripted voltage label to the (N − 1) remaining independent nodes of the network.***
2. ***The number of equations required for a complete solution is equal to the number of subscripted voltages (N − 1). Column 1 of each equation is formed by summing the admittances tied to the node of interest and multiplying the result by that subscripted nodal voltage.***
3. ***The mutual terms are always subtracted from the terms of the first column. It is possible to have more than one mutual term if the nodal voltage of interest has an element in common with more than one other nodal voltage. Each mutual term is the product of the mutual admittance and the other nodal voltage tied to that admittance.***
4. ***The column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is assigned a positive sign if it supplies current to a node, and a negative sign if it draws current from the node.***
5. ***Solve resulting simultaneous equations for the desired nodal voltages. The comments offered for mesh analysis regarding independent and dependent sources apply here also.***

EXAMPLE 17.16 Using the format approach to nodal analysis, find the voltage across the 4-Ω resistor in Fig. 17.30.

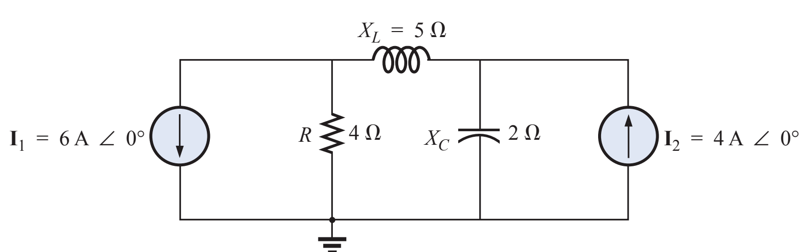


FIG. 17.30
Example 17.16.

Solution 1: Choosing nodes (Fig. 17.31) and writing the nodal equations, we have

$$\mathbf{Z}_1 = R = 4\,\Omega \qquad \mathbf{Z}_2 = j\,X_L = j\,5\,\Omega \qquad \mathbf{Z}_3 = -j\,X_C = -j\,2\,\Omega$$

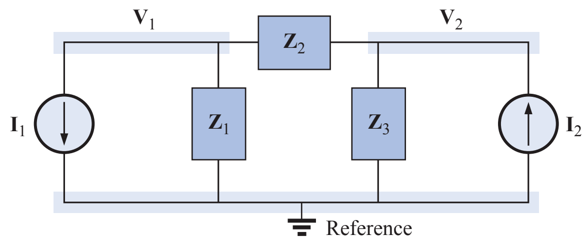


FIG. 17.31
Assigning the nodal voltages and subscripted impedances for the network of Fig. 17.30.

$$\begin{aligned}\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) - \mathbf{V}_2(\mathbf{Y}_2) &= -\mathbf{I}_1\\ \mathbf{V}_2(\mathbf{Y}_3 + \mathbf{Y}_2) - \mathbf{V}_1(\mathbf{Y}_2) &= +\mathbf{I}_2\end{aligned}$$

or

$$\begin{aligned}\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) - \mathbf{V}_2(\mathbf{Y}_2) \qquad\qquad &= -\mathbf{I}_1\\ -\mathbf{V}_1(\mathbf{Y}_2) \qquad\qquad + \mathbf{V}_2(\mathbf{Y}_3 + \mathbf{Y}_2) &= +\mathbf{I}_2\end{aligned}$$

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} \qquad \mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} \qquad \mathbf{Y}_3 = \frac{1}{\mathbf{Z}_3}$$

Using determinants yields

$$\begin{aligned}\mathbf{V}_1 &= \frac{\begin{vmatrix} -\mathbf{I}_1 & -\mathbf{Y}_2 \\ +\mathbf{I}_2 & \mathbf{Y}_3 + \mathbf{Y}_2 \end{vmatrix}}{\begin{vmatrix} \mathbf{Y}_1 + \mathbf{Y}_2 & -\mathbf{Y}_2 \\ -\mathbf{Y}_2 & \mathbf{Y}_3 + \mathbf{Y}_2 \end{vmatrix}}\\ &= \frac{-(\mathbf{Y}_3 + \mathbf{Y}_2)\mathbf{I}_1 + \mathbf{I}_2\mathbf{Y}_2}{(\mathbf{Y}_1 + \mathbf{Y}_2)(\mathbf{Y}_3 + \mathbf{Y}_2) - \mathbf{Y}_2^2}\\ &= \frac{-(\mathbf{Y}_3 + \mathbf{Y}_2)\mathbf{I}_1 + \mathbf{I}_2\mathbf{Y}_2}{\mathbf{Y}_1\mathbf{Y}_3 + \mathbf{Y}_2\mathbf{Y}_3 + \mathbf{Y}_1\mathbf{Y}_2}\end{aligned}$$

Substituting numerical values, we have

$$\begin{aligned}\mathbf{V}_1 &= \frac{-[(1/-j\,2\,\Omega) + (1/j\,5\,\Omega)]6\text{ A}\angle 0° + 4\text{ A}\angle 0°(1/j\,5\,\Omega)}{(1/4\,\Omega)(1/-j\,2\,\Omega) + (1/j\,5\,\Omega)(1/-j\,2\,\Omega) + (1/4\,\Omega)(1/j\,5\,\Omega)}\\ &= \frac{-(+j\,0.5 - j\,0.2)6\angle 0° + 4\angle 0°(-j\,0.2)}{(1/-j\,8) + (1/10) + (1/j\,20)}\\ &= \frac{(-0.3\angle 90°)(6\angle 0°) + (4\angle 0°)(0.2\angle -90°)}{j\,0.125 + 0.1 - j\,0.05}\\ &= \frac{-1.8\angle 90° + 0.8\angle -90°}{0.1 + j\,0.075}\\ &= \frac{2.6\text{ V}\angle -90°}{0.125\angle 36.87°}\end{aligned}$$

$$\mathbf{V}_1 = \mathbf{20.80\ V\angle -126.87°}$$

Mathcad Solution: For this example we will use the matrix format to find the nodal voltage $\mathbf{V}_1$. First the various parameters of the network are defined including the factor **deg** so that the phase angle will be displayed in degrees. Next the numerator is defined by **n,** and the **Matrix**

icon is selected from the **Matrix** toolbar. Within the **Insert Matrix** dialog box, the **Rows** and **Columns** are set as 2 followed by an **OK** to place the 2 × 2 matrix on the screen. The parameters are than entered as shown in Fig. 17.32 using a left click of the mouse to select the parameter to be entered. Once the numerator is set, the process is repeated to define the numerator. Finally the equation for **V1** is defined, and the result in rectangular form will appear when the equal sign is selected. The magnitude and the angle are then found in polar form as described in earlier sections of this chapter. The results are again a clear confirmation of the theoretical result.

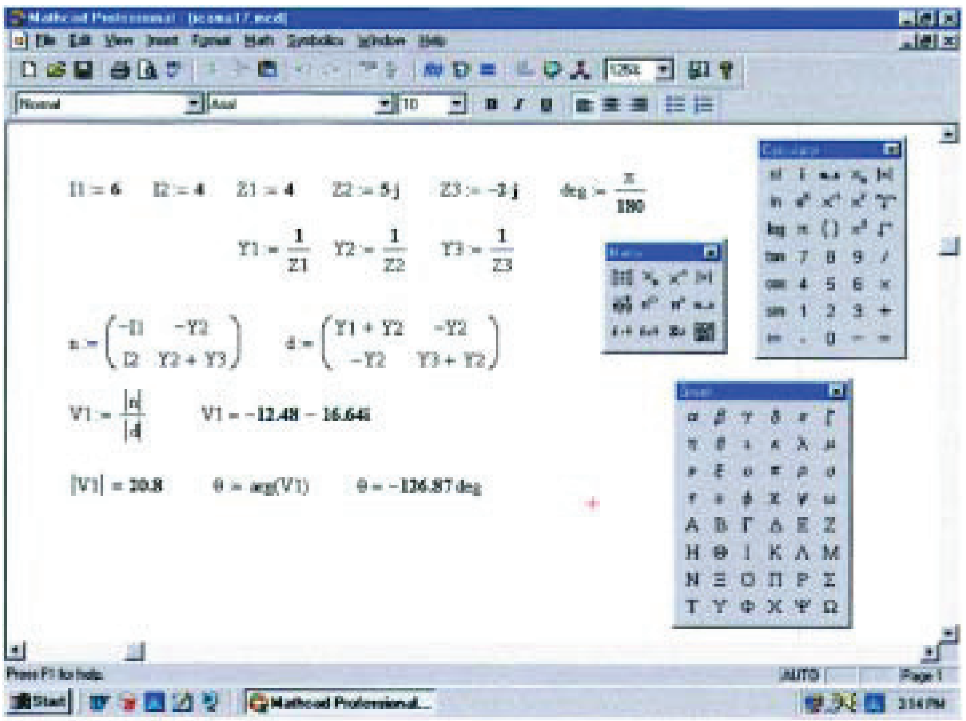


FIG. 17.32
Using Mathcad to verify the results of Example 17.16.

EXAMPLE 17.17 Using the format approach, write the nodal equations for the network of Fig. 17.33.

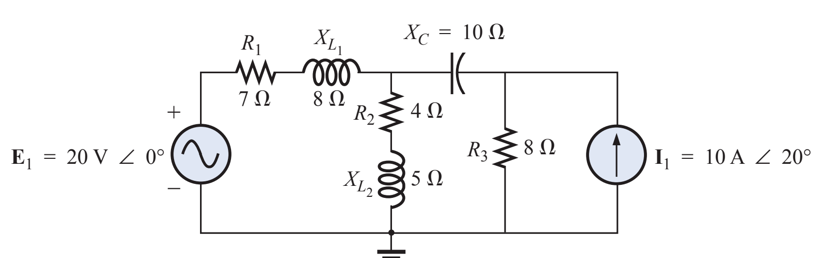


FIG. 17.33
Example 17.17.

Solution: The circuit is redrawn in Fig. 17.34, where

$$\begin{aligned}
\mathbf{Z}_1 &= R_1 + j X_{L_1} = 7\ \Omega + j\,8\ \Omega & \mathbf{E}_1 &= 20\ \mathrm{V}\ \angle 0^\circ \\
\mathbf{Z}_2 &= R_2 + j X_{L_2} = 4\ \Omega + j\,5\ \Omega & \mathbf{I}_1 &= 10\ \mathrm{A}\ \angle 20^\circ \\
\mathbf{Z}_3 &= -j X_C = -j\,10\ \Omega \\
\mathbf{Z}_4 &= R_3 = 8\ \Omega
\end{aligned}$$

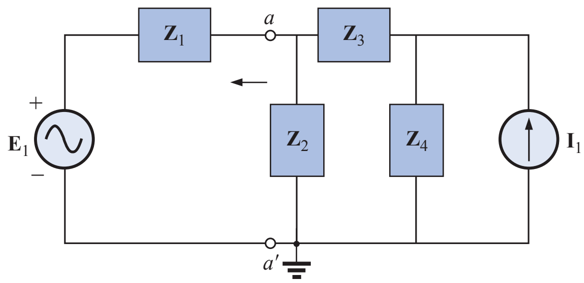


FIG. 17.34
Assigning the subscripted impedances for the network of Fig. 17.33.

Converting the voltage source to a current source and choosing nodes, we obtain Fig. 17.35. Note the "neat" appearance of the network using the subscripted impedances. Working directly with Fig. 17.33 would be more difficult and could produce errors.

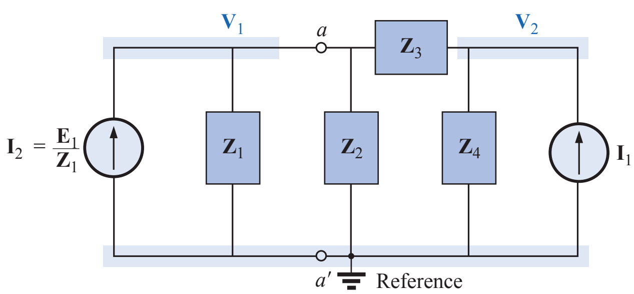


FIG. 17.35
Converting the voltage source of Fig. 17.34 to a current source and defining the nodal voltages.

Write the nodal equations:

$$\begin{aligned}
\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3) - \mathbf{V}_2(\mathbf{Y}_3) &= +\mathbf{I}_2 \\
\mathbf{V}_2(\mathbf{Y}_3 + \mathbf{Y}_4) - \mathbf{V}_1(\mathbf{Y}_3) &= +\mathbf{I}_1
\end{aligned}$$

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} \qquad \mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} \qquad \mathbf{Y}_3 = \frac{1}{\mathbf{Z}_3} \qquad \mathbf{Y}_4 = \frac{1}{\mathbf{Z}_4}$$

which are rewritten as

$$\begin{aligned}
\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3) - \mathbf{V}_2(\mathbf{Y}_3) \qquad\qquad &= +\mathbf{I}_2 \\
-\mathbf{V}_1(\mathbf{Y}_3) \qquad\qquad + \mathbf{V}_2(\mathbf{Y}_3 + \mathbf{Y}_4) &= +\mathbf{I}_1
\end{aligned}$$

EXAMPLE 17.18 Write the nodal equations for the network of Fig. 17.36. Do not solve.

Solution: Choose nodes (Fig. 17.37):

$$\begin{aligned}
\mathbf{Z}_1 &= R_1 & \mathbf{Z}_2 &= j X_{L_1} & \mathbf{Z}_3 &= R_2 - j X_{C_2} \\
\mathbf{Z}_4 &= -j X_{C_1} & \mathbf{Z}_5 &= R_3 & \mathbf{Z}_6 &= j X_{L_2}
\end{aligned}$$

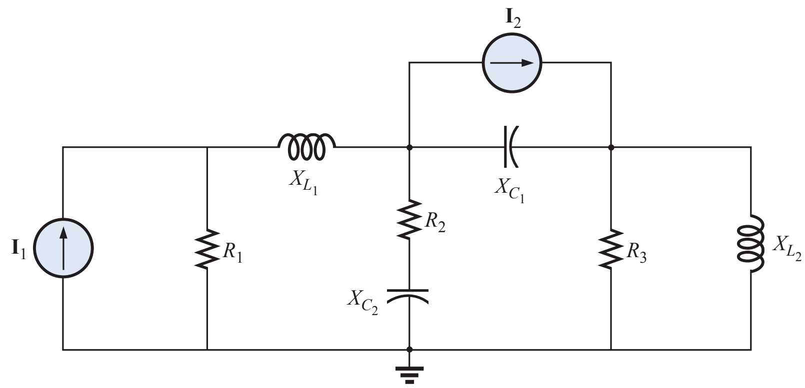


FIG. 17.36
Example 17.18.

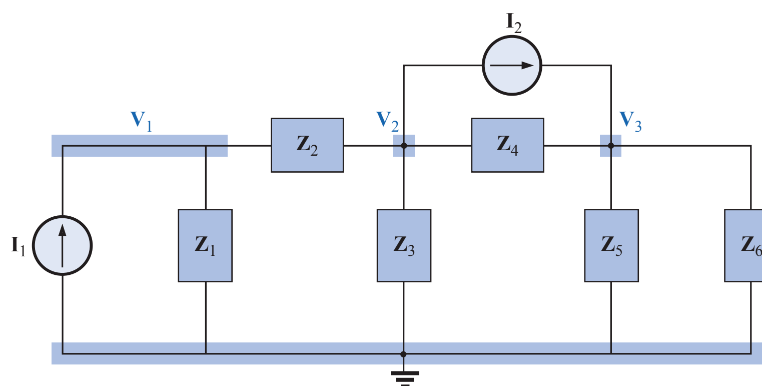


FIG. 17.37
Assigning the nodal voltages and subscripted impedances for the network of Fig. 17.36.

and write the nodal equations:

$$\begin{aligned}
\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) - \mathbf{V}_2(\mathbf{Y}_2) &= +\mathbf{I}_1 \\
\mathbf{V}_2(\mathbf{Y}_2 + \mathbf{Y}_3 + \mathbf{Y}_4) - \mathbf{V}_1(\mathbf{Y}_2) - \mathbf{V}_3(\mathbf{Y}_4) &= -\mathbf{I}_2 \\
\mathbf{V}_3(\mathbf{Y}_4 + \mathbf{Y}_5 + \mathbf{Y}_6) - \mathbf{V}_2(\mathbf{Y}_4) &= +\mathbf{I}_2
\end{aligned}$$

which are rewritten as

$$\begin{array}{llll}
\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) & - \mathbf{V}_2(\mathbf{Y}_2) & + 0 & = +\mathbf{I}_1 \\
-\mathbf{V}_1(\mathbf{Y}_2) & + \mathbf{V}_2(\mathbf{Y}_2 + \mathbf{Y}_3 + \mathbf{Y}_4) & - \mathbf{V}_3(\mathbf{Y}_4) & = -\mathbf{I}_2 \\
0 & - \mathbf{V}_2(\mathbf{Y}_4) & + \mathbf{V}_3(\mathbf{Y}_4 + \mathbf{Y}_5 + \mathbf{Y}_6) & = +\mathbf{I}_2
\end{array}$$

$$\mathbf{Y}_1 = \frac{1}{R_1} \qquad \mathbf{Y}_2 = \frac{1}{j\,X_{L_1}} \qquad \mathbf{Y}_3 = \frac{1}{R_2 - j\,X_{C_2}}$$

$$\mathbf{Y}_4 = \frac{1}{-j\,X_{C_1}} \qquad \mathbf{Y}_5 = \frac{1}{R_3} \qquad \mathbf{Y}_6 = \frac{1}{j\,X_{L_2}}$$

Note the symmetry about the diagonal for this example and those preceding it in this section.

EXAMPLE 17.19 Apply nodal analysis to the network of Fig. 17.38. Determine the voltage $\mathbf{V}_L$.

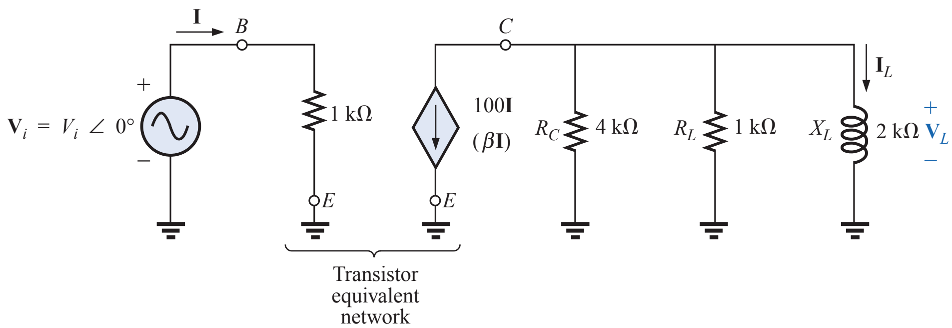


FIG. 17.38
Example 17.19.

Solution: In this case there is no need for a source conversion. The network is redrawn in Fig. 17.39 with the chosen nodal voltage and subscripted impedances.

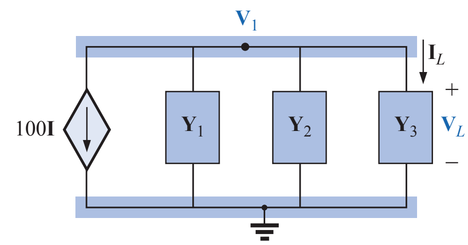


FIG. 17.39
Assigning the nodal voltage and subscripted impedances for the network of Fig. 17.38.

Apply the format approach:

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} = \frac{1}{4\text{ k}\Omega} = 0.25\text{ mS}\angle 0° = G_1\angle 0°$$

$$\mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} = \frac{1}{1\text{ k}\Omega} = 1\text{ mS}\angle 0° = G_2\angle 0°$$

$$\mathbf{Y}_3 = \frac{1}{\mathbf{Z}_3} = \frac{1}{2\text{ k}\Omega\angle 90°} = 0.5\text{ mS}\angle -90°$$

$$= -j\,0.5\text{ mS} = -j\,B_L$$

$$\mathbf{V}_1\text{:}\quad (\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3)\mathbf{V}_1 = -100\mathbf{I}$$

and
$$\mathbf{V}_1 = \frac{-100\mathbf{I}}{\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3}$$

$$= \frac{-100\mathbf{I}}{0.25\text{ mS} + 1\text{ mS} - j\,0.5\text{ mS}}$$

$$= \frac{-100\times 10^3\mathbf{I}}{1.25 - j\,0.5} = \frac{-100\times 10^3\mathbf{I}}{1.3463\angle -21.80°}$$

$$= -74.28\times 10^3\mathbf{I}\angle 21.80°$$

$$= -74.28\times 10^3\left(\frac{\mathbf{V}_i}{1\text{ k}\Omega}\right)\angle 21.80°$$

$$\mathbf{V}_1 = \mathbf{V}_L = \mathbf{-(74.28V_\mathit{i})\ V\angle 21.80°}$$

17.6 BRIDGE NETWORKS (ac)

The basic bridge configuration was discussed in some detail in Section 8.11 for dc networks. We now continue to examine **bridge networks** by considering those that have reactive components and a sinusoidal ac voltage or current applied.

We will first analyze various familiar forms of the bridge network using mesh analysis and nodal analysis (the format approach). The balance conditions will be investigated throughout the section.

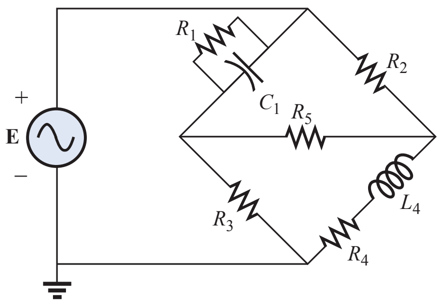


FIG. 17.40
Maxwell bridge.

Apply **mesh analysis** to the network of Fig. 17.40. The network is redrawn in Fig. 17.41, where

$$\mathbf{Z}_1 = \frac{1}{\mathbf{Y}_1} = \frac{1}{G_1 + j B_C} = \frac{G_1}{G_1^2 + B_C^2} - j\frac{B_C}{G_1^2 + B_C^2}$$

$$\mathbf{Z}_2 = R_2 \qquad \mathbf{Z}_3 = R_3 \qquad \mathbf{Z}_4 = R_4 + j X_L \qquad \mathbf{Z}_5 = R_5$$

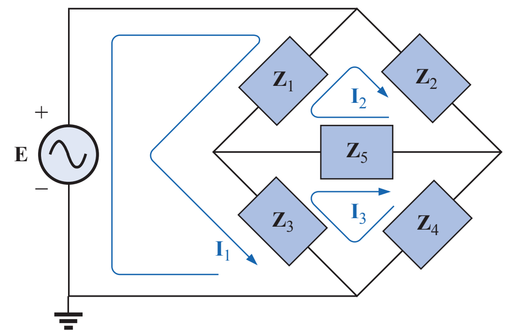


FIG. 17.41
Assigning the mesh currents and subscripted impedances for the network of Fig. 17.40.

Applying the format approach:

$$\begin{aligned}
(\mathbf{Z}_1 + \mathbf{Z}_3)\mathbf{I}_1 - (\mathbf{Z}_1)\mathbf{I}_2 - (\mathbf{Z}_3)\mathbf{I}_3 &= \mathbf{E}\\
(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5)\mathbf{I}_2 - (\mathbf{Z}_1)\mathbf{I}_1 - (\mathbf{Z}_5)\mathbf{I}_3 &= 0\\
(\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5)\mathbf{I}_3 - (\mathbf{Z}_3)\mathbf{I}_1 - (\mathbf{Z}_5)\mathbf{I}_2 &= 0
\end{aligned}$$

which are rewritten as

$$\begin{aligned}
\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_3) - \mathbf{I}_2\mathbf{Z}_1 \qquad\qquad\qquad\quad - \mathbf{I}_3\mathbf{Z}_3 \qquad\qquad &= \mathbf{E}\\
-\mathbf{I}_1\mathbf{Z}_1 \qquad + \mathbf{I}_2(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5) - \mathbf{I}_3\mathbf{Z}_5 \qquad\qquad &= 0\\
-\mathbf{I}_1\mathbf{Z}_3 \qquad - \mathbf{I}_2\mathbf{Z}_5 \qquad\qquad\quad + \mathbf{I}_3(\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) &= 0
\end{aligned}$$

Note the symmetry about the diagonal of the above equations. For balance, $\mathbf{I}_{\mathbf{Z}_5} = 0$ A, and

$$\mathbf{I}_{\mathbf{Z}_5} = \mathbf{I}_2 - \mathbf{I}_3 = 0$$

From the above equations,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_3 & \mathbf{E} & -\mathbf{Z}_3 \\ -\mathbf{Z}_1 & 0 & -\mathbf{Z}_5 \\ -\mathbf{Z}_3 & 0 & (\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_3 & -\mathbf{Z}_1 & -\mathbf{Z}_3 \\ -\mathbf{Z}_1 & (\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5) & -\mathbf{Z}_5 \\ -\mathbf{Z}_3 & -\mathbf{Z}_5 & (\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) \end{vmatrix}}$$

$$= \frac{\mathbf{E}(\mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_4 + \mathbf{Z}_1\mathbf{Z}_5 + \mathbf{Z}_3\mathbf{Z}_5)}{\Delta}$$

where Δ signifies the determinant of the denominator (or coefficients). Similarly,

$$\mathbf{I}_3 = \frac{\mathbf{E}(\mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_5 + \mathbf{Z}_3\mathbf{Z}_5)}{\Delta}$$

and $$\mathbf{I}_{\mathbf{Z}_5} = \mathbf{I}_2 - \mathbf{I}_3 = \frac{\mathbf{E}(\mathbf{Z}_1\mathbf{Z}_4 - \mathbf{Z}_3\mathbf{Z}_2)}{\Delta}$$

For $\mathbf{I}_{\mathbf{Z}_5} = 0$, the following must be satisfied (for a finite Δ not equal to zero):

$$\boxed{\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_3\mathbf{Z}_2} \qquad \mathbf{I}_{\mathbf{Z}_5} = 0 \qquad \textbf{(17.3)}$$

This condition will be analyzed in greater depth later in this section.

Applying **nodal analysis** to the network of Fig. 17.42 will result in the configuration of Fig. 17.43, where

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} = \frac{1}{R_1 - j X_C} \qquad \mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} = \frac{1}{R_2}$$

$$\mathbf{Y}_3 = \frac{1}{\mathbf{Z}_3} = \frac{1}{R_3} \qquad \mathbf{Y}_4 = \frac{1}{\mathbf{Z}_4} = \frac{1}{R_4 + j X_L} \qquad \mathbf{Y}_5 = \frac{1}{R_5}$$

and

$$\begin{aligned}(\mathbf{Y}_1 + \mathbf{Y}_2)\mathbf{V}_1 - (\mathbf{Y}_1)\mathbf{V}_2 - (\mathbf{Y}_2)\mathbf{V}_3 &= \mathbf{I}\\ (\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5)\mathbf{V}_2 - (\mathbf{Y}_1)\mathbf{V}_1 - (\mathbf{Y}_5)\mathbf{V}_3 &= 0\\ (\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5)\mathbf{V}_3 - (\mathbf{Y}_2)\mathbf{V}_1 - (\mathbf{Y}_5)\mathbf{V}_2 &= 0\end{aligned}$$

which are rewritten as

$$\begin{array}{rrrl}\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) & - \mathbf{V}_2\mathbf{Y}_1 & - \mathbf{V}_3\mathbf{Y}_2 & = \mathbf{I}\\ -\mathbf{V}_1\mathbf{Y}_1 & + \mathbf{V}_2(\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5) & - \mathbf{V}_3\mathbf{Y}_5 & = 0\\ -\mathbf{V}_1\mathbf{Y}_2 & - \mathbf{V}_2\mathbf{Y}_5 & + \mathbf{V}_3(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5) & = 0\end{array}$$

Again, note the symmetry about the diagonal axis. For balance, $\mathbf{V}_{\mathbf{Z}_5} = 0$ V, and

$$\mathbf{V}_{\mathbf{Z}_5} = \mathbf{V}_2 - \mathbf{V}_3 = 0$$

From the above equations,

$$\mathbf{V}_2 = \frac{\begin{vmatrix}\mathbf{Y}_1 + \mathbf{Y}_2 & \mathbf{I} & -\mathbf{Y}_2\\ -\mathbf{Y}_1 & 0 & -\mathbf{Y}_5\\ -\mathbf{Y}_2 & 0 & (\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5)\end{vmatrix}}{\begin{vmatrix}\mathbf{Y}_1 + \mathbf{Y}_2 & -\mathbf{Y}_1 & -\mathbf{Y}_2\\ -\mathbf{Y}_1 & (\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5) & -\mathbf{Y}_5\\ -\mathbf{Y}_2 & -\mathbf{Y}_5 & (\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5)\end{vmatrix}}$$

$$= \frac{\mathbf{I}(\mathbf{Y}_1\mathbf{Y}_3 + \mathbf{Y}_1\mathbf{Y}_4 + \mathbf{Y}_1\mathbf{Y}_5 + \mathbf{Y}_3\mathbf{Y}_5)}{\Delta}$$

Similarly,

$$\mathbf{V}_3 = \frac{\mathbf{I}(\mathbf{Y}_1\mathbf{Y}_3 + \mathbf{Y}_3\mathbf{Y}_2 + \mathbf{Y}_1\mathbf{Y}_5 + \mathbf{Y}_3\mathbf{Y}_5)}{\Delta}$$

Note the similarities between the above equations and those obtained for mesh analysis. Then

$$\mathbf{V}_{\mathbf{Z}_5} = \mathbf{V}_2 - \mathbf{V}_3 = \frac{\mathbf{I}(\mathbf{Y}_1\mathbf{Y}_4 - \mathbf{Y}_3\mathbf{Y}_2)}{\Delta}$$

For $\mathbf{V}_{\mathbf{Z}_5} = 0$, the following must be satisfied for a finite Δ not equal to zero:

$$\boxed{\mathbf{Y}_1\mathbf{Y}_4 = \mathbf{Y}_3\mathbf{Y}_2} \qquad \mathbf{V}_{\mathbf{Z}_5} = 0 \qquad \textbf{(17.4)}$$

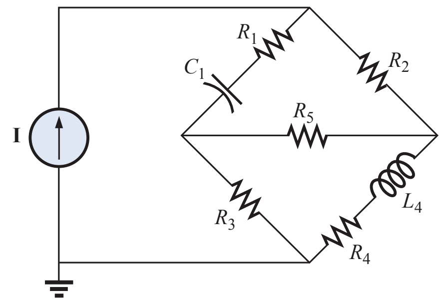


FIG. 17.42
Hay bridge.

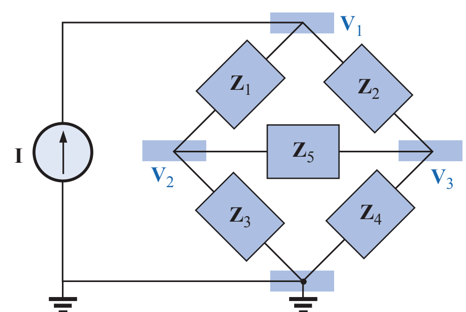


FIG. 17.43
Assigning the nodal voltages and subscripted impedances for the network of Fig. 17.42.

However, substituting $\mathbf{Y}_1 = 1/\mathbf{Z}_1$, $\mathbf{Y}_2 = 1/\mathbf{Z}_2$, $\mathbf{Y}_3 = 1/\mathbf{Z}_3$, and $\mathbf{Y}_4 = 1/\mathbf{Z}_4$, we have

$$\frac{1}{\mathbf{Z}_1\mathbf{Z}_4} = \frac{1}{\mathbf{Z}_3\mathbf{Z}_2}$$

or $$\boxed{\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_3\mathbf{Z}_2} \qquad \mathbf{V}_{\mathbf{Z}_5} = 0$$

corresponding with Eq. (17.3) obtained earlier.

Let us now investigate the balance criteria in more detail by considering the network of Fig. 17.44, where it is specified that **I** and **V** = 0.

Since **I** = 0,

$$\boxed{\mathbf{I}_1 = \mathbf{I}_3} \tag{17.5a}$$

and $$\boxed{\mathbf{I}_2 = \mathbf{I}_4} \tag{17.5b}$$

In addition, for **V** = 0,

$$\boxed{\mathbf{I}_1\mathbf{Z}_1 = \mathbf{I}_2\mathbf{Z}_2} \tag{17.5c}$$

and $$\boxed{\mathbf{I}_3\mathbf{Z}_3 = \mathbf{I}_4\mathbf{Z}_4} \tag{17.5d}$$

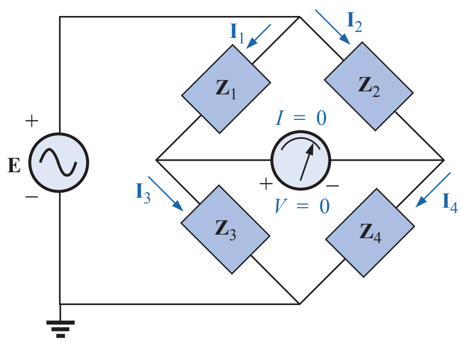


FIG. 17.44
Investigating the balance criteria for an ac bridge configuration.

Substituting the preceding current relations into Eq. (17.5d), we have

$$\mathbf{I}_1\mathbf{Z}_3 = \mathbf{I}_2\mathbf{Z}_4$$

and $$\mathbf{I}_2 = \frac{\mathbf{Z}_3}{\mathbf{Z}_4}\mathbf{I}_1$$

Substituting this relationship for $\mathbf{I}_2$ into Eq. (17.5c) yields

$$\mathbf{I}_1\mathbf{Z}_1 = \left(\frac{\mathbf{Z}_3}{\mathbf{Z}_4}\mathbf{I}_1\right)\mathbf{Z}_2$$

and $$\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_2\mathbf{Z}_3$$

as obtained earlier. Rearranging, we have

$$\boxed{\frac{\mathbf{Z}_1}{\mathbf{Z}_3} = \frac{\mathbf{Z}_2}{\mathbf{Z}_4}} \tag{17.6}$$

corresponding with Eq. (8.4) for dc resistive networks.

For the network of Fig. 17.42, which is referred to as a **Hay bridge** when $\mathbf{Z}_5$ is replaced by a sensitive galvanometer,

$$\mathbf{Z}_1 = R_1 - j\,X_C$$
$$\mathbf{Z}_2 = R_2$$
$$\mathbf{Z}_3 = R_3$$
$$\mathbf{Z}_4 = R_4 + j\,X_L$$

This particular network is used for measuring the resistance and inductance of coils in which the resistance is a small fraction of the reactance X_L.

Substitute into Eq. (17.6) in the following form:

$$\mathbf{Z}_2\mathbf{Z}_3 = \mathbf{Z}_4\mathbf{Z}_1$$
$$R_2R_3 = (R_4 + j\,X_L)(R_1 - j\,X_C)$$

or
$$R_2R_3 = R_1R_4 + j\,(R_1X_L - R_4X_C) + X_CX_L$$

so that

$$R_2R_3 + j\,0 = (R_1R_4 + X_CX_L) + j\,(R_1X_L - R_4X_C)$$

For the equations to be equal, *the real and imaginary parts must be equal.* Therefore, for a balanced Hay bridge,

$$\boxed{R_2R_3 = R_1R_4 + X_CX_L} \qquad \textbf{(17.7a)}$$

and
$$\boxed{0 = R_1X_L - R_4X_C} \qquad \textbf{(17.7b)}$$

or substituting $X_L = \omega L \quad \text{and} \quad X_C = \dfrac{1}{\omega C}$

we have $X_CX_L = \left(\dfrac{1}{\omega C}\right)(\omega L) = \dfrac{L}{C}$

and $R_2R_3 = R_1R_4 + \dfrac{L}{C}$

with $R_1\omega L = \dfrac{R_4}{\omega C}$

Solving for R_4 in the last equation yields

$$R_4 = \omega^2LCR_1$$

and substituting into the previous equation, we have

$$R_2R_3 = R_1(\omega^2LCR_1) + \frac{L}{C}$$

Multiply through by C and factor:

$$CR_2R_3 = L(\omega^2C^2R_1^2 + 1)$$

and
$$\boxed{L = \frac{CR_2R_3}{1 + \omega^2C^2R_1^2}} \qquad \textbf{(17.8a)}$$

With additional algebra this yields:

$$\boxed{R_4 = \frac{\omega^2C^2R_1R_2R_3}{1 + \omega^2C^2R_1^2}} \qquad \textbf{(17.8b)}$$

Equations (17.7) and (17.8) are the balance conditions for the Hay bridge. Note that each is frequency dependent. For different frequencies, the resistive and capacitive elements must vary for a particular coil to achieve balance. For a coil placed in the Hay bridge as shown in Fig. 17.42, the resistance and inductance of the coil can be determined by Eqs. (17.8a) and (17.8b) when balance is achieved.

The bridge of Fig. 17.40 is referred to as a **Maxwell bridge** when $\mathbf{Z}_5$ is replaced by a sensitive galvanometer. This setup is used for inductance measurements when the resistance of the coil is large enough not to require a Hay bridge.

Application of Eq. (17.6) in the form:

$$\mathbf{Z}_2\mathbf{Z}_3 = \mathbf{Z}_4\mathbf{Z}_1$$

and substituting

$$\mathbf{Z}_1 = R_1 \angle 0° \parallel X_{C_1} \angle -90° = \frac{(R_1 \angle 0°)(X_{C_1} \angle -90°)}{R_1 - j X_{C_1}}$$

$$= \frac{R_1X_{C_1} \angle -90°}{R_1 - j X_{C_1}} = \frac{-j R_1X_{C_1}}{R_1 - j X_{C_1}}$$

$$\mathbf{Z}_2 = R_2$$
$$\mathbf{Z}_3 = R_3$$

and $\quad \mathbf{Z}_4 = R_4 + j X_{L_4}$

we have $\quad (R_2)(R_3) = (R_4 + j X_{L_4})\left(\dfrac{-j R_1X_{C_1}}{R_1 - j X_{C_1}}\right)$

$$R_2R_3 = \frac{-j R_1R_4X_{C_1} + R_1X_{C_1}X_{L_4}}{R_1 - j X_{C_1}}$$

or $\quad (R_2R_3)(R_1 - j X_{C_1}) = R_1X_{C_1}X_{L_4} - j R_1R_4X_{C_1}$

and $\quad R_1R_2R_3 - j R_2R_3X_{C_1} = R_1X_{C_1}X_{L_4} - j R_1R_4X_{C_1}$

so that for balance

$$\cancel{R_1}R_2R_3 = \cancel{R_1}X_{C_1}X_{L_4}$$

$$R_2R_3 = \left(\frac{1}{\cancel{2\pi} fC_1}\right)(\cancel{2\pi} fL_4)$$

and
$$\boxed{L_4 = C_1R_2R_3} \tag{17.9}$$

and
$$R_2R_3\cancel{X_{C_1}} = R_1R_4\cancel{X_{C_1}}$$

so that
$$\boxed{R_4 = \frac{R_2R_3}{R_1}} \tag{17.10}$$

Note the absence of frequency in Eqs. (17.9) and (17.10).

One remaining popular bridge is the **capacitance comparison bridge** of Fig. 17.45. An unknown capacitance and its associated resistance can be determined using this bridge. Application of Eq. (17.6) will yield the following results:

$$\boxed{C_4 = C_3\frac{R_1}{R_2}} \tag{17.11}$$

$$\boxed{R_4 = \frac{R_2R_3}{R_1}} \tag{17.12}$$

The derivation of these equations will appear as a problem at the end of the chapter.

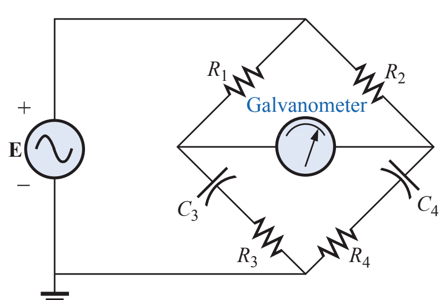


FIG. 17.45
Capacitance comparison bridge.

17.7 Δ-Y, Y-Δ CONVERSIONS

The Δ-Y, Y-Δ (or π-T, T-π as defined in Section 8.12) conversions for ac circuits will not be derived here since the development corresponds exactly with that for dc circuits. Taking the **Δ-Y configuration** shown in Fig. 17.46, we find the general equations for the impedances of the Y in terms of those for the Δ:

$$\boxed{\mathbf{Z}_1 = \frac{\mathbf{Z}_B\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}} \tag{17.13}$$

$$\boxed{\mathbf{Z}_2 = \frac{\mathbf{Z}_A\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}} \tag{17.14}$$

$$\boxed{\mathbf{Z}_3 = \frac{\mathbf{Z}_A\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}} \tag{17.15}$$

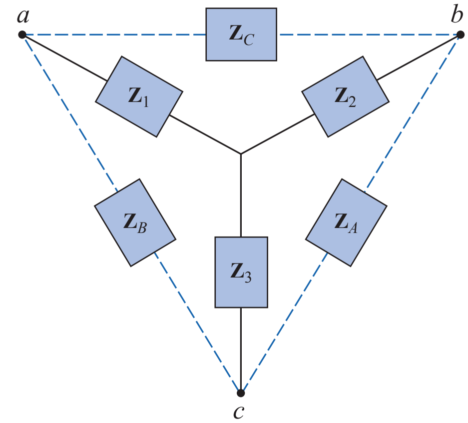


FIG. 17.46
Δ-Y configuration.

For the impedances of the Δ in terms of those for the Y, the equations are

$$\boxed{\mathbf{Z}_B = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2}} \tag{17.16}$$

$$\boxed{\mathbf{Z}_A = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1}} \tag{17.17}$$

$$\boxed{\mathbf{Z}_C = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_3}} \tag{17.18}$$

Note that each impedance of the Y is equal to the product of the impedances in the two closest branches of the Δ, divided by the sum of the impedances in the Δ.

Further, the value of each impedance of the Δ is equal to the sum of the possible product combinations of the impedances of the Y, divided by the impedances of the Y farthest from the impedance to be determined.

Drawn in different forms (Fig. 17.47), they are also referred to as the T and π configurations.

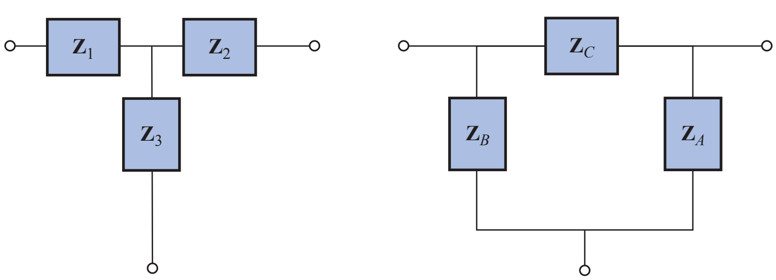


FIG. 17.47
The T and π configurations.

In the study of dc networks, we found that if all of the resistors of the Δ or Y were the same, the conversion from one to the other could be accomplished using the equation

$$R_\Delta = 3R_Y \quad \text{or} \quad R_Y = \frac{R_\Delta}{3}$$

For ac networks,

$$\boxed{\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \quad \text{or} \quad \mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}} \qquad \textbf{(17.19)}$$

Be careful when using this simplified form. It is not sufficient for all the impedances of the Δ or Y to be of the same magnitude: *The angle associated with each must also be the same.*

EXAMPLE 17.20 Find the total impedance $\mathbf{Z}_T$ of the network of Fig. 17.48.

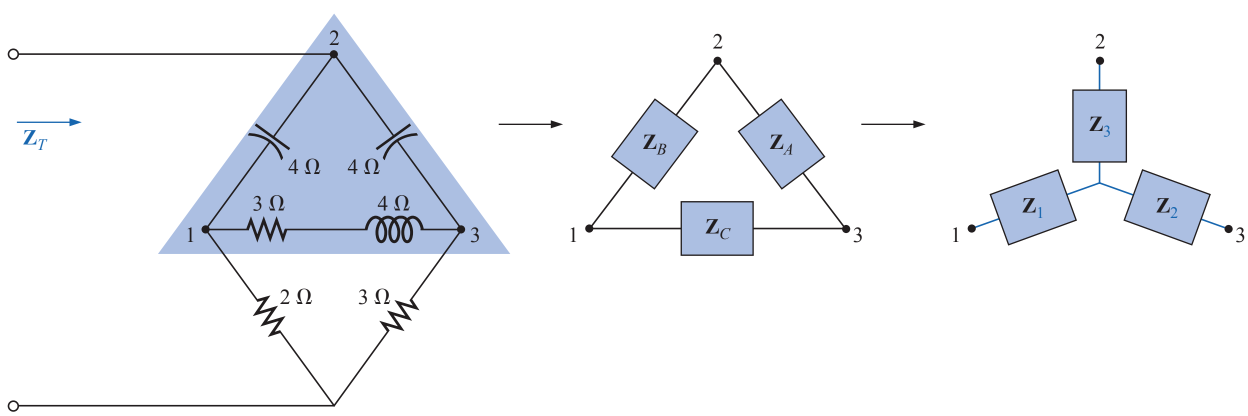


FIG. 17.48
Converting the upper Δ of a bridge configuration to a Y.

Solution:

$$\mathbf{Z}_B = -j\,4 \qquad \mathbf{Z}_A = -j\,4 \qquad \mathbf{Z}_C = 3 + j\,4$$

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_B\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} = \frac{(-j\,4\ \Omega)(3\ \Omega + j\,4\ \Omega)}{(-j\,4\ \Omega) + (-j\,4\ \Omega) + (3\ \Omega + j\,4\ \Omega)}$$

$$= \frac{(4\ \angle -90^\circ)(5\ \angle 53.13^\circ)}{3 - j\,4} = \frac{20\ \angle -36.87^\circ}{5\ \angle -53.13^\circ}$$

$$= 4\ \Omega\ \angle 16.13^\circ = 3.84\ \Omega + j\,1.11\ \Omega$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_A\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} = \frac{(-j\,4\ \Omega)(3\ \Omega + j\,4\ \Omega)}{5\ \Omega\ \angle -53.13^\circ}$$

$$= 4\ \Omega\ \angle 16.13^\circ = 3.84\ \Omega + j\,1.11\ \Omega$$

Recall from the study of dc circuits that if two branches of the Y or Δ were the same, the corresponding Δ or Y, respectively, would also have two similar branches. In this example, $\mathbf{Z}_A = \mathbf{Z}_B$. Therefore, $\mathbf{Z}_1 = \mathbf{Z}_2$, and

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_A\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} = \frac{(-j\,4\,\Omega)(-j\,4\,\Omega)}{5\,\Omega\,\angle -53.13^\circ}$$
$$= \frac{16\,\Omega\,\angle -180^\circ}{5\,\angle -53.13^\circ} = 3.2\,\Omega\,\angle -126.87^\circ = -1.92\,\Omega - j\,2.56\,\Omega$$

Replace the Δ by the Y (Fig. 17.49):

$$\mathbf{Z}_1 = 3.84\,\Omega + j\,1.11\,\Omega \qquad \mathbf{Z}_2 = 3.84\,\Omega + j\,1.11\,\Omega$$
$$\mathbf{Z}_3 = -1.92\,\Omega - j\,2.56\,\Omega \qquad \mathbf{Z}_4 = 2\,\Omega$$
$$\mathbf{Z}_5 = 3\,\Omega$$

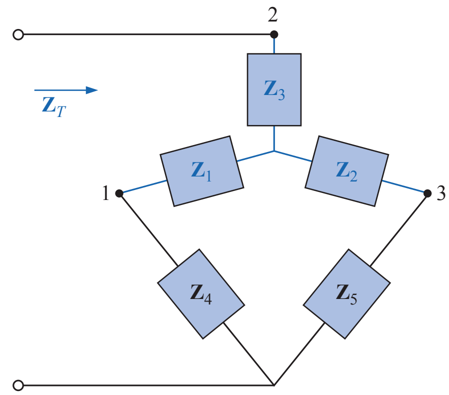


FIG. 17.49
The network of Fig. 17.48 following the substitution of the Y configuration.

Impedances $\mathbf{Z}_1$ and $\mathbf{Z}_4$ are in series:

$$\mathbf{Z}_{T_1} = \mathbf{Z}_1 + \mathbf{Z}_4 = 3.84\,\Omega + j\,1.11\,\Omega + 2\,\Omega = 5.84\,\Omega + j\,1.11\,\Omega$$
$$= 5.94\,\Omega\,\angle 10.76^\circ$$

Impedances $\mathbf{Z}_2$ and $\mathbf{Z}_5$ are in series:

$$\mathbf{Z}_{T_2} = \mathbf{Z}_2 + \mathbf{Z}_5 = 3.84\,\Omega + j\,1.11\,\Omega + 3\,\Omega = 6.84\,\Omega + j\,1.11\,\Omega$$
$$= 6.93\,\Omega\,\angle 9.22^\circ$$

Impedances $\mathbf{Z}_{T_1}$ and $\mathbf{Z}_{T_2}$ are in parallel:

$$\mathbf{Z}_{T_3} = \frac{\mathbf{Z}_{T_1}\mathbf{Z}_{T_2}}{\mathbf{Z}_{T_1} + \mathbf{Z}_{T_2}} = \frac{(5.94\,\Omega\,\angle 10.76^\circ)(6.93\,\Omega\,\angle 9.22^\circ)}{5.84\,\Omega + j\,1.11\,\Omega + 6.84\,\Omega + j\,1.11\,\Omega}$$
$$= \frac{41.16\,\Omega\,\angle 19.98^\circ}{12.68 + j\,2.22} = \frac{41.16\,\Omega\,\angle 19.98^\circ}{12.87\,\angle 9.93^\circ} = 3.198\,\Omega\,\angle 10.05^\circ$$
$$= 3.15\,\Omega + j\,0.56\,\Omega$$

Impedances $\mathbf{Z}_3$ and $\mathbf{Z}_{T_3}$ are in series. Therefore,

$$\mathbf{Z}_T = \mathbf{Z}_3 + \mathbf{Z}_{T_3} = -1.92\,\Omega - j\,2.56\,\Omega + 3.15\,\Omega + j\,0.56\,\Omega$$
$$= 1.23\,\Omega - j\,2.0\,\Omega = \mathbf{2.35\,\Omega\,\angle -58.41^\circ}$$

EXAMPLE 17.21 Using both the Δ-Y and Y-Δ transformations, find the total impedance $\mathbf{Z}_T$ for the network of Fig. 17.50.

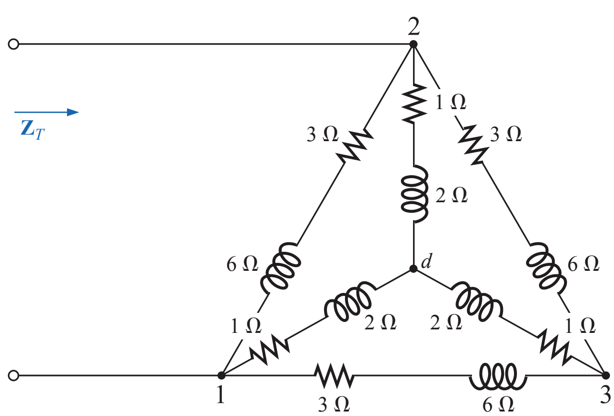


FIG. 17.50
Example 17.21.

Solution: *Using the Δ-Y transformation,* we obtain Fig. 17.51. In this case, since both systems are balanced (same impedance in each branch), the center point d' of the transformed Δ will be the same as point d of the original Y:

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = \frac{3\ \Omega + j\,6\ \Omega}{3} = 1\ \Omega + j\,2\ \Omega$$

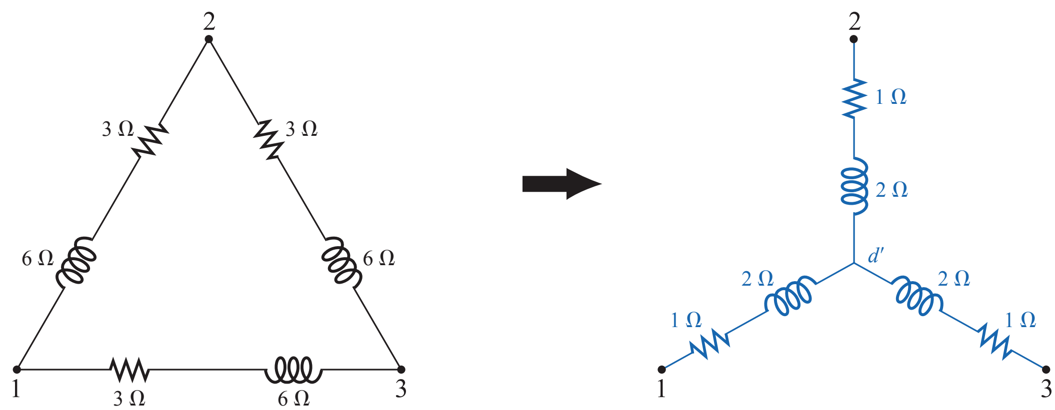


FIG. 17.51
Converting a Δ configuration to a Y configuration.

and (Fig. 17.52)

$$\mathbf{Z}_T = 2\left(\frac{1\ \Omega + j\,2\ \Omega}{2}\right) = \mathbf{1\ \Omega + j\,2\ \Omega}$$

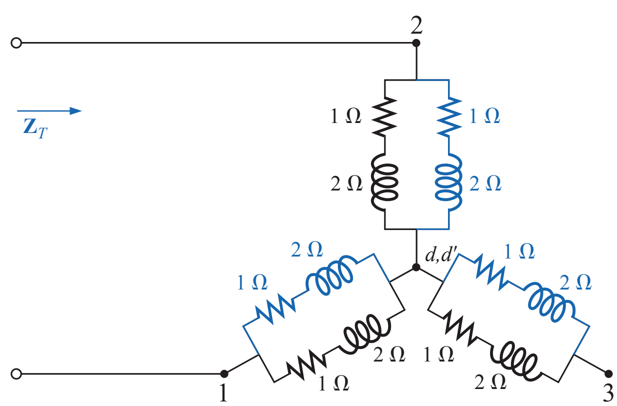


FIG. 17.52
Substituting the Y configuration of Fig. 17.51 into the network of Fig. 17.50.

Using the Y-Δ transformation (Fig. 17.53), we obtain

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y = 3(1\ \Omega + j\,2\ \Omega) = 3\ \Omega + j\,6\ \Omega$$

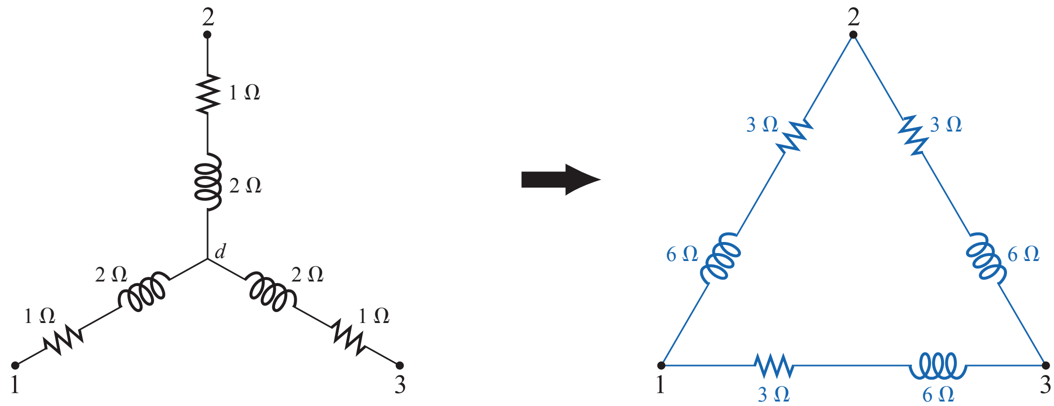


FIG. 17.53
Converting the Y configuration of Fig. 17.50 to a Δ.

Each resulting parallel combination in Fig. 17.54 will have the following impedance:

$$\mathbf{Z}' = \frac{3\ \Omega + j\,6\ \Omega}{2} = 1.5\ \Omega + j\,3\ \Omega$$

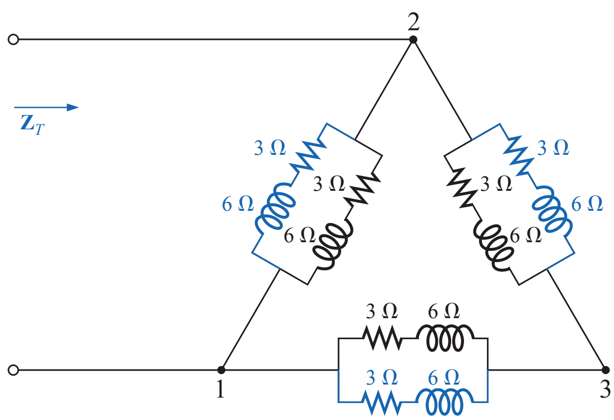


FIG. 17.54
Substituting the Δ configuration of Fig. 17.53 into the network of Fig. 17.50.

and
$$\mathbf{Z}_T = \frac{\mathbf{Z}'(2\mathbf{Z}')}{\mathbf{Z}' + 2\mathbf{Z}'} = \frac{2(\mathbf{Z}')^2}{3\mathbf{Z}'} = \frac{2\mathbf{Z}'}{3}$$
$$= \frac{2(1.5\ \Omega + j\,3\ \Omega)}{3} = \mathbf{1\ \Omega + j\,2\ \Omega}$$

which compares with the above result.

17.8 COMPUTER ANALYSIS

PSpice

Nodal Analysis The first application of PSpice will be to determine the nodal voltages for the network of Example 17.16 and compare solutions. The network will appear as shown in Fig. 17.55 using elements

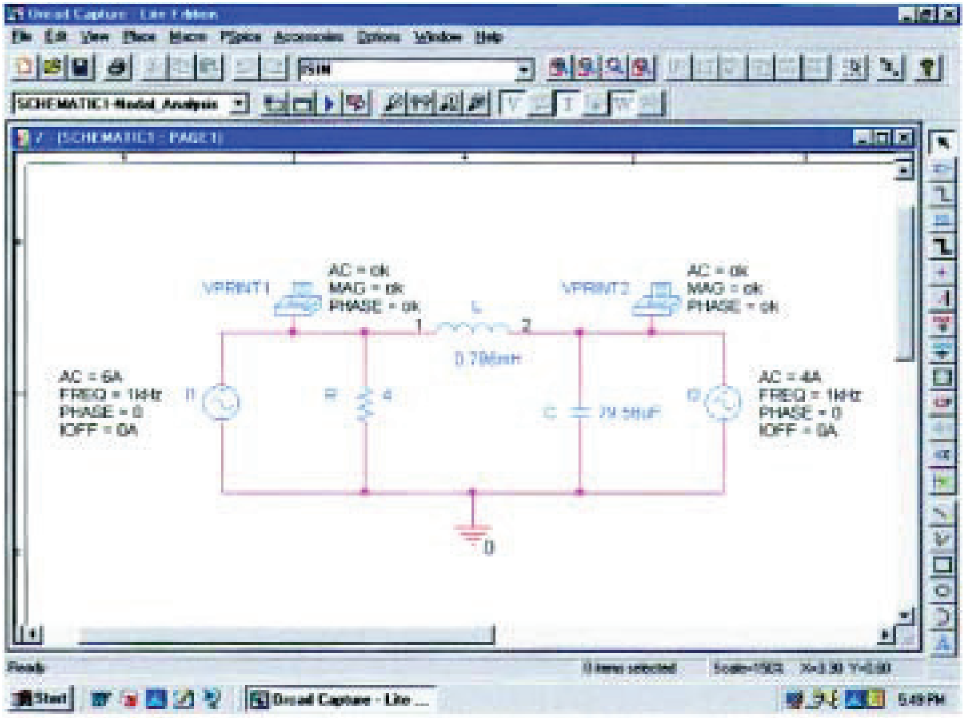


FIG. 17.55
Using PSpice to verify the results of Example 17.16.

that were determined from the reactance level at a frequency of 1 kHz. There is no need to continually use 1 kHz. Any frequency will do, but remember to use the chosen frequency to find the network components and when setting up the simulation.

For the current sources, **ISIN** was chosen so that the phase angle could be specified (even though it is 0°), although the symbol does not have the arrow used in the text material. The direction must be recognized as pointing from the + to − sign of the source. That requires that the sources $\mathbf{I}_1$ and $\mathbf{I}_2$ be set as shown in Fig. 17.55. The source $\mathbf{I}_2$ is reversed by using the **Mirror Vertically** option obtained by right-clicking the source symbol on the screen. Setting up the **ISIN** source is the same as that employed with the **VSIN** source. It can be found under the **SOURCE** library, and its attributes are the same as for the **VSIN** source. For each source, **IOFF** is set to 0 A, and the amplitude is the peak value of the source current. The frequency will be the same for each source. Then **VPRINT1** is selected from the **SPECIAL** library and placed to generate the desired nodal voltages. Finally the remaining elements are added to the network as shown in Fig. 17.55. For each source the symbol is double-clicked to generate the **Property Editor** dialog box. **AC** is set at the 6-A level for the $\mathbf{I}_1$ source and at 4 A for the $\mathbf{I}_2$ source, followed by **Display** and **Name and Value** for each. It will appear as shown in Fig. 17.55. A double-click on each **VPRINT1** option will also provide the **Property Editor,** so **OK** can be added under **AC, MAG,** and **PHASE.** For each quantity, **Display** is selected followed by **Name and Value** and **OK.** Then **Value** is selected and **VPRINT1** is displayed as **Value** only. Selecting **Apply** and leaving the dialog box will result in the listing next to each source in Fig. 17.55. For **VPRINT2** the listing on **Value** must first be changed from **VPRINT1** to **VPRINT2** before selecting **Display** and **Apply.**

Now the **New Simulation Profile** icon is selected and **ACNodal** entered as the **Name** followed by **Create.** In the **Simulation Settings** dialog box, **AC Sweep** is selected, and the **Start Frequency** and **End Frequency** are set at 1 kHz with 1 for the **Points/Decade.** Click **OK,** and select the **Run PSpice** icon; a **SCHEMATIC1** screen will result. Exiting **(X)** will bring us back to the **Orcad Capture** window. Selecting **PSpice** followed by **View Output File** will result in the display of Fig. 17.56, providing exactly the same results as obtained in Example 17.16 with $\mathbf{V}_1 = 20.8\text{ V}\angle -126.9°$. The other nodal voltage is 8.617 V $\angle -15.09°$.

```
 ** Profile: "SCHEMATIC1-Nodal_Analysis"  [ C:\PSpice\nodal_analysis-SCHEMATIC1-N
odal_Analysis.sim ]

 ****     AC ANALYSIS                          TEMPERATURE =   27.000 DEG C

******************************************************************************

  FREQ        VM(N01310)  VP(N01310)

   1.000E+03   2.080E+01  -1.269E+02

**** 07/16/01 17:40:22 ************** PSpice Lite (Mar 2000) *****************

 ** Profile: "SCHEMATIC1-Nodal_Analysis"  [ C:\PSpice\nodal_analysis-SCHEMATIC1-N
odal_Analysis.sim ]

 ****     AC ANALYSIS                          TEMPERATURE =   27.000 DEG C

******************************************************************************

  FREQ        VM(N01383)  VP(N01383)

   1.000E+03   8.617E+00  -1.509E+01
```

FIG. 17.56
Output file for the nodal voltages for the network of Fig. 17.55.

Current-Controlled Current Source (CCCS) Our interest will now turn to controlled sources in the PSpice environment. Controlled sources are not particularly difficult to apply once a few important elements of their use are understood. The network of Fig. 17.14 has a current-controlled current source in the center leg of the configuration. The magnitude of the current source is k times the current through resistor R_1, where k can be greater or less than 1. The resulting schematic, appearing in Fig. 17.57, seems quite complex in the area of the controlled source, but once the role of each component is understood, it will not be that difficult to understand. First, since it is the only new element in the schematic, let us concentrate on the controlled source. Current-controlled current sources (CCCS) are called up under the **ANALOG** library as **F** and appear as shown in the center of Fig. 17.57.

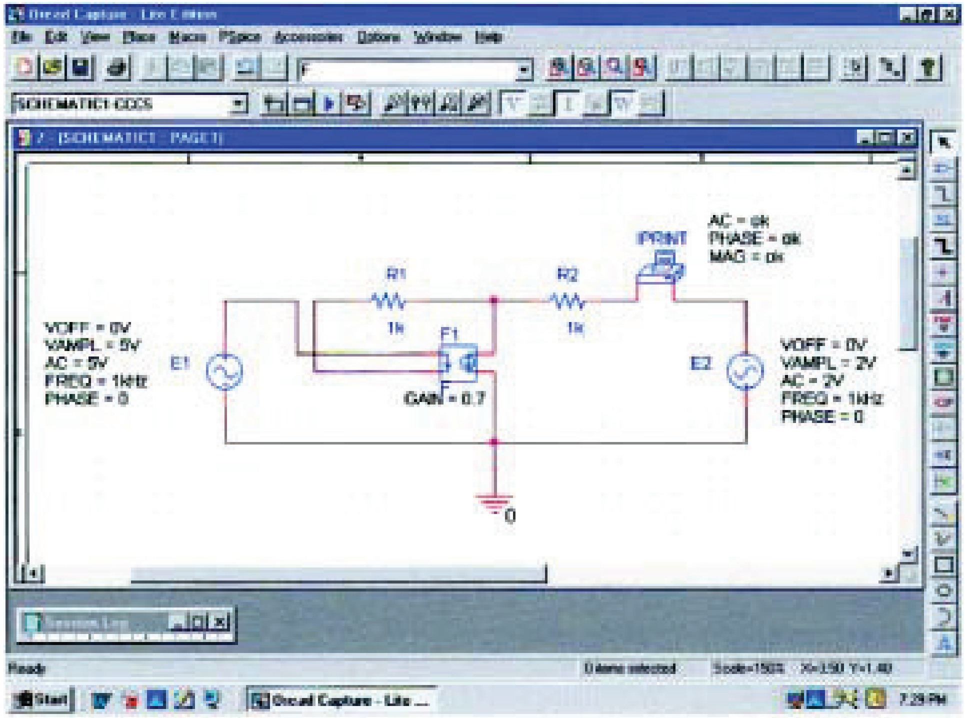


FIG. 17.57

Using PSpice to verify the results of Example 17.8.

Take special note of the direction of the current in each part of the symbol. In particular, note that the sensing current of **F** has the same direction as the defining controlling current in Fig. 17.14. In addition, note that the controlled current source also has the same direction as the source in Fig. 17.14. If we double-click on the CCCS symbol, the **Property Editor** dialog box will appear with the **GAIN** (k as described above) set at 1. In this example the gain must be set at **0.7,** so click on the region below the **GAIN** label and enter **0.7.** Then select **Display** followed by **Name and Value-OK.** Exit the **Property Editor,** and **GAIN = 0.7** will appear with the CCCS as shown in Fig. 17.57.

The other new component in this schematic is **IPRINT;** it can be found in the **SPECIAL** library. It is used to tell the program to list the current in the branch of interest in the output file. If you fail to tell the program which output data you would like, it will simply run through the simulation and list specific features of the network but will not provide any voltages or currents. In this case the current $\mathbf{I}_2$ through the resistor R_2 is desired. Double-clicking on the **IPRINT** component will result in the **Property Editor** dialog box with a number of elements that need to be defined—much like that for **VPRINT.** First enter **OK** beneath **AC** and follow with **Display-Name and Value-OK.** Repeat for **MAG** and **PHASE,** and then select **Apply** before leaving the dialog box. The **OK** is designed simply to tell the software program that these are the quantities that it is "ok" to generate and provide. The purpose of the **Apply** at the end of each visit to the **Property Editor** dialog box is to "apply" the changes made to the network under investigation. When you exit the **Property Editor,** the three chosen parameters will appear on the schematic with the **OK** directive. You may find that the labels will appear all over the **IPRINT** symbol. No problem—just click on each, and move to a more convenient location.

The remaining components of the network should be fairly familiar, but don't forget to **Mirror Vertically** the voltage source **E2.** In addition, do not forget to call up the **Property Editor** for each source and set the level of **AC, FREQ, VAMPL,** and **VOFF** and be sure that the **PHASE** is set on the default value of 0°. The value appears with each parameter in Fig. 17.57 for each source. Always be sure to select **Apply** before leaving the **Property Editor.** After placing all the components on the screen, you must connect them with a **Place wire** selection. Normally, this is pretty straightforward. However, with controlled sources there is often the need to cross over wires **without** making a connection. In general, when you're placing a wire over another wire and you don't want a connection to be made, click a spot on one side of the wire to be crossed to create the temporary red square. Then cross the wire, and make another click to establish another red square. If the connection is done properly, the crossed wire should not show a connection point (a small red dot). In this example the top of the controlling current was connected first from the **E1** source. Then a wire was connected from the lower end of the sensing current to the point where a 90° turn up the page was to be made. The wire was clicked in place at this point before crossing the original wire and clicked again before making the right turn to resistor R_1. You will not find a small red dot where the wires cross.

Now for the simulation. In the **Simulation Settings** dialog box, select **AC Sweep/Noise** with a **Start** and **End Frequency** of 1 kHz. There will be 1 **Point/Decade.** Click **OK,** and select the **Run Spice** key; a **SCHEMATIC1** will result that should be exited to obtain the **Orcad Capture** screen. Select **PSpice** followed by **View Output File,** and scroll down until you read **AC ANALYSIS** such as appearing in Fig. 17.58. The magnitude of the desired current is 1.615 mA with a phase angle of 0°, a perfect match with the theoretical analysis to follow. One would expect a phase angle of 0° since the network is composed solely of resistive elements.

The equations obtained earlier using the supermesh approach were

$$\mathbf{E} - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_2 + \mathbf{E}_2 = 0 \quad \text{or} \quad \mathbf{I}_1\mathbf{Z}_1 + \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1 + \mathbf{E}_2$$

```

 ** Profile: "SCHEMATIC1-CCCS"  [ C:\PSpice\cccs-SCHEMATIC1-CCCS.sim ]


 ****     AC ANALYSIS                        TEMPERATURE =   27.000 DEG C


******************************************************************************



  FREQ        IM(V_PRINT1)IP(V_PRINT1)


   1.000E+03   1.615E-03   0.000E+00

```

FIG. 17.58

The output file for the mesh current $\mathbf{I}_2$ *of Fig. 17.14.*

and $$k\mathbf{I} = k\mathbf{I}_1 = \mathbf{I}_1 - \mathbf{I}_2$$

resulting in $$\mathbf{I}_1 = \frac{\mathbf{I}_2}{1-k} = \frac{\mathbf{I}_2}{1-0.7} = \frac{\mathbf{I}_2}{0.3} = 3.333\mathbf{I}_2$$

so that $$\mathbf{I}_1(1\text{ k}\Omega) + \mathbf{I}_2(1\text{ k}\Omega) = 7\text{ V} \qquad \text{(from above)}$$

becomes $$(3.333\mathbf{I}_2)1\text{ k}\Omega + \mathbf{I}_2(1\text{ k}\Omega) = 7\text{ V}$$

or $$(4.333\text{ k}\Omega)\mathbf{I}_2 = 7\text{ V}$$

and $$\mathbf{I}_2 = \frac{7\text{ V}}{4.333\text{ k}\Omega} = \mathbf{1.615\ mA\ \angle 0°}$$

confirming the computer solution.

PROBLEMS

SECTION 17.2 Independent versus Dependent (Controlled) Sources

1. Discuss, in your own words, the difference between a controlled and an independent source.

SECTION 17.3 Source Conversions

2. Convert the voltage sources of Fig. 17.59 to current sources.

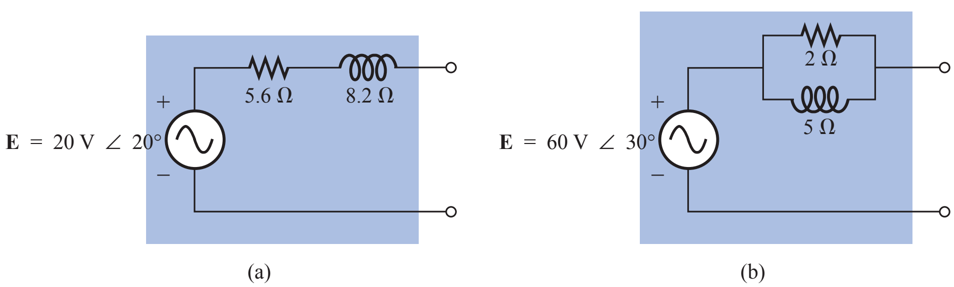


FIG. 17.59
Problem 2.

3. Convert the current sources of Fig. 17.60 to voltage sources.

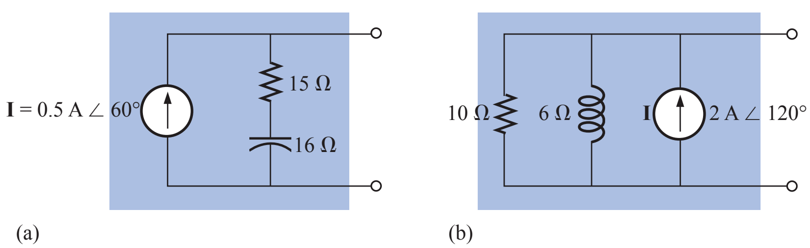


FIG. 17.60
Problem 3.

4. Convert the voltage source of Fig. 17.61(a) to a current source and the current source of Fig. 17.61(b) to a voltage source.

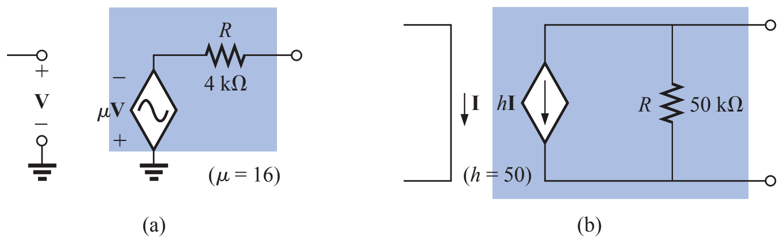


FIG. 17.61
Problem 4.

SECTION 17.4 Mesh Analysis

5. Write the mesh equations for the networks of Fig. 17.62. Determine the current through the resistor R.

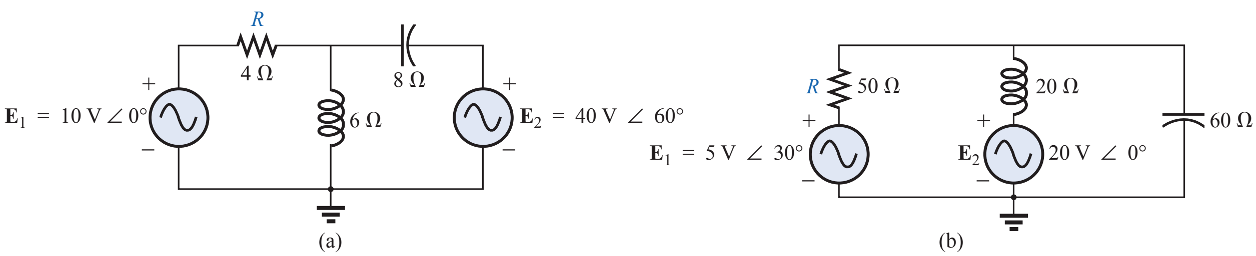


FIG. 17.62
Problems 5 and 34.

6. Write the mesh equations for the networks of Fig. 17.63. Determine the current through the resistor R_1.

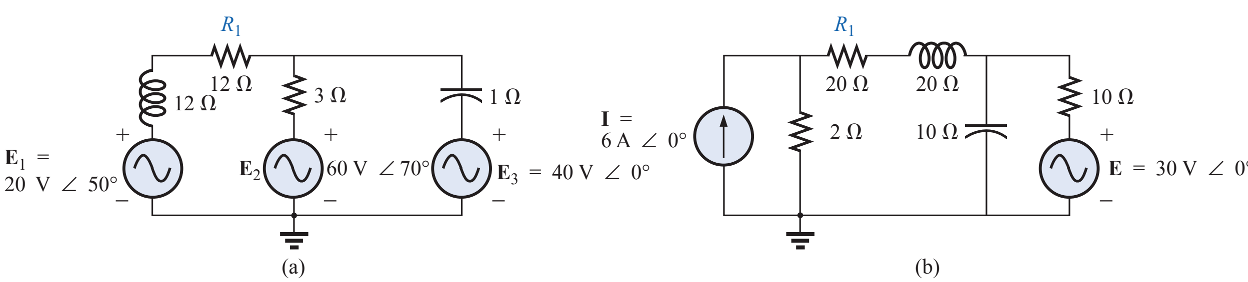


FIG. 17.63
Problems 6 and 16.

***7.** Write the mesh equations for the networks of Fig. 17.64. Determine the current through the resistor R_1.

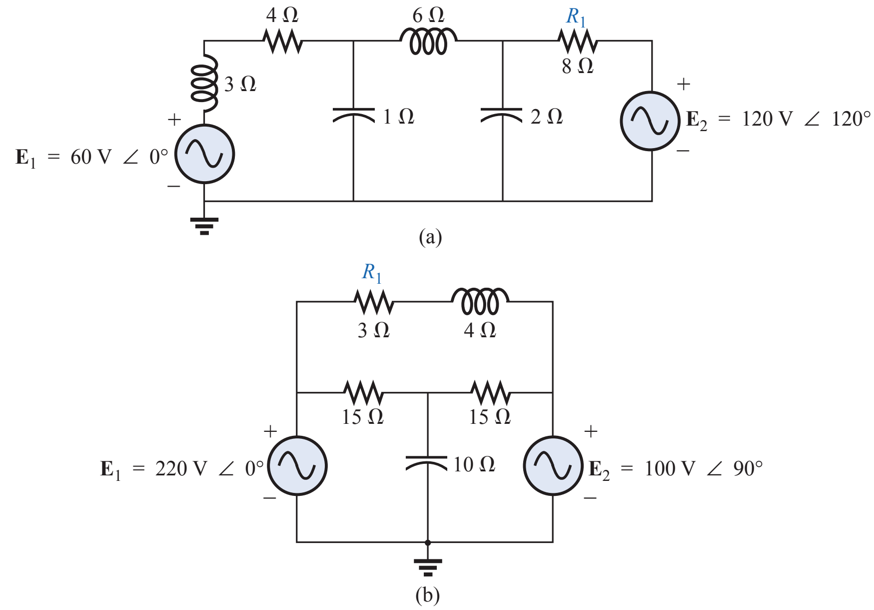


FIG. 17.64
Problems 7, 17, and 35.

***8.** Write the mesh equations for the networks of Fig. 17.65. Determine the current through the resistor R_1.

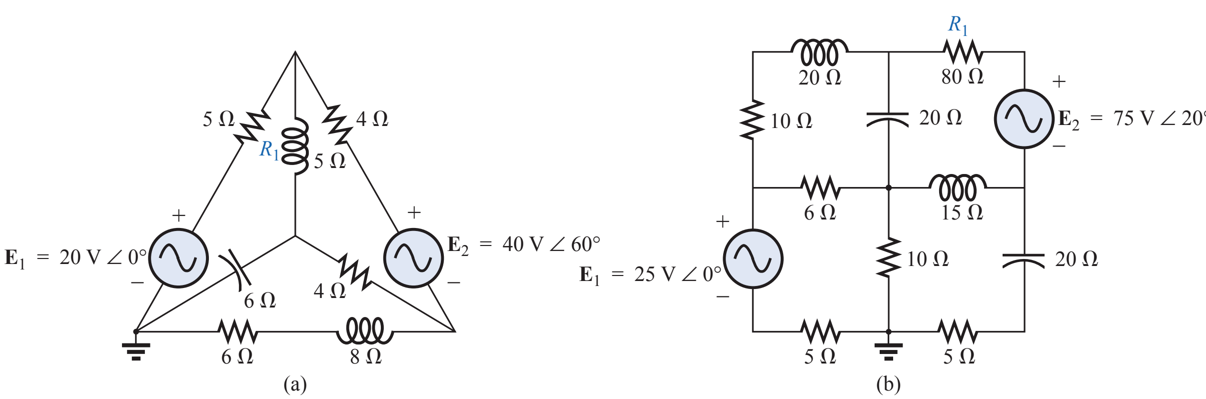


FIG. 17.65
Problems 8, 18, and 19.

9. Using mesh analysis, determine the current $\mathbf{I}_L$ (in terms of $\mathbf{V}$) for the network of Fig. 17.66.

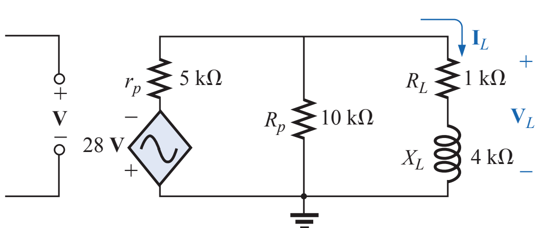


FIG. 17.66
Problem 9.

*10. Using mesh analysis, determine the current $\mathbf{I}_L$ (in terms of $\mathbf{I}$) for the network of Fig. 17.67.

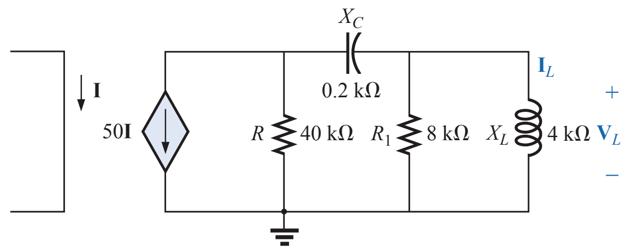


FIG. 17.67
Problem 10.

*11. Write the mesh equations for the network of Fig. 17.68, and determine the current through the 1-kΩ and 2-kΩ resistors.

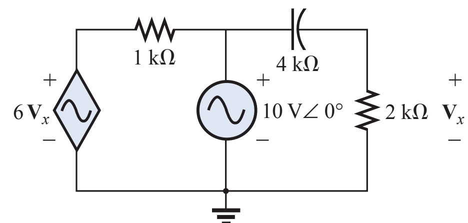


FIG. 17.68
Problems 11 and 36.

*12. Write the mesh equations for the network of Fig. 17.69, and determine the current through the 10-kΩ resistor.

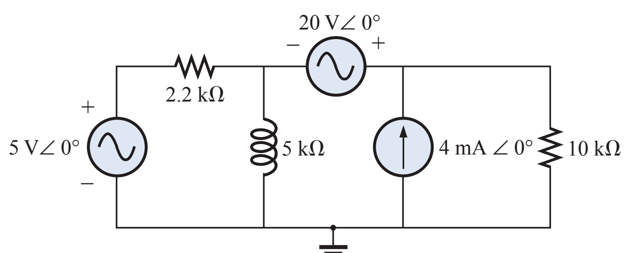


FIG. 17.69
Problems 12 and 37.

*13. Write the mesh equations for the network of Fig. 17.70, and determine the current through the inductive element.

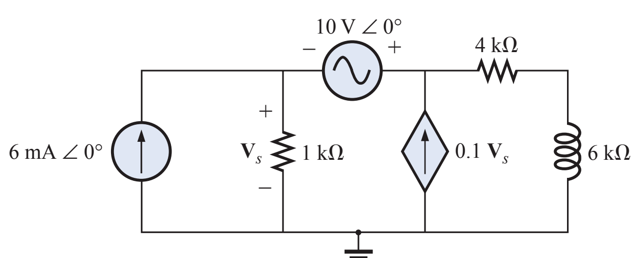


FIG. 17.70
Problems 13 and 38.

SECTION 17.5 Nodal Analysis

14. Determine the nodal voltages for the networks of Fig. 17.71.

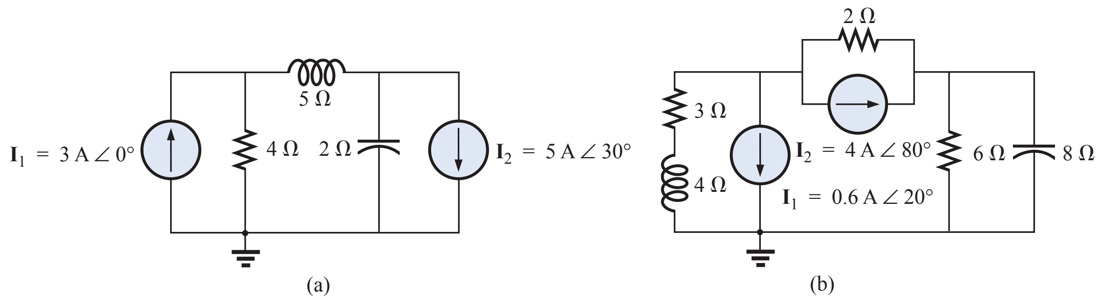


FIG. 17.71
Problems 14 and 39.

15. Determine the nodal voltages for the networks of Fig. 17.72.

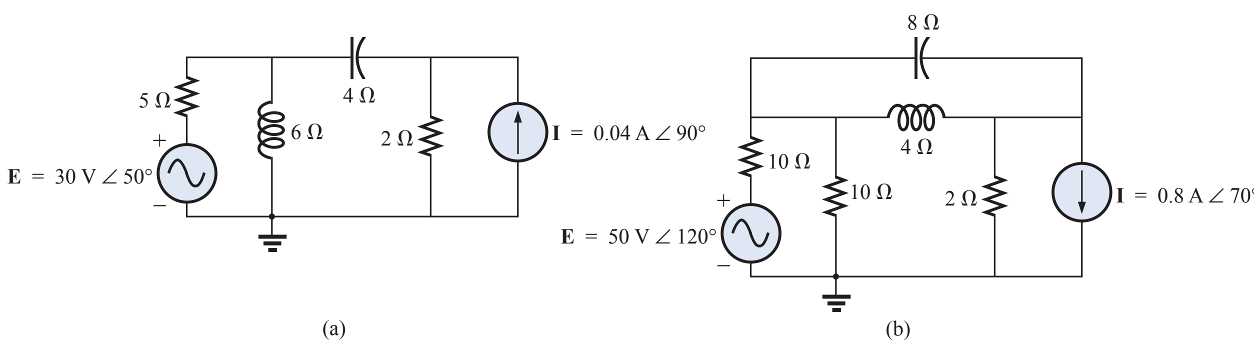


FIG. 17.72
Problem 15.

16. Determine the nodal voltages for the network of Fig. 17.63(b).

17. Determine the nodal voltages for the network of Fig. 17.64(b).

***18.** Determine the nodal voltages for the network of Fig. 17.65(a).

***19.** Determine the nodal voltages for the network of Fig. 17.65(b).

***20.** Determine the nodal voltages for the networks of Fig. 17.73.

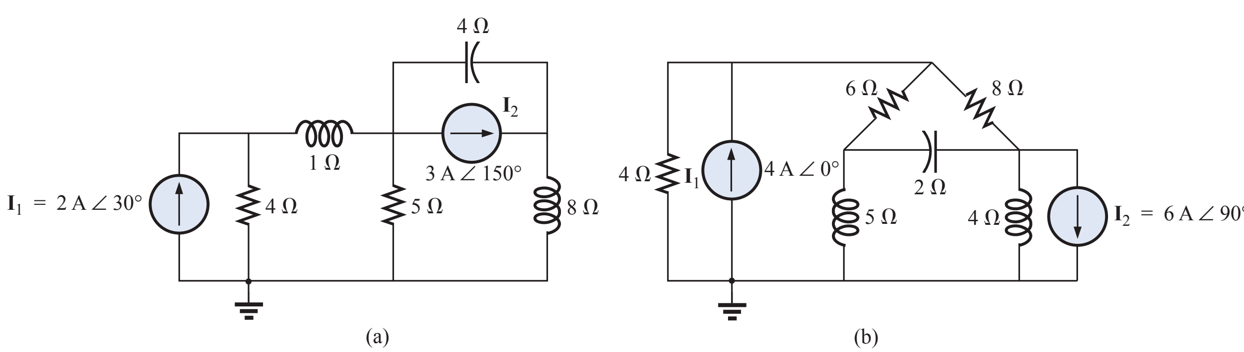


FIG. 17.73
Problem 20.

***21.** Write the nodal equations for the network of Fig. 17.74, and find the voltage across the 1-kΩ resistor.

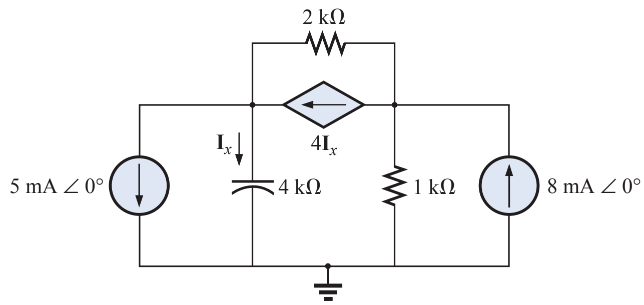


FIG. 17.74
Problems 21 and 40.

***22.** Write the nodal equations for the network of Fig. 17.75, and find the voltage across the capacitive element.

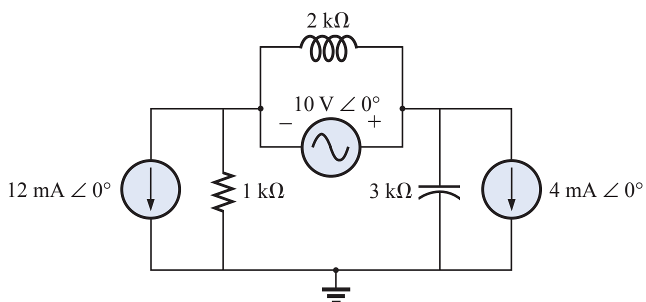


FIG. 17.75
Problems 22 and 41.

***23.** Write the nodal equations for the network of Fig. 17.76, and find the voltage across the 2-kΩ resistor.

***24.** Write the nodal equations for the network of Fig. 17.77, and find the voltage across the 2-kΩ resistor.

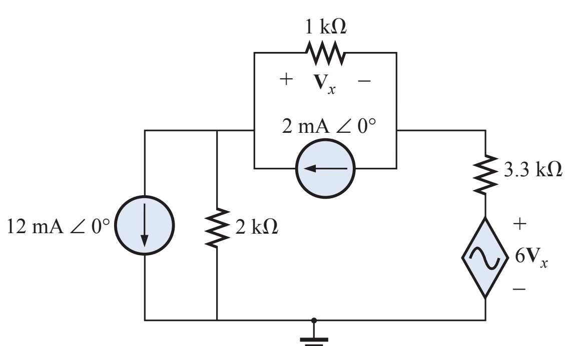


FIG. 17.76
Problems 23 and 42.

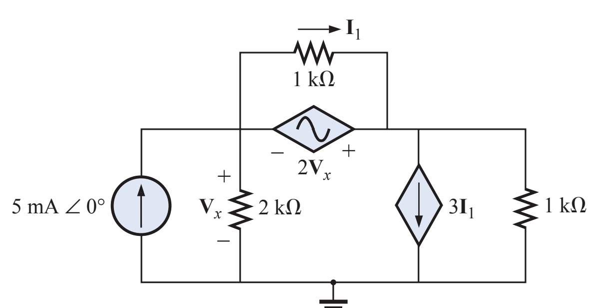


FIG. 17.77
Problems 24 and 43.

*25. For the network of Fig. 17.78, determine the voltage $\mathbf{V}_L$ in terms of the voltage $\mathbf{E}_i$.

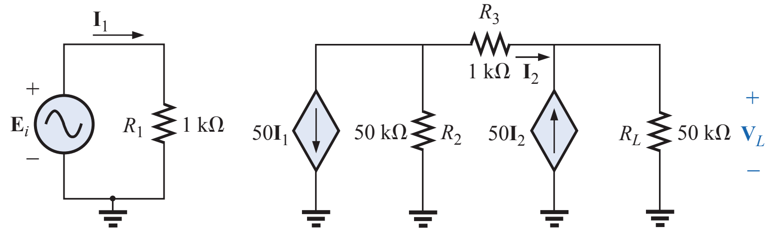


FIG. 17.78
Problem 25.

SECTION 17.6 Bridge Networks (ac)

26. For the bridge network of Fig. 17.79:
   a. Is the bridge balanced?
   b. Using mesh analysis, determine the current through the capacitive reactance.
   c. Using nodal analysis, determine the voltage across the capacitive reactance.

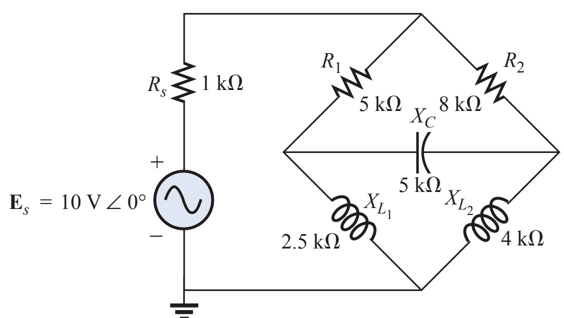


FIG. 17.79
Problem 26.

27. For the bridge network of Fig. 17.80:
   a. Is the bridge balanced?
   b. Using mesh analysis, determine the current through the capacitive reactance.
   c. Using nodal analysis, determine the voltage across the capacitive reactance.

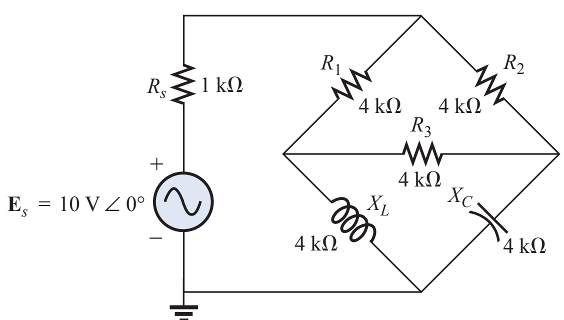


FIG. 17.80
Problem 27.

28. The Hay bridge of Fig. 17.81 is balanced. Using Eq. (17.3), determine the unknown inductance L_x and resistance R_x.

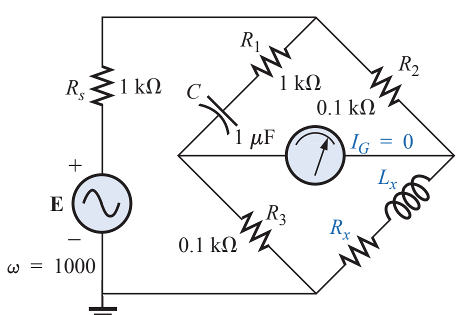


FIG. 17.81
Problem 28.

29. Determine whether the Maxwell bridge of Fig. 17.82 is balanced ($\omega = 1000$ rad/s).

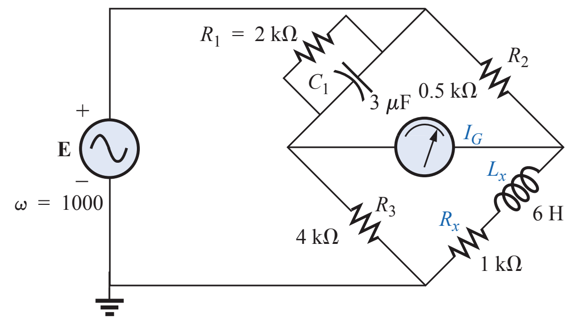


FIG. 17.82
Problem 29.

30. Derive the balance equations (17.11) and (17.12) for the capacitance comparison bridge.

31. Determine the balance equations for the inductance bridge of Fig. 17.83.

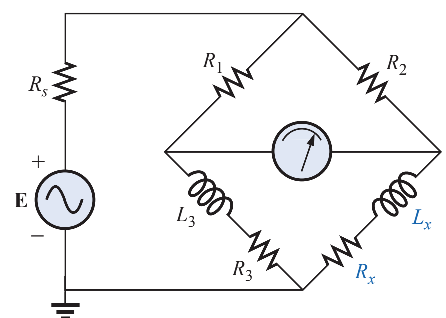


FIG. 17.83
Problem 31.

SECTION 17.7 Δ-Y, Y-Δ Conversions

32. Using the Δ-Y or Y-Δ conversion, determine the current **I** for the networks of Fig. 17.84.

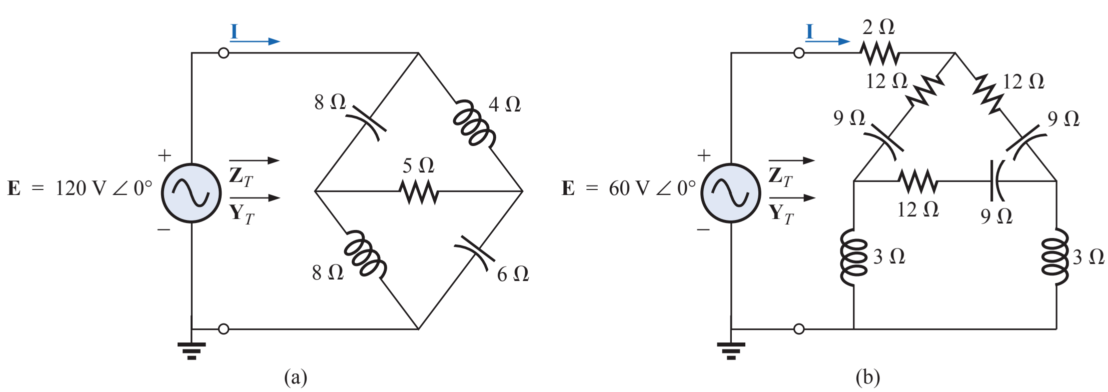


FIG. 17.84
Problem 32.

33. Using the Δ-Y or Y-Δ conversion, determine the current **I** for the networks of Fig. 17.85. (**E** = 100 V ∠0° in each case.)

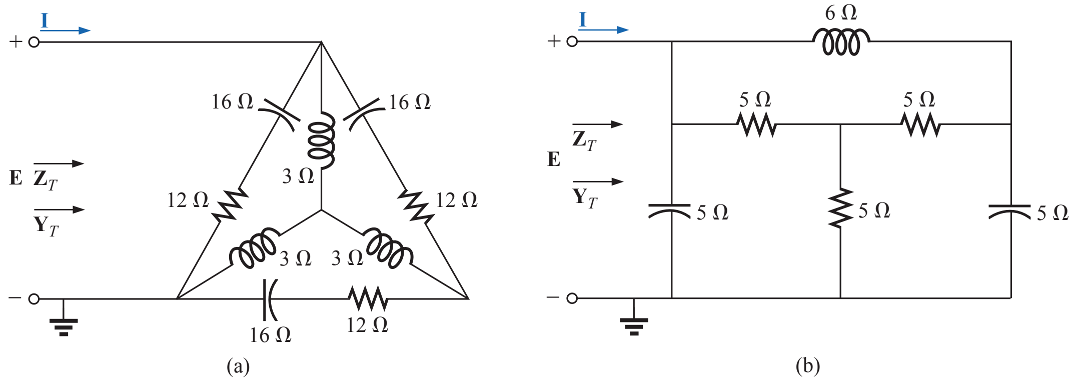


FIG. 17.85
Problem 33.

SECTION 17.8 Computer Analysis

PSpice or Electronics Workbench

34. Determine the mesh currents for the network of Fig. 17.62(a).

35. Determine the mesh currents for the network of Fig. 17.64(a).

***36.** Determine the mesh currents for the network of Fig. 17.68.

***37.** Determine the mesh currents for the network of Fig. 17.69.

***38.** Determine the mesh currents for the network of Fig. 17.70.

39. Determine the nodal voltages for the network of Fig. 17.71(b).

***40.** Determine the nodal voltages for the network of Fig. 17.74.

***41.** Determine the nodal voltages for the network of Fig. 17.75.

***42.** Determine the nodal voltages for the network of Fig. 17.76.

***43.** Determine the nodal voltages for the network of Fig. 17.77.

Programming Language (C++, QBASIC, Pascal, etc.)

44. Write a computer program that will provide a general solution for the network of Fig. 17.10. That is, given the reactance of each element and the parameters of the source voltages, generate a solution in phasor form for both mesh currents.

45. Repeat Problem 35 for the nodal voltages of Fig. 17.30.

46. Given a bridge composed of series impedances in each branch, write a program to test the balance condition as defined by Eq. (17.6).

GLOSSARY

Bridge network A network configuration having the appearance of a diamond in which no two branches are in series or parallel.

Capacitance comparison bridge A bridge configuration having a galvanometer in the bridge arm that is used to determine an unknown capacitance and associated resistance.

Delta (Δ) configuration A network configuration having the appearance of the capital Greek letter delta.

Dependent (controlled) source A source whose magnitude and/or phase angle is determined (controlled) by a current or voltage of the system in which it appears.

Hay bridge A bridge configuration used for measuring the resistance and inductance of coils in those cases where the resistance is a small fraction of the reactance of the coil.

Independent source A source whose magnitude is independent of the network to which it is applied. It displays its terminal characteristics even if completely isolated.

Maxwell bridge A bridge configuration used for inductance measurements when the resistance of the coil is large enough not to require a Hay bridge.

Mesh analysis A method through which the loop (or mesh) currents of a network can be determined. The branch currents of the network can then be determined directly from the loop currents.

Nodal analysis A method through which the nodal voltages of a network can be determined. The voltage across each element can then be determined through application of Kirchhoff's voltage law.

Source conversion The changing of a voltage source to a current source, or vice versa, which will result in the same terminal behavior of the source. In other words, the external network is unaware of the change in sources.

Wye (Y) configuration A network configuration having the appearance of the capital letter Y.

18

Network Theorems (ac)

18.1 INTRODUCTION

This chapter will parallel Chapter 9, which dealt with network theorems as applied to dc networks. It would be time well spent to review each theorem in Chapter 9 before beginning this chapter because many of the comments offered there will not be repeated.

Due to the need for developing confidence in the application of the various theorems to networks with controlled (dependent) sources, some sections have been divided into two parts: independent sources and dependent sources.

Theorems to be considered in detail include the superposition theorem, Thévenin's and Norton's theorems, and the maximum power theorem. The substitution and reciprocity theorems and Millman's theorem are not discussed in detail here because a review of Chapter 9 will enable you to apply them to sinusoidal ac networks with little difficulty.

18.2 SUPERPOSITION THEOREM

You will recall from Chapter 9 that the **superposition theorem** eliminated the need for solving simultaneous linear equations by considering the effects of each source independently. To consider the effects of each source, we had to remove the remaining sources. This was accomplished by setting voltage sources to zero (short-circuit representation) and current sources to zero (open-circuit representation). The current through, or voltage across, a portion of the network produced by each source was then added algebraically to find the total solution for the current or voltage.

The only variation in applying this method to ac networks with independent sources is that we will now be working with impedances and phasors instead of just resistors and real numbers.

The superposition theorem is not applicable to power effects in ac networks since we are still dealing with a nonlinear relationship. It can be applied to networks with sources of different frequencies only if the total response for *each* frequency is found independently and the results are expanded in a nonsinusoidal expression, as appearing in Chapter 25.

One of the most frequent applications of the superposition theorem is to electronic systems in which the dc and ac analyses are treated separately and the total solution is the sum of the two. It is an important application of the theorem because the impact of the reactive elements changes dramatically in response to the two types of independent sources. In addition, the dc analysis of an electronic system can often define important parameters for the ac analysis. Example 18.4 will demonstrate the impact of the applied source on the general configuration of the network.

We will first consider networks with only independent sources to provide a close association with the analysis of Chapter 9.

Independent Sources

EXAMPLE 18.1 Using the superposition theorem, find the current **I** through the 4-Ω reactance (X_{L_2}) of Fig. 18.1.

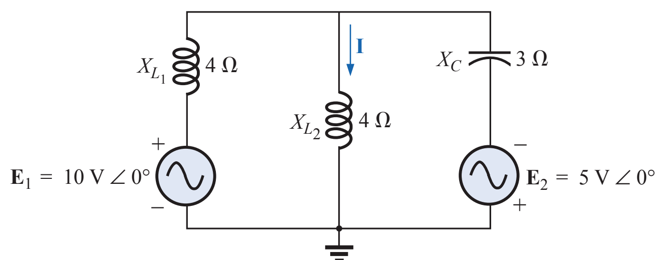


FIG. 18.1
Example 18.1.

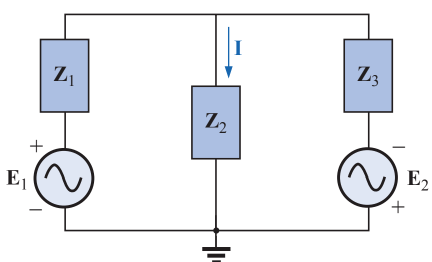


FIG. 18.2
Assigning the subscripted impedances to the network of Fig. 18.1.

Solution: For the redrawn circuit (Fig. 18.2),

$$\mathbf{Z}_1 = +j\,X_{L_1} = j\,4\,\Omega$$
$$\mathbf{Z}_2 = +j\,X_{L_2} = j\,4\,\Omega$$
$$\mathbf{Z}_3 = -j\,X_C = -j\,3\,\Omega$$

Considering the effects of the voltage source $\mathbf{E}_1$ (Fig. 18.3), we have

$$\mathbf{Z}_{2\|3} = \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(j\,4\,\Omega)(-j\,3\,\Omega)}{j\,4\,\Omega - j\,3\,\Omega} = \frac{12\,\Omega}{j} = -j\,12\,\Omega$$
$$= 12\,\Omega \angle -90°$$

$$I_{s_1} = \frac{\mathbf{E}_1}{\mathbf{Z}_{2\|3} + \mathbf{Z}_1} = \frac{10\text{ V} \angle 0°}{-j\,12\,\Omega + j\,4\,\Omega} = \frac{10\text{ V} \angle 0°}{8\,\Omega \angle -90°}$$
$$= 1.25\text{ A} \angle 90°$$

and

$$\mathbf{I}' = \frac{\mathbf{Z}_3\mathbf{I}_{s_1}}{\mathbf{Z}_2 + \mathbf{Z}_3} \qquad \text{(current divider rule)}$$
$$= \frac{(-j\,3\,\Omega\,)(\,j\,1.25\text{ A})}{j\,4\,\Omega - j\,3\,\Omega} = \frac{3.75\text{ A}}{j\,1} = 3.75\text{ A} \angle -90°$$

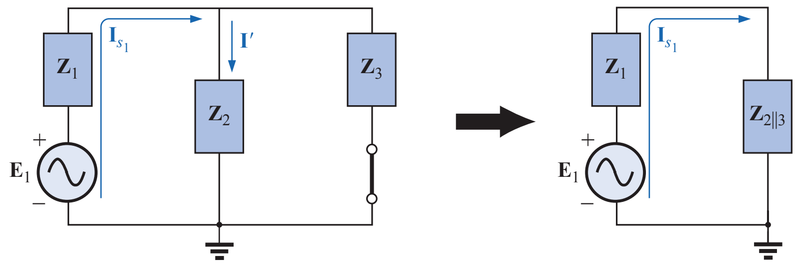


FIG. 18.3
Determining the effect of the voltage source $\mathbf{E}_1$ *on the current* $\mathbf{I}$ *of the network of Fig. 18.1.*

Considering the effects of the voltage source $\mathbf{E}_2$ (Fig. 18.4), we have

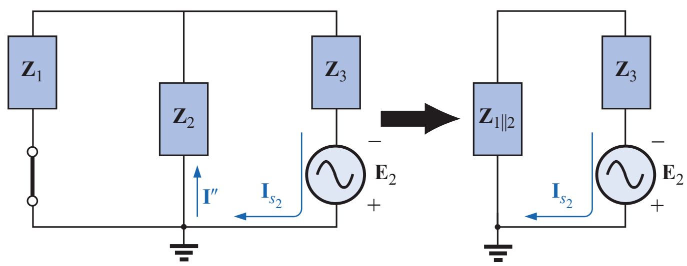


FIG. 18.4
Determining the effect of the voltage source $\mathbf{E}_2$ *on the current* $\mathbf{I}$ *of the network of Fig. 18.1.*

$$\mathbf{Z}_{1\|2} = \frac{\mathbf{Z}_1}{N} = \frac{j\,4\,\Omega}{2} = j\,2\,\Omega$$

$$\mathbf{I}_{s_2} = \frac{\mathbf{E}_2}{\mathbf{Z}_{1\|2} + \mathbf{Z}_3} = \frac{5\text{ V}\angle 0^\circ}{j\,2\,\Omega - j\,3\,\Omega} = \frac{5\text{ V}\angle 0^\circ}{1\,\Omega\angle -90^\circ} = 5\text{ A}\angle 90^\circ$$

and
$$\mathbf{I}'' = \frac{\mathbf{I}_{s_2}}{2} = 2.5\text{ A}\angle 90^\circ$$

The resultant current through the 4-Ω reactance X_{L_2} (Fig. 18.5) is

$$\begin{aligned}
\mathbf{I} &= \mathbf{I}' - \mathbf{I}'' \\
&= 3.75\text{ A}\angle -90^\circ - 2.50\text{ A}\angle 90^\circ = -j\,3.75\text{ A} - j\,2.50\text{ A} \\
&= -j\,6.25\text{ A} \\
\mathbf{I} &= \mathbf{6.25\text{ A}\angle -90^\circ}
\end{aligned}$$

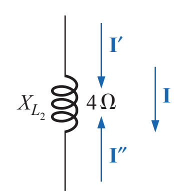


FIG. 18.5
Determining the resultant current for the network of Fig. 18.1.

EXAMPLE 18.2 Using superposition, find the current **I** through the 6-Ω resistor of Fig. 18.6.

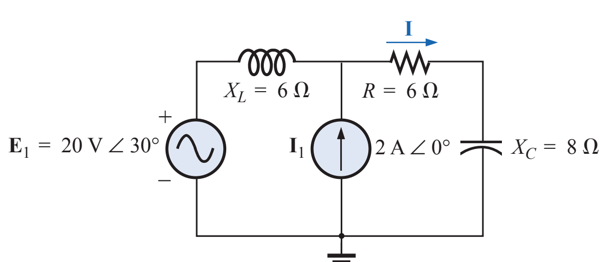


FIG. 18.6
Example 18.2.

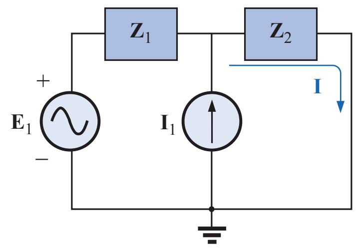


FIG. 18.7
Assigning the subscripted impedances to the network of Fig. 18.6.

Solution: For the redrawn circuit (Fig. 18.7),

$$\mathbf{Z}_1 = j\,6\,\Omega \qquad \mathbf{Z}_2 = 6 - j\,8\,\Omega$$

Consider the effects of the current source (Fig. 18.8). Applying the current divider rule, we have

$$\mathbf{I}' = \frac{\mathbf{Z}_1\mathbf{I}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,6\,\Omega)(2\text{ A})}{j\,6\,\Omega + 6\,\Omega - j\,8\,\Omega} = \frac{j\,12\text{ A}}{6 - j\,2}$$
$$= \frac{12\text{ A}\angle 90^\circ}{6.32\angle -18.43^\circ}$$
$$\mathbf{I}' = 1.9\text{ A}\angle 108.43^\circ$$

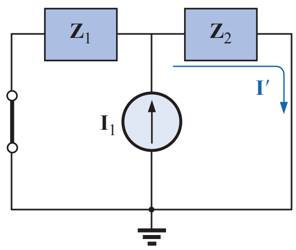


FIG. 18.8
Determining the effect of the current source $\mathbf{I}_1$ *on the current* **I** *of the network of Fig. 18.6.*

Consider the effects of the voltage source (Fig. 18.9). Applying Ohm's law gives us

$$\mathbf{I}'' = \frac{\mathbf{E}_1}{\mathbf{Z}_T} = \frac{\mathbf{E}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{20\text{ V}\angle 30^\circ}{6.32\,\Omega\angle -18.43^\circ}$$
$$= 3.16\text{ A}\angle 48.43^\circ$$

The total current through the 6-Ω resistor (Fig. 18.10) is

$$\begin{aligned}\mathbf{I} &= \mathbf{I}' + \mathbf{I}''\\ &= 1.9\text{ A}\angle 108.43^\circ + 3.16\text{ A}\angle 48.43^\circ\\ &= (-0.60\text{ A} + j\,1.80\text{ A}) + (2.10\text{ A} + j\,2.36\text{ A})\\ &= 1.50\text{ A} + j\,4.16\text{ A}\\ \mathbf{I} &= \mathbf{4.42\text{ A}\angle 70.2^\circ}\end{aligned}$$

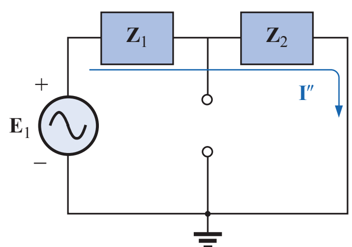


FIG. 18.9
Determining the effect of the voltage source $\mathbf{E}_1$ *on the current* **I** *of the network of Fig. 18.6.*

EXAMPLE 18.3 Using superposition, find the voltage across the 6-Ω resistor in Fig. 18.6. Check the results against $\mathbf{V}_{6\Omega} = \mathbf{I}(6\,\Omega)$, where **I** is the current found through the 6-Ω resistor in Example 18.2.

Solution: For the current source,

$$\mathbf{V}'_{6\Omega} = \mathbf{I}'(6\,\Omega) = (1.9\text{ A}\angle 108.43^\circ)(6\,\Omega) = 11.4\text{ V}\angle 108.43^\circ$$

For the voltage source,

$$\mathbf{V}''_{6\Omega} = \mathbf{I}''(6) = (3.16\text{ A}\angle 48.43^\circ)(6\,\Omega) = 18.96\text{ V}\angle 48.43^\circ$$

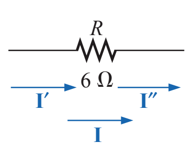


FIG. 18.10
Determining the resultant current **I** *for the network of Fig. 18.6.*

The total voltage across the 6-Ω resistor (Fig. 18.11) is

$$\begin{aligned}\mathbf{V}_{6\Omega} &= \mathbf{V}'_{6\Omega} + \mathbf{V}''_{6\Omega}\\ &= 11.4\text{ V}\angle 108.43^\circ + 18.96\text{ V}\angle 48.43^\circ\\ &= (-3.60\text{ V} + j\,10.82\text{ V}) + (12.58\text{ V} + j\,14.18\text{ V})\\ &= 8.98\text{ V} + j\,25.0\text{ V}\\ \mathbf{V}_{6\Omega} &= \mathbf{26.5\text{ V}\angle 70.2^\circ}\end{aligned}$$

Checking the result, we have

$$\begin{aligned}\mathbf{V}_{6\Omega} &= \mathbf{I}(6\,\Omega) = (4.42\text{ A}\angle 70.2^\circ)(6\,\Omega)\\ &= \mathbf{26.5\text{ V}\angle 70.2^\circ} \quad \text{(checks)}\end{aligned}$$

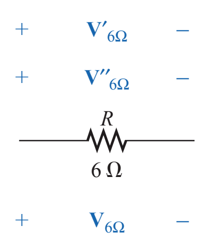


FIG. 18.11
Determining the resultant voltage $\mathbf{V}_{6\Omega}$ *for the network of Fig. 18.6.*

EXAMPLE 18.4 For the network of Fig. 18.12, determine the sinusoidal expression for the voltage v_3 using superposition.

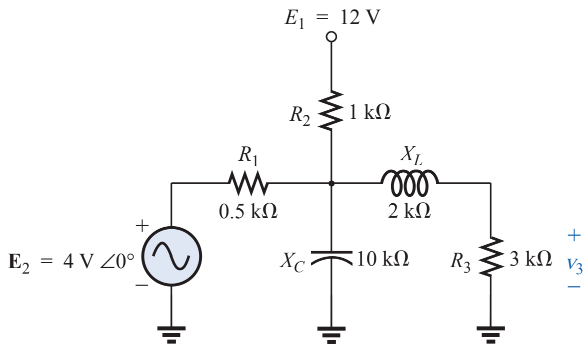


FIG. 18.12
Example 18.4.

Solution: For the dc source, recall that for dc analysis, in the steady state the capacitor can be replaced by an open-circuit equivalent, and the inductor by a short-circuit equivalent. The result is the network of Fig. 18.13.

The resistors R_1 and R_3 are then in parallel, and the voltage V_3 can be determined using the voltage divider rule:

$$R' = R_1 \parallel R_3 = 0.5\text{ k}\Omega \parallel 3\text{ k}\Omega = 0.429\text{ k}\Omega$$

and
$$V_3 = \frac{R'E_1}{R' + R_2}$$
$$= \frac{(0.429\text{ k}\Omega)(12\text{ V})}{0.429\text{ k}\Omega + 1\text{ k}\Omega} = \frac{5.148\text{ V}}{1.429}$$
$$V_3 \cong \mathbf{3.6\text{ V}}$$

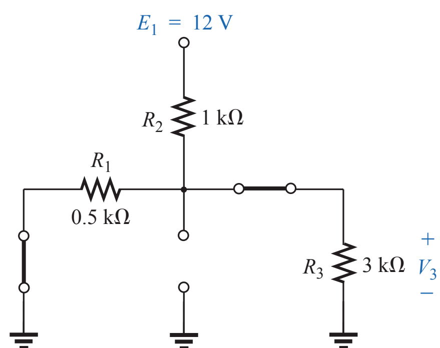


FIG. 18.13
Determining the effect of the dc voltage source E_1 on the voltage v_3 of the network of Fig. 18.12.

For ac analysis, the dc source is set to zero and the network is redrawn, as shown in Fig. 18.14.

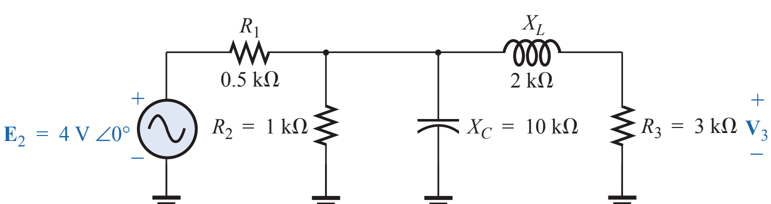


FIG. 18.14
Redrawing the network of Fig. 18.12 to determine the effect of the ac voltage source $\mathbf{E}_2$.

The block impedances are then defined as in Fig. 18.15, and series-parallel techniques are applied as follows:

$$\mathbf{Z}_1 = 0.5\text{ k}\Omega \angle 0^\circ$$
$$\mathbf{Z}_2 = (R_2 \angle 0^\circ \parallel (X_C \angle -90^\circ)$$
$$= \frac{(1\text{ k}\Omega \angle 0^\circ)(10\text{ k}\Omega \angle -90^\circ)}{1\text{ k}\Omega - j\,10\text{ k}\Omega} = \frac{10\text{ k}\Omega \angle -90^\circ}{10.05 \angle -84.29^\circ}$$
$$= 0.995\text{ k}\Omega \angle -5.71^\circ$$

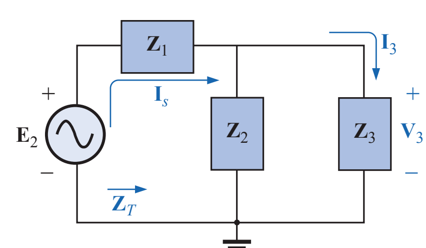


FIG. 18.15
Assigning the subscripted impedances to the network of Fig. 18.14.

$$\mathbf{Z}_3 = R_3 + j\,X_L = 3\text{ k}\Omega + j\,2\text{ k}\Omega = 3.61\text{ k}\Omega\ \angle 33.69°$$

and

$$\begin{aligned}\mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3\\ &= 0.5\text{ k}\Omega + (0.995\text{ k}\Omega\ \angle -5.71°) \parallel (3.61\text{ k}\Omega\ \angle 33.69°)\\ &= 1.312\text{ k}\Omega\ \angle 1.57°\end{aligned}$$

Calculator Performing the above on the TI-86 calculator gives the following result:

(0.5,0)+((0.995∠−5.71)*(3.61∠33.69))/((0.995∠−5.71)+(3.61∠33.69)) Enter
(1.311E0,35.373E−3)
Ans ▶ Pol
(1.312E0∠1.545E0)

CALC. 18.1

$$\mathbf{I}_s = \frac{\mathbf{E}_2}{\mathbf{Z}_T} = \frac{4\text{ V}\ \angle 0°}{1.312\text{ k}\Omega\ \angle 1.57°} = 3.05\text{ mA}\ \angle -1.57°$$

Current divider rule:

$$\begin{aligned}\mathbf{I}_3 &= \frac{\mathbf{Z}_2\mathbf{I}_s}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(0.995\text{ k}\Omega\ \angle -5.71°)(3.05\text{ mA}\ \angle -1.57°)}{0.995\text{ k}\Omega\ \angle -5.71° + 3.61\text{ k}\Omega\ \angle 33.69°}\\ &= 0.686\text{ mA}\ \angle -32.74°\end{aligned}$$

with

$$\begin{aligned}\mathbf{V}_3 &= (I_3\ \angle\theta)(R_3\ \angle 0°)\\ &= (0.686\text{ mA}\ \angle -32.74°)(3\text{ k}\Omega\ \angle 0°)\\ &= \mathbf{2.06\ V\ \angle -32.74°}\end{aligned}$$

The total solution:

$$\begin{aligned}v_3 &= v_3\,(\text{dc}) + v_3\,(\text{ac})\\ &= 3.6\text{ V} + 2.06\text{ V}\ \angle -32.74°\\ v_3 &= \mathbf{3.6 + 2.91\,sin(\omega t - 32.74°)}\end{aligned}$$

The result is a sinusoidal voltage having a peak value of 2.91 V riding on an average value of 3.6 V, as shown in Fig. 18.16.

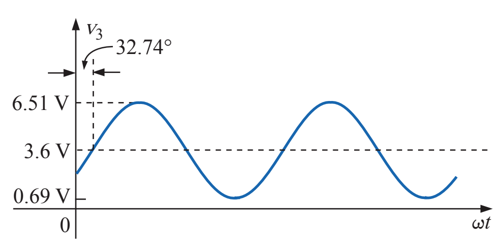


FIG. 18.16
The resultant voltage v_3 for the network of Fig. 18.12.

Dependent Sources

For dependent sources in which *the controlling variable is not determined by the network to which the superposition theorem is to be applied,* the application of the theorem is basically the same as for independent sources. The solution obtained will simply be in terms of the controlling variables.

EXAMPLE 18.5 Using the superposition theorem, determine the current $\mathbf{I}_2$ for the network of Fig. 18.17. The quantities μ and h are constants.

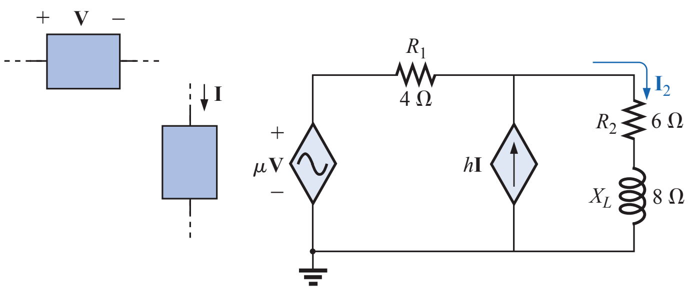


FIG. 18.17
Example 18.5.

Solution: With a portion of the system redrawn (Fig. 18.18),

$$\mathbf{Z}_1 = R_1 = 4\ \Omega \qquad \mathbf{Z}_2 = R_2 + j\,X_L = 6 + j\,8\ \Omega$$

For the voltage source (Fig. 18.19),

$$\mathbf{I}' = \frac{\mu\mathbf{V}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{\mu\mathbf{V}}{4\ \Omega + 6\ \Omega + j\,8\ \Omega} = \frac{\mu\mathbf{V}}{10\ \Omega + j\,8\ \Omega}$$
$$= \frac{\mu\mathbf{V}}{12.8\ \Omega\ \angle 38.66°} = 0.078\ \mu\mathbf{V}/\Omega\ \angle -38.66°$$

For the current source (Fig. 18.20),

$$\mathbf{I}'' = \frac{\mathbf{Z}_1(h\mathbf{I})}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(4\ \Omega)(h\mathbf{I})}{12.8\ \Omega\ \angle 38.66°} = 4(0.078)h\mathbf{I}\ \angle -38.66°$$
$$= 0.312h\mathbf{I}\ \angle -38.66°$$

The current $\mathbf{I}_2$ is

$$\mathbf{I}_2 = \mathbf{I}' + \mathbf{I}''$$
$$= 0.078\ \mu\mathbf{V}/\Omega\ \angle -38.66° + 0.312h\mathbf{I}\ \angle -38.66°$$

For $\mathbf{V} = 10\ \text{V}\ \angle 0°$, $\mathbf{I} = 20\ \text{mA}\ \angle 0°$, $\mu = 20$, and $h = 100$,

$$\mathbf{I}_2 = 0.078(20)(10\ \text{V}\ \angle 0°)/\Omega\ \angle -38.66°$$
$$+ 0.312(100)(20\ \text{mA}\ \angle 0°)\ \angle -38.66°$$
$$= 15.60\ \text{A}\ \angle -38.66° + 0.62\ \text{A}\ \angle -38.66°$$
$$\mathbf{I}_2 = \mathbf{16.22\ A\ \angle -38.66°}$$

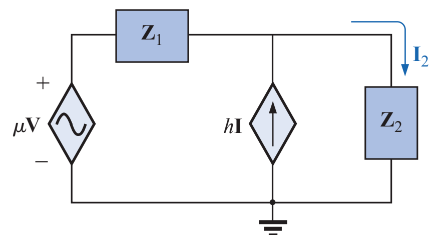


FIG. 18.18
Assigning the subscripted impedances to the network of Fig. 18.17.

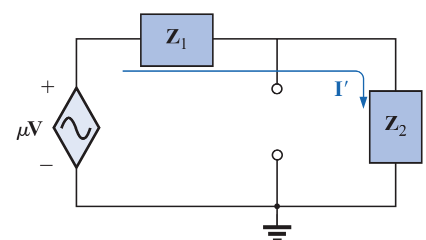


FIG. 18.19
Determining the effect of the voltage-controlled voltage source on the current $\mathbf{I}_2$ for the network of Fig. 18.17.

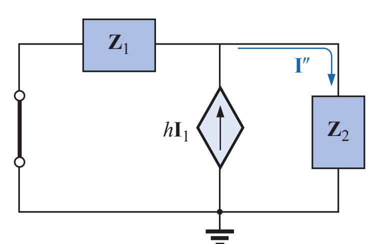


FIG. 18.20
Determining the effect of the current-controlled current source on the current $\mathbf{I}_2$ for the network of Fig. 18.17.

For dependent sources in which *the controlling variable is determined by the network to which the theorem is to be applied,* the dependent source cannot be set to zero unless the controlling variable is also zero. For networks containing dependent sources such as indicated in Example 18.5 and dependent sources of the type just introduced above, the superposition theorem is applied for each independent source and each dependent source not having a controlling variable in the portions of the network under investigation. It must be reemphasized that dependent sources are not sources of energy in the sense that, if all independent sources are removed from a system, all currents and voltages must be zero.

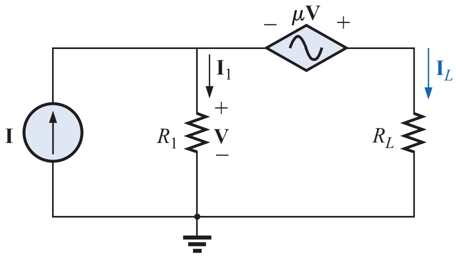


FIG. 18.21
Example 18.6.

EXAMPLE 18.6 Determine the current $\mathbf{I}_L$ through the resistor R_L of Fig. 18.21.

Solution: Note that the controlling variable **V** is determined by the network to be analyzed. From the above discussions, it is understood that the dependent source cannot be set to zero unless **V** is zero. If we set **I** to zero, the network lacks a source of voltage, and $\mathbf{V} = 0$ with $\mu\mathbf{V} = 0$. The resulting $\mathbf{I}_L$ under this condition is zero. Obviously, therefore, the network must be analyzed as it appears in Fig. 18.21, with the result that neither source can be eliminated, as is normally done using the superposition theorem.

Applying Kirchhoff's voltage law, we have

$$\mathbf{V}_L = \mathbf{V} + \mu\mathbf{V} = (1 + \mu)\mathbf{V}$$

and

$$\mathbf{I}_L = \frac{\mathbf{V}_L}{R_L} = \frac{(1 + \mu)\mathbf{V}}{R_L}$$

The result, however, must be found in terms of **I** since **V** and $\mu\mathbf{V}$ are only dependent variables.

Applying Kirchhoff's current law gives us

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_L = \frac{\mathbf{V}}{R_1} + \frac{(1 + \mu)\mathbf{V}}{R_L}$$

and

$$\mathbf{I} = \mathbf{V}\left(\frac{1}{R_1} + \frac{1 + \mu}{R_L}\right)$$

or

$$\mathbf{V} = \frac{\mathbf{I}}{(1/R_1) + [(1 + \mu)/R_L]}$$

Substituting into the above yields

$$\mathbf{I}_L = \frac{(1 + \mu)\mathbf{V}}{R_L} = \frac{(1 + \mu)}{R_L}\left(\frac{\mathbf{I}}{(1/R_1) + [(1 + \mu)/R_L]}\right)$$

Therefore,

$$\boldsymbol{\mathbf{I}_L = \frac{(1 + \mu)R_1\mathbf{I}}{R_L + (1 + \mu)R_1}}$$

18.3 THÉVENIN'S THEOREM

Thévenin's theorem, as stated for sinusoidal ac circuits, is changed only to include the term *impedance* instead of *resistance;* that is,

any two-terminal linear ac network can be replaced with an equivalent circuit consisting of a voltage source and an impedance in series, as shown in Fig. 18.22.

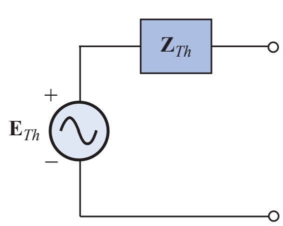


FIG. 18.22
Thévenin equivalent circuit for ac networks.

Since the reactances of a circuit are frequency dependent, the Thévenin circuit found for a particular network is applicable only at *one* frequency.

The steps required to apply this method to dc circuits are repeated here with changes for sinusoidal ac circuits. As before, the only change is the replacement of the term *resistance* with *impedance.* Again, dependent and independent sources will be treated separately.

Example 18.9, the last example of the independent source section, will include a network with dc and ac sources to establish the groundwork for possible use in the electronics area.

Independent Sources

1. ***Remove that portion of the network across which the Thévenin equivalent circuit is to be found.***
2. ***Mark (○, ●, and so on) the terminals of the remaining two-terminal network.***
3. ***Calculate $\mathbf{Z}_{Th}$ by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.***
4. ***Calculate $\mathbf{E}_{Th}$ by first replacing the voltage and current sources and then finding the open-circuit voltage between the marked terminals.***
5. ***Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Thévenin equivalent circuit.***

EXAMPLE 18.7 Find the Thévenin equivalent circuit for the network external to resistor R in Fig. 18.23.

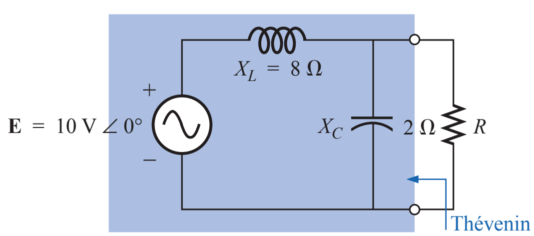


FIG. 18.23
Example 18.7.

Solution:

Steps 1 and 2 (Fig. 18.24):

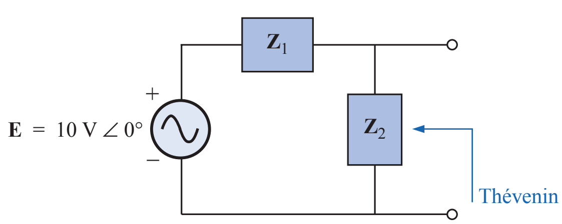


FIG. 18.24
Assigning the subscripted impedances to the network of Fig. 18.23.

$$\mathbf{Z}_1 = j\,X_L = j\,8\;\Omega \qquad \mathbf{Z}_2 = -j\,X_C = -j\,2\;\Omega$$

Step 3 (Fig. 18.25):

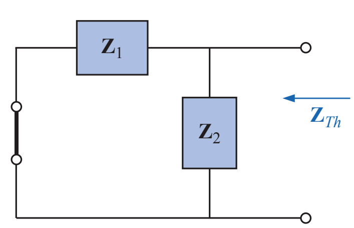


FIG. 18.25
Determining the Thévenin impedance for the network of Fig. 18.23.

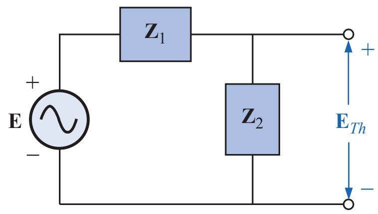


FIG. 18.26
Determining the open-circuit Thévenin voltage for the network of Fig. 18.23.

$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,8\,\Omega)(-j\,2\,\Omega)}{j\,8\,\Omega - j\,2\,\Omega} = \frac{-j^2 16\,\Omega}{j\,6} = \frac{16\,\Omega}{6\,\angle 90^\circ}$$
$$= \mathbf{2.67\,\Omega \angle -90^\circ}$$

Step 4 (Fig. 18.26):

$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} \qquad \text{(voltage divider rule)}$$
$$= \frac{(-j\,2\,\Omega)(10\text{ V})}{j\,8\,\Omega - j\,2\,\Omega} = \frac{-j\,20\text{ V}}{j\,6} = \mathbf{3.33\text{ V} \angle -180^\circ}$$

Step 5: The Thévenin equivalent circuit is shown in Fig. 18.27.

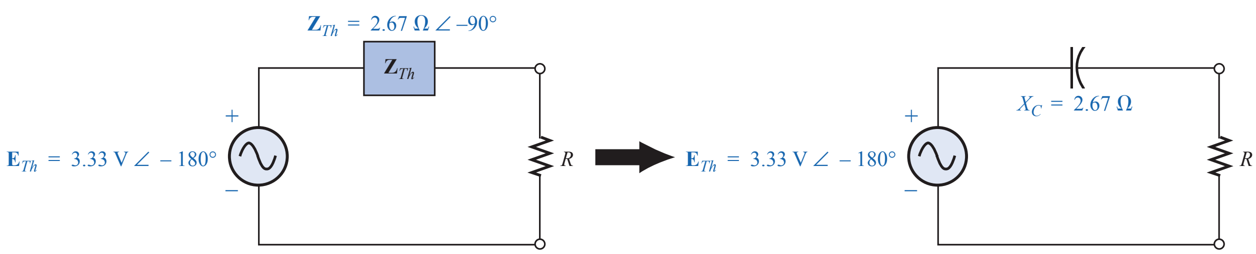


FIG. 18.27
The Thévenin equivalent circuit for the network of Fig. 18.23.

EXAMPLE 18.8 Find the Thévenin equivalent circuit for the network external to branch *a-a′* in Fig. 18.28.

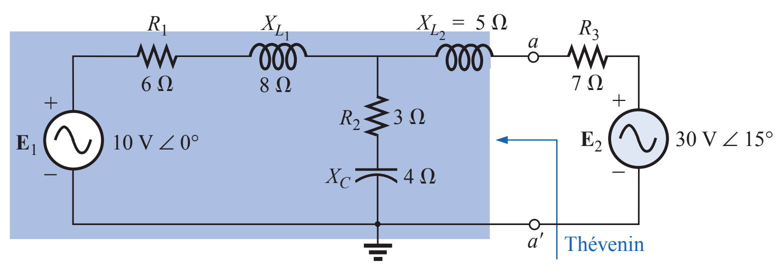


FIG. 18.28
Example 18.8.

Solution:

Steps 1 and 2 (Fig. 18.29): Note the reduced complexity with subscripted impedances:

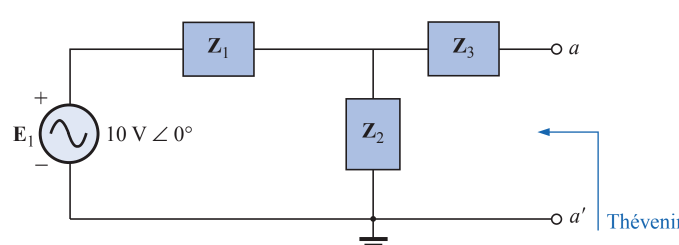


FIG. 18.29
Assigning the subscripted impedances to the network of Fig. 18.28.

$$\mathbf{Z}_1 = R_1 + j X_{L_1} = 6\ \Omega + j\ 8\ \Omega$$
$$\mathbf{Z}_2 = R_2 - j X_C = 3\ \Omega - j\ 4\ \Omega$$
$$\mathbf{Z}_3 = +j X_{L_2} = j\ 5\ \Omega$$

Step 3 (Fig. 18.30):

$$\mathbf{Z}_{Th} = \mathbf{Z}_3 + \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = j\ 5\ \Omega + \frac{(10\ \Omega \angle 53.13°)(5\ \Omega \angle -53.13°)}{(6\ \Omega + j\ 8\ \Omega) + (3\ \Omega - j\ 4\ \Omega)}$$
$$= j\ 5 + \frac{50 \angle 0°}{9 + j\ 4} = j\ 5 + \frac{50 \angle 0°}{9.85 \angle 23.96°}$$
$$= j\ 5 + 5.08 \angle -23.96° = j\ 5 + 4.64 - j\ 2.06$$
$$\mathbf{Z}_{Th} = \mathbf{4.64\ \Omega + j\ 2.94\ \Omega = 5.49\ \Omega \angle 32.36°}$$

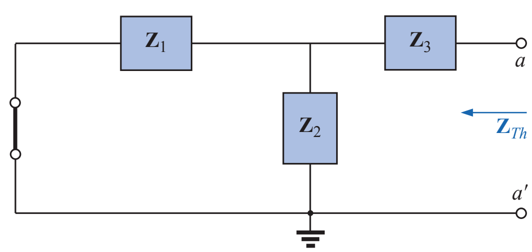


FIG. 18.30
Determining the Thévenin impedance for the network of Fig. 18.28.

Step 4 (Fig. 18.31): Since *a-a′* is an open circuit, $\mathbf{I}_{\mathbf{Z}_3} = 0$. Then $\mathbf{E}_{Th}$ is the voltage drop across $\mathbf{Z}_2$:

$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_1} \qquad \text{(voltage divider rule)}$$
$$= \frac{(5\ \Omega \angle -53.13°)(10\ \text{V} \angle 0°)}{9.85\ \Omega \angle 23.96°}$$
$$\mathbf{E}_{Th} = \frac{50\ \text{V} \angle -53.13°}{9.85 \angle 23.96°} = \mathbf{5.08\ V \angle -77.09°}$$

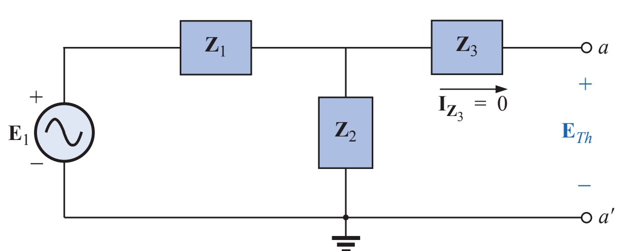


FIG. 18.31
Determining the open-circuit Thévenin voltage for the network of Fig. 18.28.

Step 5: The Thévenin equivalent circuit is shown in Fig. 18.32.

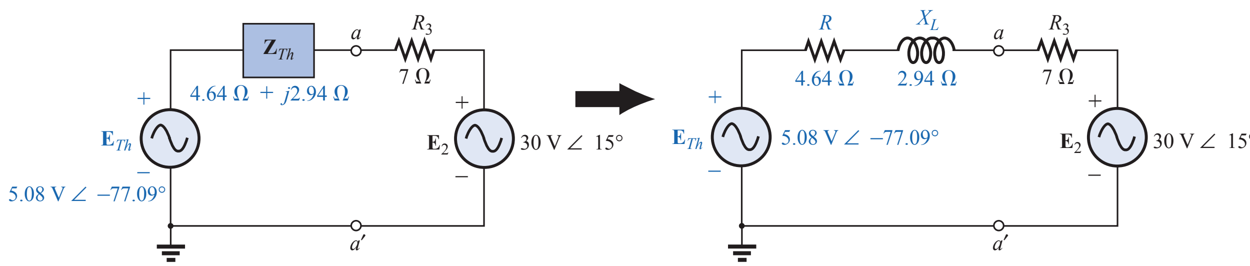


FIG. 18.32
The Thévenin equivalent circuit for the network of Fig. 18.28.

The next example demonstrates how superposition is applied to electronic circuits to permit *a separation of the dc and ac analyses.* The fact that the controlling variable in this analysis is not in the portion of the network connected directly to the terminals of interest permits an analysis of the network in the same manner as applied above for independent sources.

EXAMPLE 18.9 Determine the Thévenin equivalent circuit for the transistor network external to the resistor R_L in the network of Fig. 18.33. Then determine $\mathbf{V}_L$.

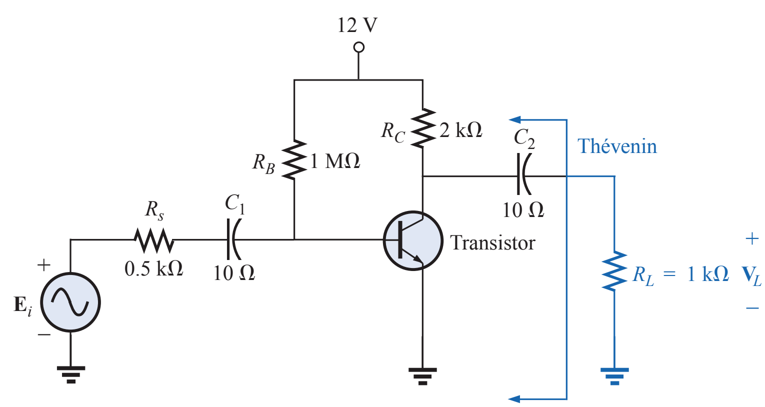


FIG. 18.33
Example 18.9.

Solution: Applying superposition.

dc Conditions Substituting the open-circuit equivalent for the coupling capacitor C_2 will isolate the dc source and the resulting currents from the load resistor. The result is that for dc conditions, $V_L = 0$ V. Although the output dc voltage is zero, the application of the dc voltage is important to the basic operation of the transistor in a number of important ways, one of which is to determine the parameters of the "equivalent circuit" to appear in the ac analysis to follow.

ac Conditions For the ac analysis, an equivalent circuit is substituted for the transistor, as established by the dc conditions above, that will behave like the actual transistor. A great deal more will be said about equivalent circuits and the operations performed to obtain the network of Fig. 18.34, but for now let us limit our attention to the manner in which the Thévenin equivalent circuit is obtained. Note in Fig. 18.34 that the equivalent circuit includes a resistor of 2.3 kΩ and a controlled current source whose magnitude is determined by the product of a factor of 100 and the current I_1 in another part of the network.

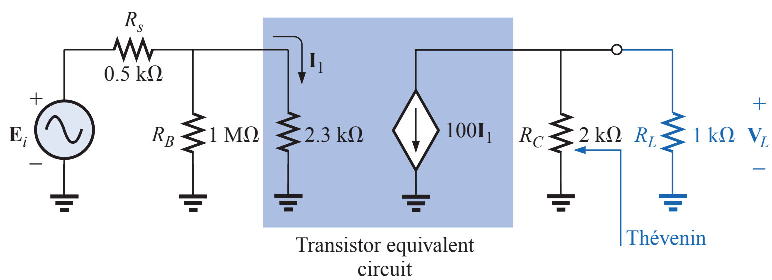


FIG. 18.34
The ac equivalent network for the transistor amplifier of Fig. 18.33.

Note in Fig. 18.34 the absence of the coupling capacitors for the ac analysis. In general, coupling capacitors are designed to be open circuits for dc analysis and short circuits for ac analysis. The short-circuit equivalent is valid because the other impedances in series with the coupling capacitors are so much larger in magnitude that the effect of the coupling capacitors can be ignored. Both R_B and R_C are now tied to ground because the dc source was set to zero volts (superposition) and replaced by a short-circuit equivalent to ground.

For the analysis to follow, the effect of the resistor R_B will be ignored since it is so much larger than the parallel 2.3-kΩ resistor.

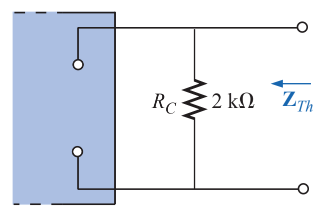


FIG. 18.35
Determining the Thévenin impedance for the network of Fig. 18.34.

$\mathbf{Z}_{Th}$ When $\mathbf{E}_i$ is set to zero volts, the current $\mathbf{I}_1$ will be zero amperes, and the controlled source $100\mathbf{I}_1$ will be zero amperes also. The result is an open-circuit equivalent for the source, as appearing in Fig. 18.35.

It is fairly obvious from Fig. 18.35 that

$$\mathbf{Z}_{Th} = \mathbf{2\ k\Omega}$$

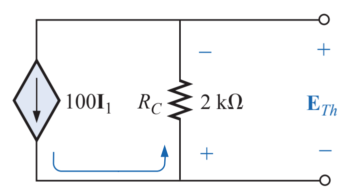


FIG. 18.36
Determining the Thévenin voltage for the network of Fig. 18.34.

$\mathbf{E}_{Th}$ For $\mathbf{E}_{Th}$, the current $\mathbf{I}_1$ of Fig. 18.34 will be

$$\mathbf{I}_1 = \frac{\mathbf{E}_i}{R_s + 2.3\ \text{k}\Omega} = \frac{\mathbf{E}_i}{0.5\ \text{k}\Omega + 2.3\ \text{k}\Omega} = \frac{\mathbf{E}_i}{2.8\ \text{k}\Omega}$$

and
$$100\mathbf{I}_1 = (100)\left(\frac{\mathbf{E}_i}{2.8\ \text{k}\Omega}\right) = 35.71 \times 10^{-3}/\Omega\ \mathbf{E}_i$$

Referring to Fig. 18.36, we find that

$$\begin{aligned}\mathbf{E}_{Th} &= -(100\mathbf{I}_1)R_C\\ &= -(35.71 \times 10^{-3}/\Omega\ \mathbf{E}_i)(2 \times 10^3\ \Omega)\\ \mathbf{E}_{Th} &= \mathbf{-71.42E}_{\boldsymbol{i}}\end{aligned}$$

The Thévenin equivalent circuit appears in Fig. 18.37 with the original load R_L.

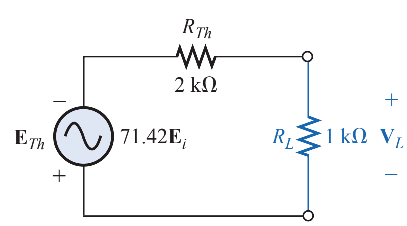


FIG. 18.37
The Thévenin equivalent circuit for the network of Fig. 18.34.

Output Voltage V_L

$$\mathbf{V}_L = \frac{-R_L\mathbf{E}_{Th}}{R_L + \mathbf{Z}_{Th}} = \frac{-(1\text{ k}\Omega)(71.42\mathbf{E}_i)}{1\text{ k}\Omega + 2\text{ k}\Omega}$$

and
$$\mathbf{V}_L = \mathbf{-23.81E}_i$$

revealing that the output voltage is 23.81 times the applied voltage with a phase shift of 180° due to the minus sign.

Dependent Sources

For dependent sources with a *controlling variable not in the network under investigation,* the procedure indicated above can be applied. However, for dependent sources of the other type, where the *controlling variable is part of the network to which the theorem is to be applied,* another approach must be employed. The necessity for a different approach will be demonstrated in an example to follow. The method is *not limited to dependent sources* of the latter type. It can also be applied to any dc or sinusoidal ac network. However, for networks of independent sources, the method of application employed in Chapter 9 and presented in the first portion of this section is generally more direct, with the usual savings in time and errors.

The new approach to Thévenin's theorem can best be introduced at this stage in the development by considering the Thévenin equivalent circuit of Fig. 18.38(a). As indicated in Fig. 18.38(b), the open-circuit terminal voltage ($\mathbf{E}_{oc}$) of the Thévenin equivalent circuit is the Thévenin equivalent voltage; that is,

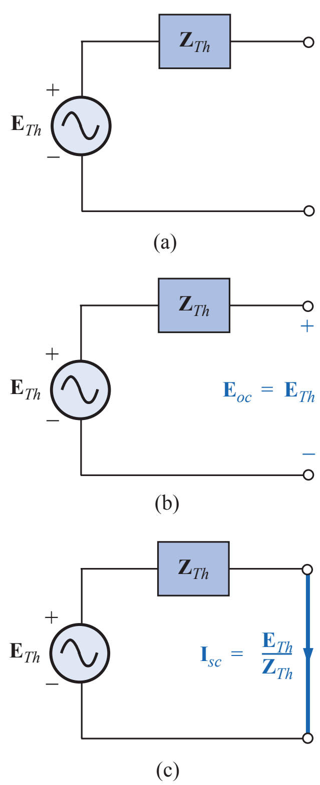


FIG. 18.38
Defining an alternative approach for determining the Thévenin impedance.

$$\boxed{\mathbf{E}_{oc} = \mathbf{E}_{Th}} \tag{18.1}$$

If the external terminals are short circuited as in Fig. 18.38(c), the resulting short-circuit current is determined by

$$\boxed{\mathbf{I}_{sc} = \frac{\mathbf{E}_{Th}}{\mathbf{Z}_{Th}}} \tag{18.2}$$

or, rearranged,

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{Th}}{\mathbf{I}_{sc}}$$

and
$$\boxed{\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}}} \tag{18.3}$$

Equations (18.1) and (18.3) indicate that for any linear bilateral dc or ac network with or without dependent sources of any type, if the open-circuit terminal voltage of a portion of a network can be determined along with the short-circuit current between the same two terminals, the Thévenin equivalent circuit is effectively known. A few examples will make the method quite clear. The advantage of the method, which was stressed earlier in this section for independent sources, should now be more obvious. The current $\mathbf{I}_{sc}$, which is necessary to find $\mathbf{Z}_{Th}$, is in general more difficult to obtain since all of the sources are present.

There is a third approach to the Thévenin equivalent circuit that is also useful from a practical viewpoint. The Thévenin voltage is found as in the two previous methods. However, the Thévenin impedance is obtained by applying a source of voltage to the terminals of interest and determining the source current as indicated in Fig. 18.39. For this method, the source voltage of the original network is set to zero. The Thévenin impedance is then determined by the following equation:

$$\boxed{\mathbf{Z}_{Th} = \frac{\mathbf{E}_g}{\mathbf{I}_g}} \tag{18.4}$$

Note that for each technique, $\mathbf{E}_{Th} = \mathbf{E}_{oc}$, but the Thévenin impedance is found in different ways.

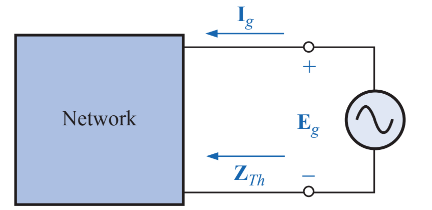


FIG. 18.39
Determining $\mathbf{Z}_{Th}$ using the approach $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$.

EXAMPLE 18.10 Using each of the three techniques described in this section, determine the Thévenin equivalent circuit for the network of Fig. 18.40.

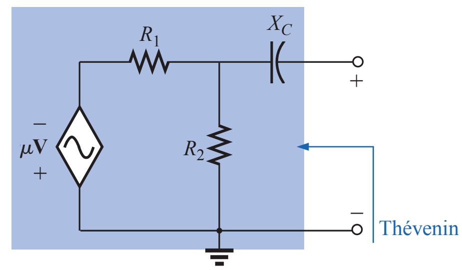


FIG. 18.40
Example 18.10.

Solution: Since for each approach the Thévenin voltage is found in exactly the same manner, it will be determined first. From Fig. 18.40, where $\mathbf{I}_{X_C} = 0$,

Due to the polarity for **V** and defined terminal polarities

$$\mathbf{V}_{R_1} = \mathbf{E}_{Th} = \mathbf{E}_{oc} = -\frac{R_2(\mu\mathbf{V})}{R_1 + R_2} = -\frac{\boldsymbol{\mu R_2 \mathbf{V}}}{\boldsymbol{R_1 + R_2}}$$

The following three methods for determining the Thévenin impedance appear in the order in which they were introduced in this section.

Method 1: See Fig. 18.41.

$$\mathbf{Z}_{Th} = \boldsymbol{R_1 \parallel R_2 - j X_C}$$

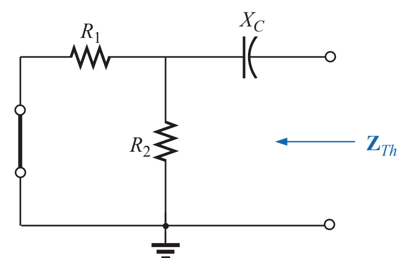


FIG. 18.41
Determining the Thévenin impedance for the network of Fig. 18.40.

Method 2: See Fig. 18.42. Converting the voltage source to a current source (Fig. 18.43), we have (current divider rule)

$$\mathbf{I}_{sc} = \frac{-(R_1 \parallel R_2)\dfrac{\mu\mathbf{V}}{R_1}}{(R_1 \parallel R_2) - j X_C} = \frac{-\dfrac{R_1 R_2}{R_1 + R_2}\left(\dfrac{\mu\mathbf{V}}{R_1}\right)}{(R_1 \parallel R_2) - j X_C}$$

$$= \frac{\dfrac{-\mu R_2 \mathbf{V}}{R_1 + R_2}}{(R_1 \parallel R_2) - j X_C}$$

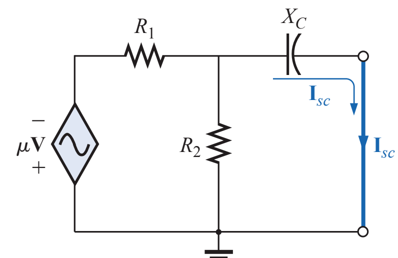


FIG. 18.42
Determining the short-circuit current for the network of Fig. 18.40.

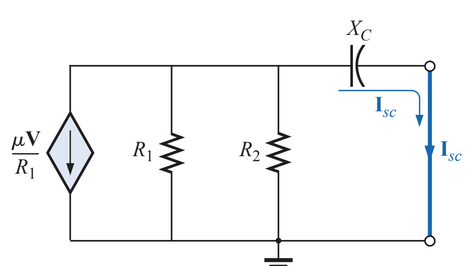


FIG. 18.43
Converting the voltage source of Fig. 18.42 to a current source.

and

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{\dfrac{-\mu R_2\mathbf{V}}{R_1 + R_2}}{\dfrac{\dfrac{-\mu R_2\mathbf{V}}{R_1 + R_2}}{(R_1 \parallel R_2) - j\,X_C}} = \frac{1}{\dfrac{1}{(R_1 \parallel R_2) - j\,X_C}}$$

$$= \boldsymbol{R_1 \parallel R_2 - j\,X_C}$$

Method 3: See Fig. 18.44.

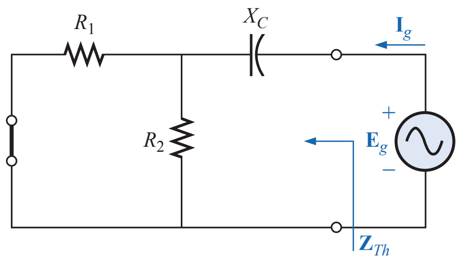


FIG. 18.44
Determining the Thévenin impedance for the network of Fig. 18.40 using the approach $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$.

$$\mathbf{I}_g = \frac{\mathbf{E}_g}{(R_1 \parallel R_2) - j\,X_C}$$

and $$\mathbf{Z}_{Th} = \frac{\mathbf{E}_g}{\mathbf{I}_g} = \boldsymbol{R_1 \parallel R_2 - j\,X_C}$$

In each case, the Thévenin impedance is the same. The resulting Thévenin equivalent circuit is shown in Fig. 18.45.

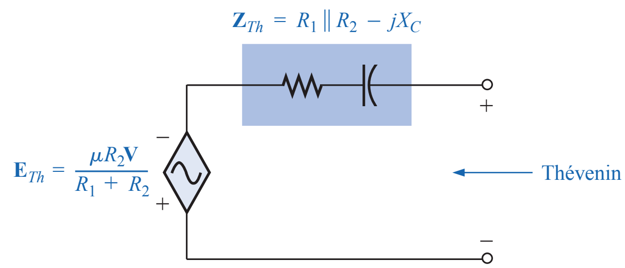


FIG. 18.45
The Thévenin equivalent circuit for the network of Fig. 18.40.

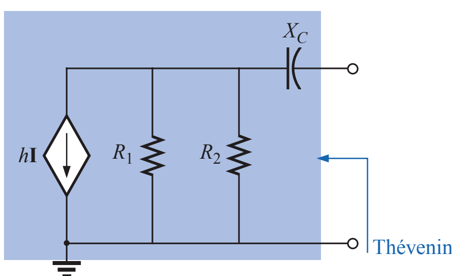


FIG. 18.46
Example 18.11.

EXAMPLE 18.11 Repeat Example 18.10 for the network of Fig. 18.46.

Solution: From Fig. 18.46, $\mathbf{E}_{Th}$ is

$$\mathbf{E}_{Th} = \mathbf{E}_{oc} = -h\mathbf{I}(R_1 \parallel R_2) = \boldsymbol{-\frac{hR_1R_2\mathbf{I}}{R_1 + R_2}}$$

Method 1: See Fig. 18.47.

$$\mathbf{Z}_{Th} = \boldsymbol{R_1 \parallel R_2 - j\,X_C}$$

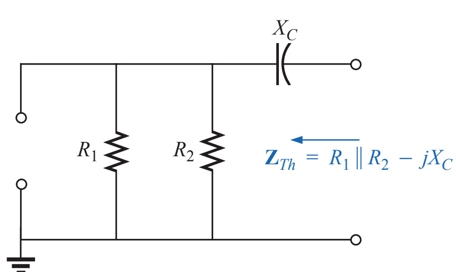


FIG. 18.47
Determining the Thévenin impedance for the network of Fig. 18.46.

Note the similarity between this solution and that obtained for the previous example.

Method 2: See Fig. 18.48.

$$\mathbf{I}_{sc} = \frac{-(R_1 \parallel R_2)h\mathbf{I}}{(R_1 \parallel R_2) - j\,X_C}$$

and $$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{-h\mathbf{I}(R_1 \parallel R_2)}{\dfrac{-(R_1 \parallel R_2)h\mathbf{I}}{(R_1 \parallel R_2) - j\,X_C}} = \boldsymbol{R_1 \parallel R_2 - j\,X_C}$$

Method 3: See Fig. 18.49.

$$\mathbf{I}_g = \frac{\mathbf{E}_g}{(R_1 \parallel R_2) - j\,X_C}$$

and

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_g}{\mathbf{I}_g} = \boldsymbol{R_1 \parallel R_2 - j\,X_C}$$

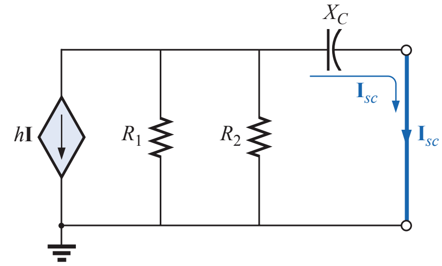


FIG. 18.48
Determining the short-circuit current for the network of Fig. 18.46.

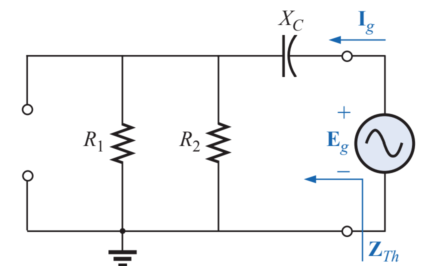


FIG. 18.49
Determining the Thévenin impedance using the approach $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$.

The following example has a dependent source that will not permit the use of the method described at the beginning of this section for independent sources. All three methods will be applied, however, so that the results can be compared.

EXAMPLE 18.12 For the network of Fig. 18.50 (introduced in Example 18.6), determine the Thévenin equivalent circuit between the indicated terminals using each method described in this section. Compare your results.

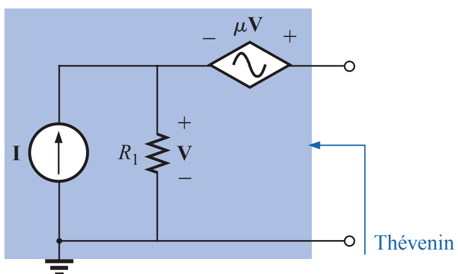


FIG. 18.50
Example 18.12.

Solution: First, using Kirchhoff's voltage law, $\mathbf{E}_{Th}$ (which is the same for each method) is written

$$\mathbf{E}_{Th} = \mathbf{V} + \mu\mathbf{V} = (1 + \mu)\mathbf{V}$$

However, $\mathbf{V} = \mathbf{I}R_1$

so $\mathbf{E}_{Th} = \boldsymbol{(1 + \mu)\mathbf{I}R_1}$

$\mathbf{Z}_{Th}$

Method 1: See Fig. 18.51. Since $\mathbf{I} = 0$, $\mathbf{V}$ and $\mu\mathbf{V} = 0$, and

$$\cancel{\mathbf{Z}_{Th} = R_1} \qquad \text{(incorrect)}$$

Method 2: See Fig. 18.52. Kirchhoff's voltage law around the indicated loop gives us

$$\mathbf{V} + \mu\mathbf{V} = 0$$

and

$$\mathbf{V}(1 + \mu) = 0$$

Since μ is a positive constant, the above equation can be satisfied only when $\mathbf{V} = 0$. Substitution of this result into Fig. 18.52 will yield the configuration of Fig. 18.53, and

$$\mathbf{I}_{sc} = \mathbf{I}$$

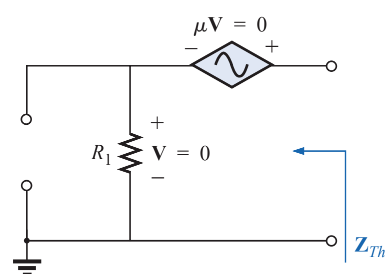


FIG. 18.51
Determining $\mathbf{Z}_{Th}$ incorrectly.

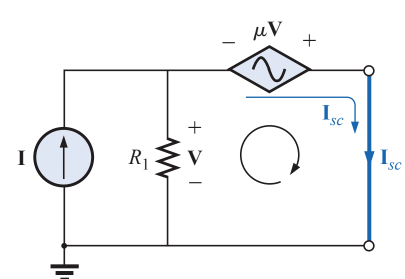


FIG. 18.52
Determining $\mathbf{I}_{sc}$ for the network of Fig. 18.50.

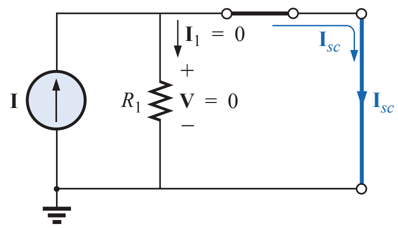


FIG. 18.53
Substituting **V** *= 0 into the network of Fig. 18.52.*

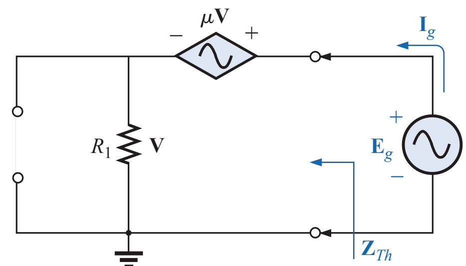


FIG. 18.54
Determining $\mathbf{Z}_{Th}$ *using the approach* $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$.

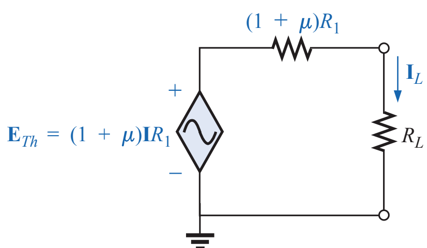


FIG. 18.55
The Thévenin equivalent circuit for the network of Fig. 18.50.

with

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{(1 + \mu)\mathbf{I}R_1}{\mathbf{I}} = \boldsymbol{(1 + \mu)R_1} \qquad \text{(correct)}$$

Method 3: See Fig. 18.54.

$$\mathbf{E}_g = \mathbf{V} + \mu\mathbf{V} = (1 + \mu)\mathbf{V}$$

or

$$\mathbf{V} = \frac{\mathbf{E}_g}{1 + \mu}$$

and

$$\mathbf{I}_g = \frac{\mathbf{V}}{R_1} = \frac{\mathbf{E}_g}{(1 + \mu)R_1}$$

and

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_g}{\mathbf{I}_g} = \boldsymbol{(1 + \mu)R_1} \qquad \text{(correct)}$$

The Thévenin equivalent circuit appears in Fig. 18.55, and

$$\mathbf{I}_L = \frac{\boldsymbol{(1 + \mu)R_1\mathbf{I}}}{\boldsymbol{R_L + (1 + \mu)R_1}}$$

which compares with the result of Example 18.6.

The network of Fig. 18.56 is the basic configuration of the transistor equivalent circuit applied most frequently today (although most texts in electronics will use the circle rather than the diamond outline for the source). Obviously, it is necessary to know its characteristics and to be adept in its use. Note that there are both a controlled voltage and a controlled current source, each controlled by variables in the configuration.

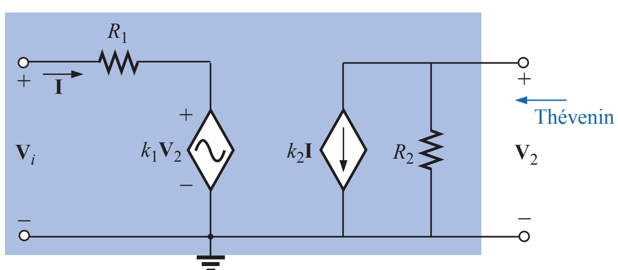


FIG. 18.56
Example 18.13: Transistor equivalent network.

EXAMPLE 18.13 Determine the Thévenin equivalent circuit for the indicated terminals of the network of Fig. 18.56.

Solution: Apply the second method introduced in this section.

E_{Th}

$$\mathbf{E}_{oc} = \mathbf{V}_2$$

$$\mathbf{I} = \frac{\mathbf{V}_i - k_1\mathbf{V}_2}{R_1} = \frac{\mathbf{V}_i - k_1\mathbf{E}_{oc}}{R_1}$$

and

$$\mathbf{E}_{oc} = -k_2\mathbf{I}R_2 = -k_2R_2\left(\frac{\mathbf{V}_i - k_1\mathbf{E}_{oc}}{R_1}\right)$$

$$= \frac{-k_2R_2\mathbf{V}_i}{R_1} + \frac{k_1k_2R_2\mathbf{E}_{oc}}{R_1}$$

or $$\mathbf{E}_{oc}\left(1 - \frac{k_1k_2R_2}{R_1}\right) = \frac{-k_2R_2\mathbf{V}_i}{R_1}$$

and $$\mathbf{E}_{oc}\left(\frac{R_1 - k_1k_2R_2}{\cancel{R_1}}\right) = \frac{-k_2R_2\mathbf{V}_i}{\cancel{R_1}}$$

so $$\boxed{\mathbf{E}_{oc} = \frac{-k_2R_2\mathbf{V}_i}{R_1 - k_1k_2R_2} = \mathbf{E}_{Th}} \qquad \textbf{(18.5)}$$

$\mathbf{I}_{sc}$ For the network of Fig. 18.57, where

$$\mathbf{V}_2 = 0 \qquad k_1\mathbf{V}_2 = 0 \qquad \mathbf{I} = \frac{\mathbf{V}_i}{R_1}$$

and $$\mathbf{I}_{sc} = -k_2\mathbf{I} = \frac{-k_2\mathbf{V}_i}{R_1}$$

so $$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{\dfrac{-k_2R_2\mathbf{V}_i}{R_1 - k_1k_2R_2}}{\dfrac{-k_2\mathbf{V}_i}{R_1}} = \frac{R_1R_2}{R_1 - k_1k_2R_2}$$

and $$\boxed{\mathbf{Z}_{Th} = \frac{R_2}{1 - \dfrac{k_1k_2R_2}{R_1}}} \qquad \textbf{(18.6)}$$

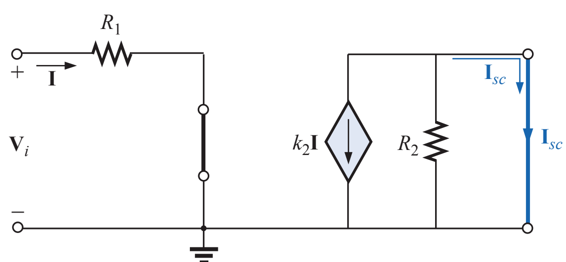


FIG. 18.57
Determining $\mathbf{I}_{sc}$ for the network of Fig. 18.56.

Frequently, the approximation $k_1 \cong 0$ is applied. Then the Thévenin voltage and impedance are

$$\boxed{\mathbf{E}_{Th} = \frac{-k_2R_2\mathbf{V}_i}{R_1}} \qquad k_1 = 0 \qquad \textbf{(18.7)}$$

$$\boxed{\mathbf{Z}_{Th} = R_2} \qquad k_1 = 0 \qquad \textbf{(18.8)}$$

Apply $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$ to the network of Fig. 18.58, where

$$\mathbf{I} = \frac{-k_1\mathbf{V}_2}{R_1}$$

But $$\mathbf{V}_2 = \mathbf{E}_g$$

so $$\mathbf{I} = \frac{-k_1\mathbf{E}_g}{R_1}$$

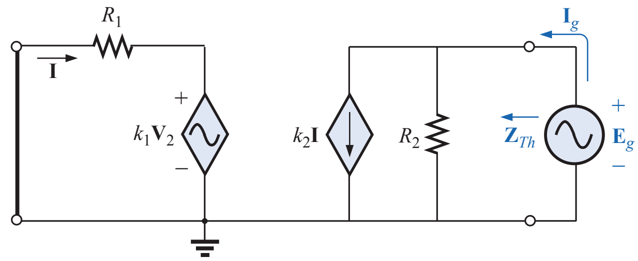


FIG. 18.58
Determining $\mathbf{Z}_{Th}$ using the procedure $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$.

Applying Kirchhoff's current law, we have

$$\mathbf{I}_g = k_2\mathbf{I} + \frac{\mathbf{E}_g}{R_2} = k_2\left(-\frac{k_1\mathbf{E}_g}{R_1}\right) + \frac{\mathbf{E}_g}{R_2}$$

$$= \mathbf{E}_g\left(\frac{1}{R_2} - \frac{k_1k_2}{R_1}\right)$$

and

$$\frac{\mathbf{I}_g}{\mathbf{E}_g} = \frac{R_1 - k_1k_2R_2}{R_1R_2}$$

or

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_g}{\mathbf{I}_g} = \boldsymbol{\frac{R_1R_2}{R_1 - k_1k_2R_2}}$$

as obtained above.

The last two methods presented in this section were applied only to networks in which the magnitudes of the controlled sources were dependent on a variable within the network for which the Thévenin equivalent circuit was to be obtained. Understand that both of these methods can also be applied to any dc or sinusoidal ac network containing only independent sources or dependent sources of the other kind.

18.4 NORTON'S THEOREM

The three methods described for Thévenin's theorem will each be altered to permit their use with **Norton's theorem.** Since the Thévenin and Norton impedances are the same for a particular network, certain portions of the discussion will be quite similar to those encountered in the previous section. We will first consider independent sources and the approach developed in Chapter 9, followed by dependent sources and the new techniques developed for Thévenin's theorem.

You will recall from Chapter 9 that Norton's theorem allows us to replace any two-terminal linear bilateral ac network with an equivalent circuit consisting of a current source and an impedance, as in Fig. 18.59.

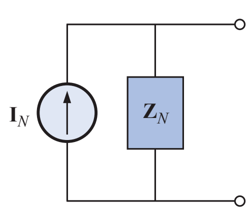


FIG. 18.59
The Norton equivalent circuit for ac networks.

The Norton equivalent circuit, like the Thévenin equivalent circuit, is applicable at only one frequency since the reactances are frequency dependent.

Independent Sources

The procedure outlined below to find the Norton equivalent of a sinusoidal ac network is changed (from that in Chapter 9) in only one respect: the replacement of the term *resistance* with the term *impedance.*

1. ***Remove that portion of the network across which the Norton equivalent circuit is to be found.***
2. ***Mark (○, •, and so on) the terminals of the remaining two-terminal network.***
3. ***Calculate $\mathbf{Z}_N$ by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.***
4. ***Calculate $\mathbf{I}_N$ by first replacing the voltage and current sources and then finding the short-circuit current between the marked terminals.***
5. ***Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Norton equivalent circuit.***

The Norton and Thévenin equivalent circuits can be found from each other by using the source transformation shown in Fig. 18.60. The source transformation is applicable for any Thévenin or Norton equivalent circuit determined from a network with any combination of independent or dependent sources.

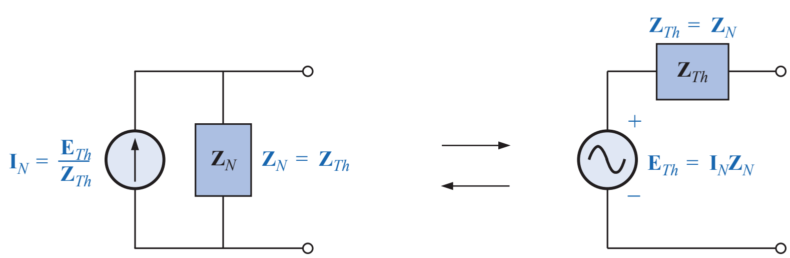


FIG. 18.60
Conversion between the Thévenin and Norton equivalent circuits.

EXAMPLE 18.14 Determine the Norton equivalent circuit for the network external to the 6-Ω resistor of Fig. 18.61.

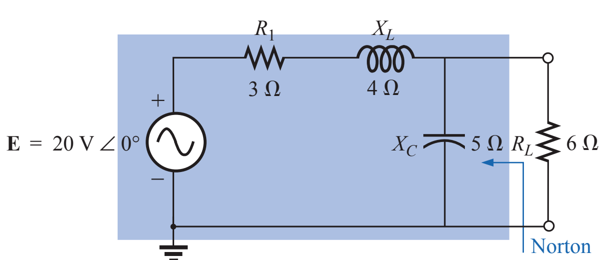


FIG. 18.61
Example 18.14.

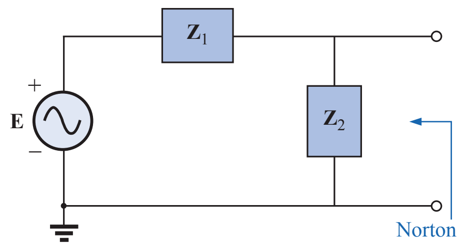


FIG. 18.62
Assigning the subscripted impedances to the network of Fig. 18.61.

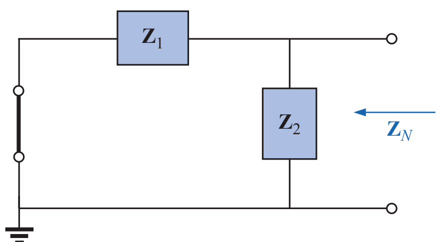


FIG. 18.63
Determining the Norton impedance for the network of Fig. 18.61.

Solution:

Steps 1 and 2 (Fig. 18.62):

$$\mathbf{Z}_1 = R_1 + j\,X_L = 3\,\Omega + j\,4\,\Omega = 5\,\Omega\,\angle 53.13^\circ$$
$$\mathbf{Z}_2 = -j\,X_C = -j\,5\,\Omega$$

Step 3 (Fig. 18.63):

$$\mathbf{Z}_N = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(5\,\Omega\,\angle 53.13^\circ)(5\,\Omega\,\angle -90^\circ)}{3\,\Omega + j\,4\,\Omega - j\,5\,\Omega} = \frac{25\,\Omega\,\angle -36.87^\circ}{3 - j\,1}$$
$$= \frac{25\,\Omega\,\angle -36.87^\circ}{3.16\,\angle -18.43^\circ} = 7.91\,\Omega\,\angle -18.44^\circ = \mathbf{7.50\,\Omega - \mathit{j}\,2.50\,\Omega}$$

Step 4 (Fig. 18.64):

$$\mathbf{I}_N = \mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{20\,\text{V}\,\angle 0^\circ}{5\,\Omega\,\angle 53.13^\circ} = \mathbf{4\,A\,\angle -53.13^\circ}$$

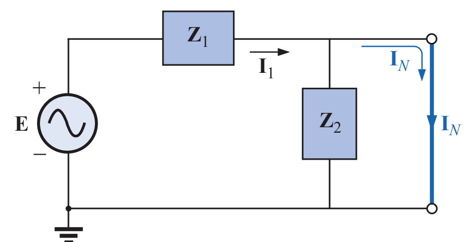


FIG. 18.64
Determining $\mathbf{I}_N$ for the network of Fig. 18.61.

Step 5: The Norton equivalent circuit is shown in Fig. 18.65.

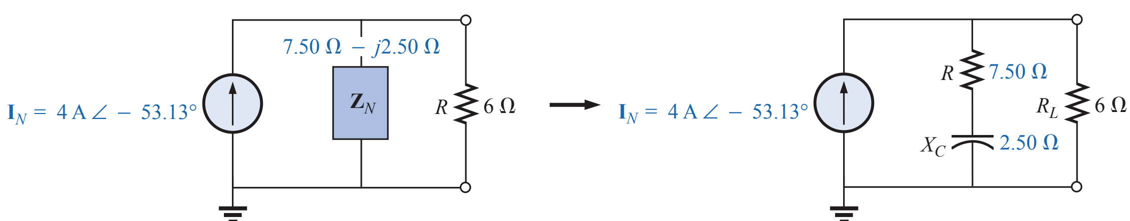


FIG. 18.65
The Norton equivalent circuit for the network of Fig. 18.61.

EXAMPLE 18.15 Find the Norton equivalent circuit for the network external to the 7-Ω capacitive reactance in Fig. 18.66.

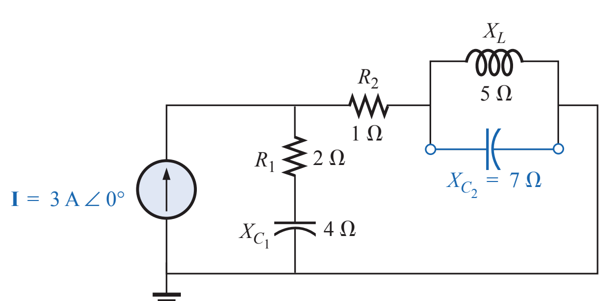


FIG. 18.66
Example 18.15.

Solution:

Steps 1 and 2 (Fig. 18.67):

$$\mathbf{Z}_1 = R_1 - j\,X_{C_1} = 2\ \Omega - j\,4\ \Omega$$
$$\mathbf{Z}_2 = R_2 = 1\ \Omega$$
$$\mathbf{Z}_3 = +j\,X_L = j\,5\ \Omega$$

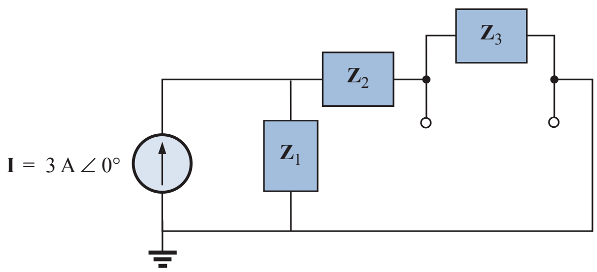


FIG. 18.67
Assigning the subscripted impedances to the network of Fig. 18.66.

Step 3 (Fig. 18.68):

$$\mathbf{Z}_N = \frac{\mathbf{Z}_3(\mathbf{Z}_1 + \mathbf{Z}_2)}{\mathbf{Z}_3 + (\mathbf{Z}_1 + \mathbf{Z}_2)}$$
$$\mathbf{Z}_1 + \mathbf{Z}_2 = 2\ \Omega - j\,4\ \Omega + 1\ \Omega = 3\ \Omega - j\,4\ \Omega = 5\ \Omega\ \angle -53.13^\circ$$
$$\mathbf{Z}_N = \frac{(5\ \Omega\ \angle 90^\circ)(5\ \Omega\ \angle -53.13^\circ)}{j\,5\ \Omega + 3\ \Omega - j\,4\ \Omega} = \frac{25\ \Omega\ \angle 36.87^\circ}{3 + j\,1}$$
$$= \frac{25\ \Omega\ \angle 36.87^\circ}{3.16\ \angle +18.43^\circ}$$
$$\mathbf{Z}_N = 7.91\ \Omega\ \angle 18.44^\circ = \mathbf{7.50\ \Omega + \mathit{j}\,2.50\ \Omega}$$

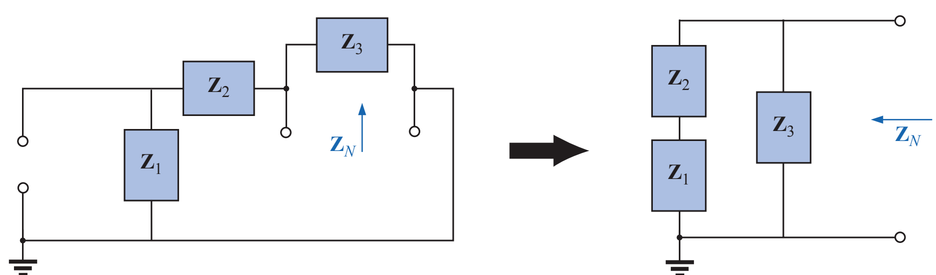


FIG. 18.68
Finding the Norton impedance for the network of Fig. 18.66.

Calculator Performing the above on the TI-86 calculator, we obtain the following:

```
((0,5)*((2,−4)+(1,0)))/((0,5)+((2,−4)+(1,0)))
  (7.500E0,2.500E0)
Ans ▶ Pol
  (7.906E0∠18.435E0)
```

CALC. 18.2

Step 4 (Fig. 18.69):

$$\mathbf{I}_N = \mathbf{I}_1 = \frac{\mathbf{Z}_1\mathbf{I}}{\mathbf{Z}_1 + \mathbf{Z}_2} \qquad \text{(current divider rule)}$$

$$= \frac{(2\ \Omega - j\,4\ \Omega)(3\ \text{A})}{3\ \Omega - j\,4\ \Omega} = \frac{6\ \text{A} - j\,12\ \text{A}}{5\ \angle -53.13^\circ} = \frac{13.4\ \text{A}\ \angle -63.43^\circ}{5\ \angle -53.13^\circ}$$

$$\mathbf{I}_N = \mathbf{2.68\ A\ \angle -10.3^\circ}$$

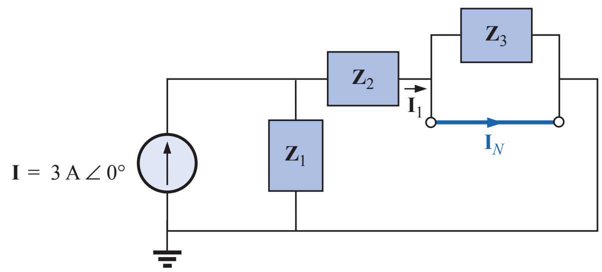


FIG. 18.69
Determining $\mathbf{I}_N$ for the network of Fig. 18.66.

Step 5: The Norton equivalent circuit is shown in Fig. 18.70.

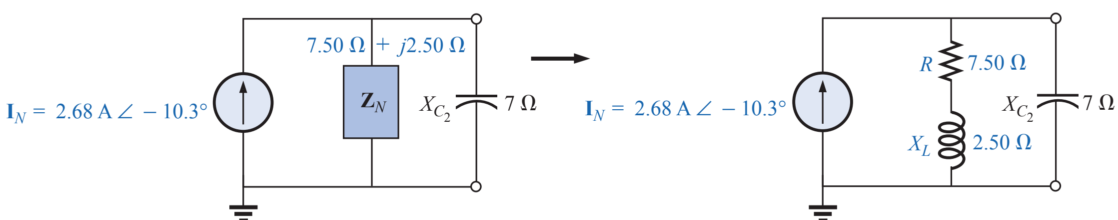


FIG. 18.70
The Norton equivalent circuit for the network of Fig. 18.66.

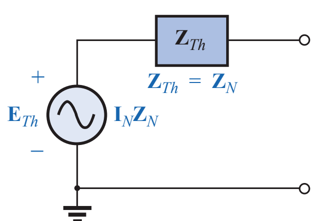


FIG. 18.71
Determining the Thévenin equivalent circuit for the Norton equivalent of Fig. 18.70.

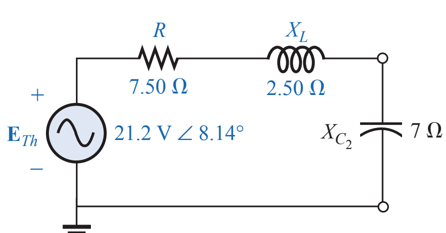


FIG. 18.72
The Thévenin equivalent circuit for the network of Fig. 18.66.

EXAMPLE 18.16 Find the Thévenin equivalent circuit for the network external to the 7-Ω capacitive reactance in Fig. 18.66.

Solution: Using the conversion between sources (Fig. 18.71), we obtain

$$\mathbf{Z}_{Th} = \mathbf{Z}_N = \mathbf{7.50\ \Omega + j\,2.50\ \Omega}$$

$$\mathbf{E}_{Th} = \mathbf{I}_N\mathbf{Z}_N = (2.68\ \text{A}\ \angle -10.3^\circ)(7.91\ \Omega\ \angle 18.44^\circ)$$

$$= \mathbf{21.2\ V\ \angle 8.14^\circ}$$

The Thévenin equivalent circuit is shown in Fig. 18.72.

Dependent Sources

As stated for Thévenin's theorem, *dependent sources in which the controlling variable is not determined by the network* for which the Norton equivalent circuit is to be found do not alter the procedure outlined above.

For dependent sources of the other kind, one of the following procedures must be applied. Both of these procedures can also be applied to networks with any combination of independent sources and dependent sources not controlled by the network under investigation.

The Norton equivalent circuit appears in Fig. 18.73(a). In Fig. 18.73(b), we find that

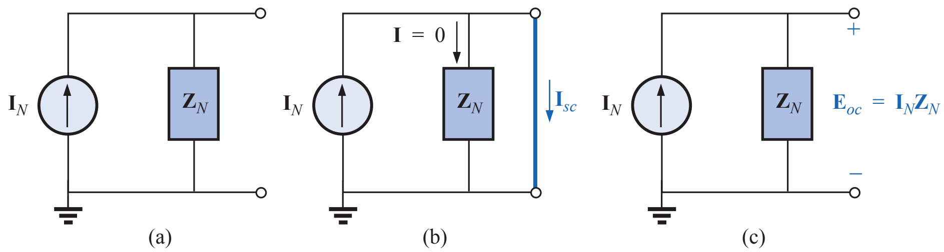


FIG. 18.73
Defining an alternative approach for determining $\mathbf{Z}_N$.

$$\boxed{\mathbf{I}_{sc} = \mathbf{I}_N} \tag{18.9}$$

and in Fig. 18.73(c) that

$$\mathbf{E}_{oc} = \mathbf{I}_N\mathbf{Z}_N$$

Or, rearranging, we have

$$\mathbf{Z}_N = \frac{\mathbf{E}_{oc}}{\mathbf{I}_N}$$

and

$$\boxed{\mathbf{Z}_N = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}}} \tag{18.10}$$

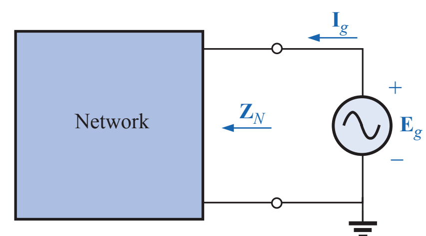


FIG. 18.74
Determining the Norton impedance using the approach $\mathbf{Z}_N = \mathbf{E}_g/\mathbf{I}_g$.

The Norton impedance can also be determined by applying a source of voltage $\mathbf{E}_g$ to the terminals of interest and finding the resulting $\mathbf{I}_g$, as shown in Fig. 18.74. All independent sources and dependent sources not controlled by a variable in the network of interest are set to zero, and

$$\boxed{\mathbf{Z}_N = \frac{\mathbf{E}_g}{\mathbf{I}_g}} \tag{18.11}$$

For this latter approach, the Norton current is still determined by the short-circuit current.

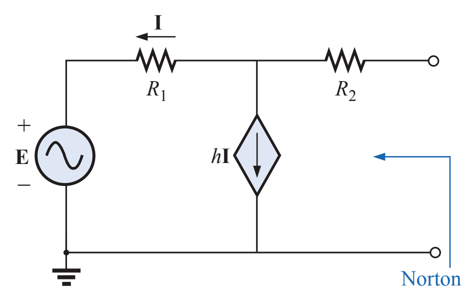


FIG. 18.75
Example 18.17.

EXAMPLE 18.17 Using each method described for dependent sources, find the Norton equivalent circuit for the network of Fig. 18.75.

Solution:

$\mathbf{I}_N$ For each method, $\mathbf{I}_N$ is determined in the same manner. From Fig. 18.76, using Kirchhoff's current law, we have

$$0 = \mathbf{I} + h\mathbf{I} + \mathbf{I}_{sc}$$

or

$$\mathbf{I}_{sc} = -(1 + h)\mathbf{I}$$

Applying Kirchhoff's voltage law gives us

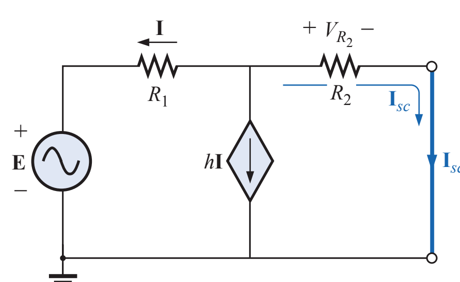


FIG. 18.76
Determining $\mathbf{I}_{sc}$ for the network of Fig. 18.75.

$$\mathbf{E} + \mathbf{I}R_1 - \mathbf{I}_{sc}R_2 = 0$$

and
$$\mathbf{I}R_1 = \mathbf{I}_{sc}R_2 - \mathbf{E}$$

or
$$\mathbf{I} = \frac{\mathbf{I}_{sc}R_2 - \mathbf{E}}{R_1}$$

so
$$\mathbf{I}_{sc} = -(1 + h)\mathbf{I} = -(1 + h)\left(\frac{\mathbf{I}_{sc}R_2 - \mathbf{E}}{R_1}\right)$$

or
$$R_1\mathbf{I}_{sc} = -(1 + h)\mathbf{I}_{sc}R_2 + (1 + h)\mathbf{E}$$
$$\mathbf{I}_{sc}[R_1 + (1 + h)R_2] = (1 + h)\mathbf{E}$$
$$\mathbf{I}_{sc} = \boldsymbol{\frac{(1 + h)\mathbf{E}}{R_1 + (1 + h)R_2}} = \mathbf{I}_N$$

$\mathbf{Z}_N$

Method 1: $\mathbf{E}_{oc}$ is determined from the network of Fig. 18.77. By Kirchhoff's current law,

$$0 = \mathbf{I} + h\mathbf{I} \quad \text{or} \quad \mathbf{I}(h + 1) = 0$$

For h, a positive constant $\mathbf{I}$ must equal zero to satisfy the above. Therefore,

$$\mathbf{I} = 0 \quad \text{and} \quad h\mathbf{I} = 0$$

and
$$\mathbf{E}_{oc} = \mathbf{E}$$

with
$$\mathbf{Z}_N = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{\mathbf{E}}{\dfrac{(1 + h)\mathbf{E}}{R_1 + (1 + h)R_2}} = \boldsymbol{\frac{R_1 + (1 + h)R_2}{(1 + h)}}$$

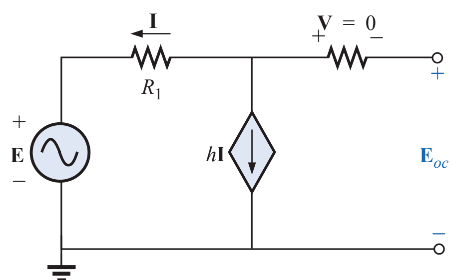


FIG. 18.77
Determining $\mathbf{E}_{oc}$ for the network of Fig. 18.75.

Method 2: Note Fig. 18.78. By Kirchhoff's current law,

$$\mathbf{I}_g = \mathbf{I} + h\mathbf{I} = (1 + h)\mathbf{I}$$

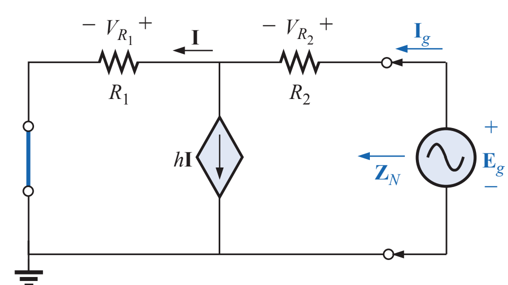


FIG. 18.78
Determining the Norton impedance using the approach $\mathbf{Z}_N = \mathbf{E}_g/\mathbf{E}_g$.

By Kirchhoff's voltage law,

$$\mathbf{E}_g - \mathbf{I}_gR_2 - \mathbf{I}R_1 = 0$$

or
$$\mathbf{I} = \frac{\mathbf{E}_g - \mathbf{I}_gR_2}{R_1}$$

Substituting, we have

$$\mathbf{I}_g = (1 + h)\mathbf{I} = (1 + h)\left(\frac{\mathbf{E}_g - \mathbf{I}_gR_2}{R_1}\right)$$

and
$$\mathbf{I}_gR_1 = (1 + h)\mathbf{E}_g - (1 + h)\mathbf{I}_gR_2$$

so $$\mathbf{E}_g(1 + h) = \mathbf{I}_g[R_1 + (1 + h)R_2]$$

or $$\mathbf{Z}_N = \frac{\mathbf{E}_g}{\mathbf{I}_g} = \frac{\boldsymbol{R_1 + (1 + h)R_2}}{\mathbf{1 + \boldsymbol{h}}}$$

which agrees with the above.

EXAMPLE 18.18 Find the Norton equivalent circuit for the network configuration of Fig. 18.56.

Solution: By source conversion,

$$\mathbf{I}_N = \frac{\mathbf{E}_{Th}}{\mathbf{Z}_{Th}} = \frac{\dfrac{-k_2R_2\mathbf{V}_i}{R_1 - k_1k_2R_2}}{\dfrac{R_1R_2}{R_1 - k_1k_2R_2}}$$

and $$\boxed{\mathbf{I}_N = \frac{-k_2\mathbf{V}_i}{R_1}} \qquad \mathbf{(18.12)}$$

which is $\mathbf{I}_{sc}$ as determined in Example 18.13, and

$$\boxed{\mathbf{Z}_N = \mathbf{Z}_{Th} = \frac{R_2}{1 - \dfrac{k_1k_2R_2}{R_1}}} \qquad \mathbf{(18.13)}$$

For $k_1 \cong 0$, we have

$$\boxed{\mathbf{I}_N = \frac{-k_2\mathbf{V}_i}{R_1}} \qquad k_1 = 0 \qquad \mathbf{(18.14)}$$

$$\boxed{\mathbf{Z}_N = R_2} \qquad k_1 = 0 \qquad \mathbf{(18.15)}$$

18.5 MAXIMUM POWER TRANSFER THEOREM

When applied to ac circuits, the **maximum power transfer theorem** states that

maximum power will be delivered to a load when the load impedance is the conjugate of the Thévenin impedance across its terminals.

That is, for Fig. 18.79, for maximum power transfer to the load,

$$\boxed{Z_L = Z_{Th} \quad \text{and} \quad \theta_L = -\theta_{Th_Z}} \qquad \mathbf{(18.16)}$$

or, in rectangular form,

$$\boxed{R_L = R_{Th} \quad \text{and} \quad \pm j\, X_{\text{load}} = \mp j\, X_{Th}} \qquad \mathbf{(18.17)}$$

The conditions just mentioned will make the total impedance of the circuit appear purely resistive, as indicated in Fig. 18.80:

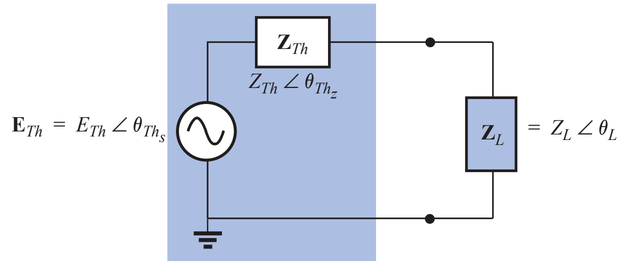


FIG. 18.79
Defining the conditions for maximum power transfer to a load.

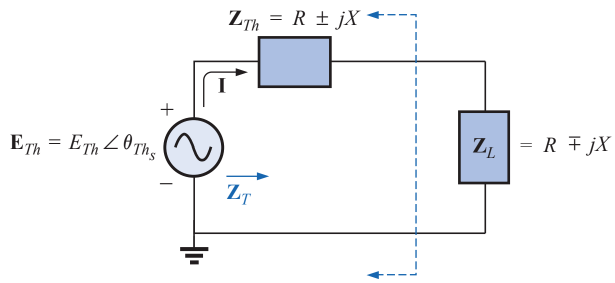


FIG. 18.80
Conditions for maximum power transfer to $\mathbf{Z}_L$.

$$\mathbf{Z}_T = (R \pm j\,X) + (R \mp j\,X)$$

and
$$\boxed{\mathbf{Z}_T = 2R} \tag{18.18}$$

Since the circuit is purely resistive, the power factor of the circuit under maximum power conditions is 1; that is,

$$\boxed{F_p = 1} \qquad \text{(maximum power transfer)} \tag{18.19}$$

The magnitude of the current **I** of Fig. 18.80 is

$$I = \frac{E_{Th}}{Z_T} = \frac{E_{Th}}{2R}$$

The maximum power to the load is

$$P_{\max} = I^2R = \left(\frac{E_{Th}}{2R}\right)^2 R$$

and
$$\boxed{P_{\max} = \frac{E_{Th}^2}{4R}} \tag{18.20}$$

EXAMPLE 18.19 Find the load impedance in Fig. 18.81 for maximum power to the load, and find the maximum power.

Solution: Determine $\mathbf{Z}_{Th}$ [Fig. 18.82(a)]:

$$\mathbf{Z}_1 = R - j\,X_C = 6\,\Omega - j\,8\,\Omega = 10\,\Omega \angle -53.13^\circ$$
$$\mathbf{Z}_2 = +j\,X_L = j\,8\,\Omega$$

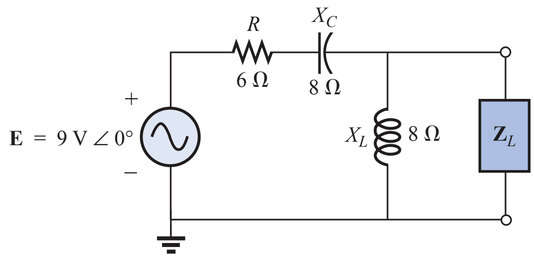


FIG. 18.81
Example 18.19.

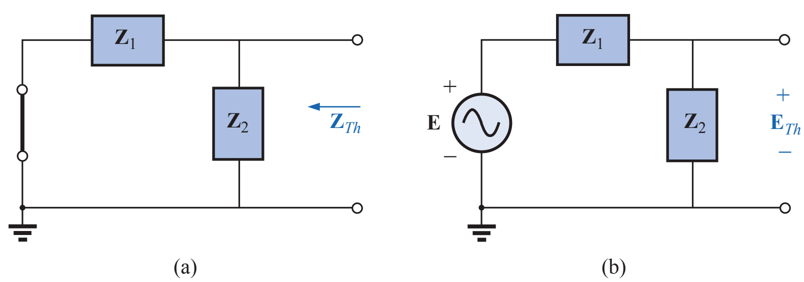


FIG. 18.82
Determining (a) $\mathbf{Z}_{Th}$ and (b) $\mathbf{E}_{Th}$ for the network external to the load in Fig. 18.81.

$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(10\ \Omega\ \angle -53.13^\circ)(8\ \Omega\ \angle 90^\circ)}{6\ \Omega - j\,8\ \Omega + j\,8\ \Omega} = \frac{80\ \Omega\ \angle 36.87^\circ}{6\ \angle 0^\circ}$$
$$= 13.33\ \Omega\ \angle 36.87^\circ = 10.66\ \Omega + j\,8\ \Omega$$

and $\qquad \mathbf{Z}_L = 13.3\ \Omega\ \angle -36.87^\circ = \mathbf{10.66\ \Omega - \mathit{j}\,8\ \Omega}$

To find the maximum power, we must first find $\mathbf{E}_{Th}$ [Fig. 18.82(b)], as follows:

$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_1} \qquad \text{(voltage divider rule)}$$
$$= \frac{(8\ \Omega\ \angle 90^\circ)(9\ \text{V}\ \angle 0^\circ)}{j\,8\ \Omega + 6\ \Omega - j\,8\ \Omega} = \frac{72\ \text{V}\ \angle 90^\circ}{6\ \angle 0^\circ} = 12\ \text{V}\ \angle 90^\circ$$

Then $\qquad P_{\text{max}} = \dfrac{E_{Th}^2}{4R} = \dfrac{(12\ \text{V})^2}{4(10.66\ \Omega)} = \dfrac{144}{42.64} = \mathbf{3.38\ W}$

EXAMPLE 18.20 Find the load impedance in Fig. 18.83 for maximum power to the load, and find the maximum power.

Solution: First we must find $\mathbf{Z}_{Th}$ (Fig. 18.84).

$$\mathbf{Z}_1 = +j\,X_L = j\,9\ \Omega \qquad \mathbf{Z}_2 = R = 8\ \Omega$$

Converting from a Δ to a Y (Fig. 18.85), we have

$$\mathbf{Z}'_1 = \frac{\mathbf{Z}_1}{3} = j\,3\ \Omega \qquad \mathbf{Z}_2 = 8\ \Omega$$

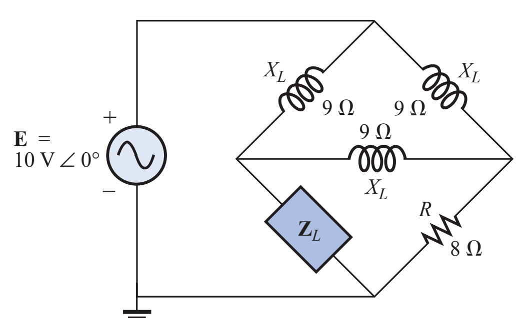


FIG. 18.83
Example 18.20.

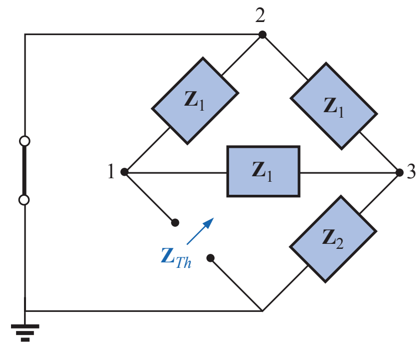


FIG. 18.84
Defining the subscripted impedances for the network of Fig. 18.83.

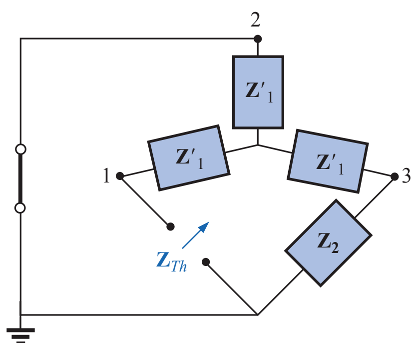


FIG. 18.85
Substituting the Y equivalent for the upper Δ configuration of Fig. 18.84.

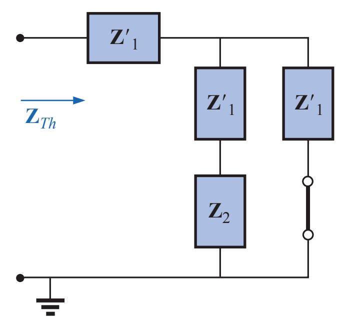


FIG. 18.86
Determining $\mathbf{Z}_{Th}$ for the network of Fig. 18.83.

The redrawn circuit (Fig. 18.86) shows

$$\mathbf{Z}_{Th} = \mathbf{Z}'_1 + \frac{\mathbf{Z}'_1(\mathbf{Z}'_1 + \mathbf{Z}_2)}{\mathbf{Z}'_1 + (\mathbf{Z}'_1 + \mathbf{Z}_2)}$$
$$= j\,3\ \Omega + \frac{3\ \Omega\ \angle 90°(j\,3\ \Omega + 8\ \Omega)}{j\,6\ \Omega + 8\ \Omega}$$
$$= j\,3 + \frac{(3\ \angle 90°)(8.54\ \angle 20.56°)}{10\ \angle 36.87°}$$
$$= j\,3 + \frac{25.62\ \angle 110.56°}{10\ \angle 36.87°} = j\,3 + 2.56\ \angle 73.69°$$
$$= j\,3 + 0.72 + j\,2.46$$
$$\mathbf{Z}_{Th} = 0.72\ \Omega + j\,5.46\ \Omega$$

and
$$\mathbf{Z}_L = \mathbf{0.72\ \Omega - j\,5.46\ \Omega}$$

For $\mathbf{E}_{Th}$, use the modified circuit of Fig. 18.87 with the voltage source replaced in its original position. Since $I_1 = 0$, $\mathbf{E}_{Th}$ is the voltage across the series impedance of $\mathbf{Z}'_1$ and $\mathbf{Z}_2$. Using the voltage divider rule gives us

$$\mathbf{E}_{Th} = \frac{(\mathbf{Z}'_1 + \mathbf{Z}_2)\mathbf{E}}{\mathbf{Z}'_1 + \mathbf{Z}_2 + \mathbf{Z}'_1} = \frac{(j\,3\ \Omega + 8\ \Omega)(10\ \text{V}\ \angle 0°)}{8\ \Omega + j\,6\ \Omega}$$
$$= \frac{(8.54\ \angle 20.56°)(10\ \text{V}\ \angle 0°)}{10\ \angle 36.87°}$$
$$\mathbf{E}_{Th} = 8.54\ \text{V}\ \angle -16.31°$$

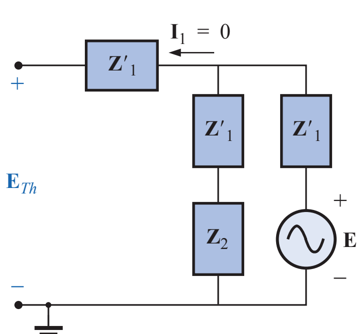


FIG. 18.87
Finding the Thévenin voltage for the network of Fig. 18.83.

and $$P_{max} = \frac{E_{Th}^2}{4R} = \frac{(8.54\text{ V})^2}{4(0.72\ \Omega)} = \frac{72.93}{2.88}\text{ W}$$
$$= \mathbf{25.32\ W}$$

If the load resistance is adjustable but the magnitude of the load reactance cannot be set equal to the magnitude of the Thévenin reactance, then the maximum power *that can be delivered* to the load will occur when the load reactance is made as close to the Thévenin reactance as possible and the load resistance is set to the following value:

$$\boxed{R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_{load})^2}} \quad \textbf{(18.21)}$$

where each reactance carries a positive sign if inductive and a negative sign if capacitive.

The power delivered will be determined by

$$\boxed{P = E_{Th}^2/4R_{av}} \quad \textbf{(18.22)}$$

where $$\boxed{R_{av} = \frac{R_{Th} + R_L}{2}} \quad \textbf{(18.23)}$$

The derivation of the above equations is given in Appendix G of the text. The following example demonstrates the use of the above.

EXAMPLE 18.21 For the network of Fig. 18.88:

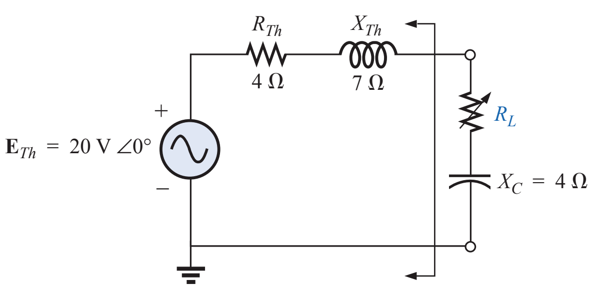


FIG. 18.88
Example 18.21.

a. Determine the value of R_L for maximum power to the load if the load reactance is fixed at 4 Ω.
b. Find the power delivered to the load under the conditions of part (a).
c. Find the maximum power to the load if the load reactance is made adjustable to any value, and compare the result to part (b) above.

Solutions:

a. Eq. (18.21): $$R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_{load})^2}$$
$$= \sqrt{(4\ \Omega)^2 + (7\ \Omega - 4\ \Omega)^2}$$

$$= \sqrt{16 + 9} = \sqrt{25}$$

$$R_L = \mathbf{5\ \Omega}$$

b. Eq. (18.23): $$R_{av} = \frac{R_{Th} + R_L}{2} = \frac{4\ \Omega + 5\ \Omega}{2}$$

$$= \mathbf{4.5\ \Omega}$$

Eq. (18.22): $$P = \frac{E_{Th}^2}{4R_{av}}$$

$$= \frac{(20\ \text{V})^2}{4(4.5\ \Omega)} = \frac{400}{18}\ \text{W}$$

$$\cong \mathbf{22.22\ W}$$

c. For $\mathbf{Z}_L = 4\ \Omega - j\ 7\ \Omega$,

$$P_{max} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(20\ \text{V})^2}{4(4\ \Omega)}$$

$$= \mathbf{25\ W}$$

exceeding the result of part (b) by 2.78 W.

18.6 SUBSTITUTION, RECIPROCITY, AND MILLMAN'S THEOREMS

As indicated in the introduction to this chapter, the **substitution** and **reciprocity theorems** and **Millman's theorem** will not be considered here in detail. A careful review of Chapter 9 will enable you to apply these theorems to sinusoidal ac networks with little difficulty. A number of problems in the use of these theorems appear in the problems section at the end of the chapter.

18.7 APPLICATIONS

Soldering Gun

Soldering and welding are two operations that are best performed by the application of heat that is unaffected by the thermal characteristics of the materials involved. In other words, the heat applied should not be sensitive to the changing parameters of the welding materials, the metals involved, or the welding conditions. The arc (a heavy current) established in the welding process should remain fixed in magnitude to ensure an even weld. This is best accomplished by ensuring a fixed current through the system even though the load characteristics may change—that is, by ensuring a constant current supply of sufficient amperage to establish the required arc for the welding equipment or even heating of the soldering iron tip. A further requirement for the soldering process is that the heat developed be sufficient to raise the solder to its melting point of about 800°F.

The soldering gun of Fig. 18.89(a) employs a unique approach to establishing a fixed current through the soldering tip. The soldering tip is actually part of a secondary winding of a transformer (Chapter 21) having only one turn as its secondary as shown in Fig. 18.89(b). Because of the heavy currents that will be established in this single-turn secondary, it is quite large in size to ensure that it can handle the current and to minimize its resistance level. The primary of the transformer

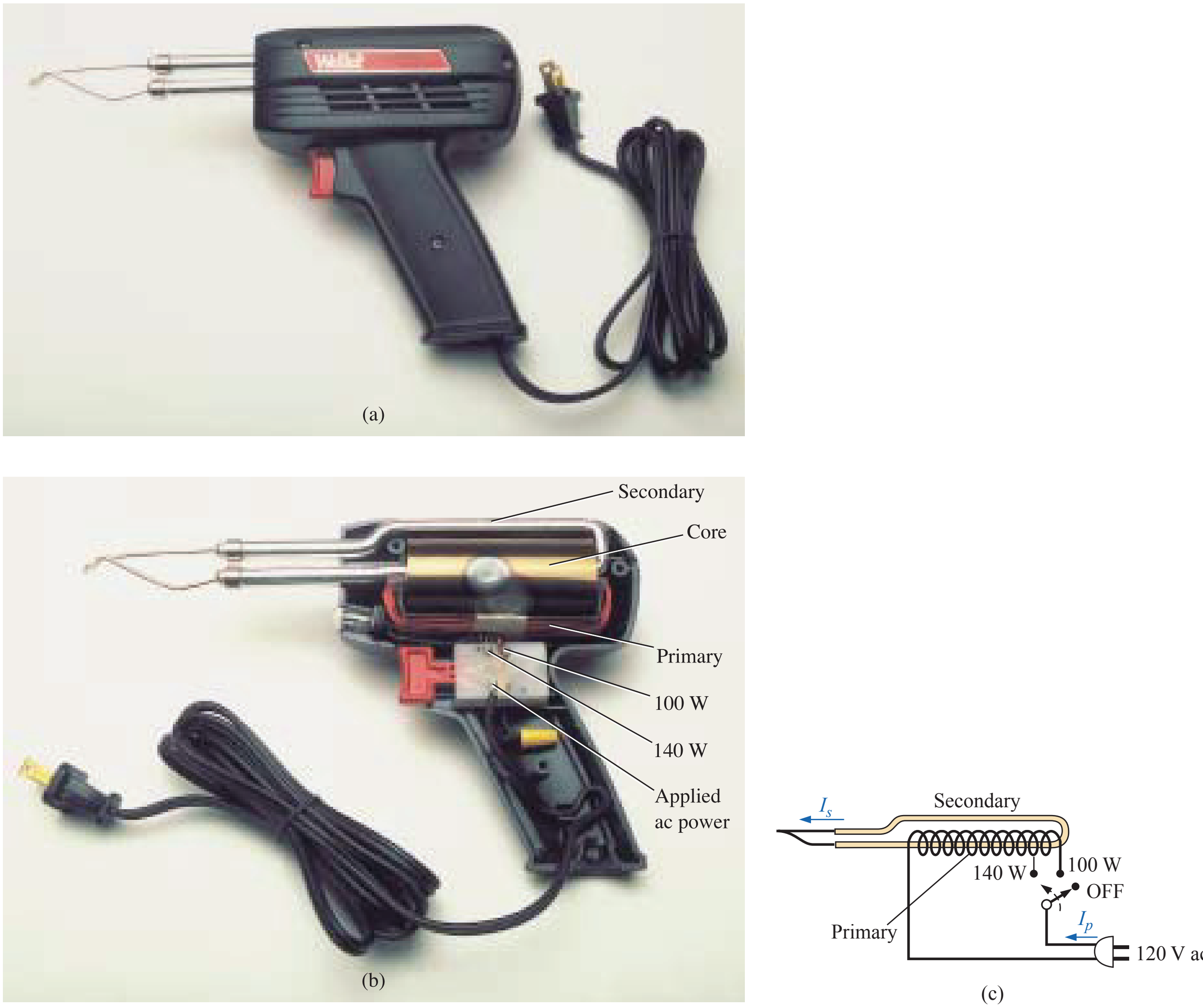


FIG. 18.89
Soldering gun: (a) appearance; (b) internal construction; (c) turns ratio control.

has many turns of thinner wire to establish the turns ratio necessary to establish the required current in the secondary. The Universal® unit of Fig. 18.89 is rated 140 W/100 W, indicating that it has two levels of power controlled by the trigger. As you pull the trigger, the first setting will be at 140 W, and a fully depressed trigger will provide 100 W of power. The inductance of the primary is 285 mH at the 140-W setting and 380 mH at the 100-W setting, indicating that the switch controls how many windings of the primary will be part of the transformer action for each wattage rating, as shown in Fig. 18.89(c). Since inductance is a direct function of the number of turns, the 140-W setting has fewer turns than the 100-W setting. The dc resistance of the primary was found to be about 11.2 Ω for the 140-W setting and 12.8 Ω for the 100-W setting, which makes sense also since more turns will require a longer wire and the resistance should increase accordingly.

Under rated operating conditions, the primary current for each setting can be determined using Ohm's law in the following manner:

For 140 W,

$$I_p = \frac{P}{V_p} = \frac{140\text{ W}}{120\text{ V}} = \mathbf{1.17\text{ A}}$$

For 100 W,

$$I_p = \frac{P}{V_p} = \frac{100\text{ W}}{120\text{ V}} = \mathbf{0.83\text{ A}}$$

As expected, the current demand is more for the 140-W setting than for the 100-W setting. Using the measured values of input inductance and resistance for the 140-W setting, the equivalent circuit of Fig. 18.90(a) will result. Using the applied 60 Hz to determine the reactance of the coil and then determining the total impedance seen by the primary will result in the following for the source current:

$$X_L = 2\pi f L = 2\pi(60\text{ Hz})(285\text{ mH}) = 107.44\ \Omega$$

and $$\mathbf{Z}_T = R + j\,X_L = 11.2\ \Omega + j\,107.44\ \Omega = 108.02\ \Omega \angle 84.05^\circ$$

so that $$|I_p| = \left|\frac{E}{Z_T}\right| = \frac{120\text{ V}}{108.02\ \Omega} = \mathbf{1.11\text{ A}}$$

which is a close match with the rated level.

For the 100-W level of Fig. 18.90(b), the following analysis would result:

$$X_L = 2\pi f L = 2\pi(60\text{ Hz})(380\text{ mH}) = 143.26\ \Omega$$

and $$\mathbf{Z}_T = R + j\,X_L = 12.8\ \Omega + j\,143.26\ \Omega = 143.83\ \Omega \angle 84.89^\circ$$

so that $$|I_p| = \left|\frac{E}{Z_T}\right| = \frac{120\text{ V}}{143.83\ \Omega} = \mathbf{0.83\text{ A}}$$

which is a match to hundredths place with the value calculated from rated conditions.

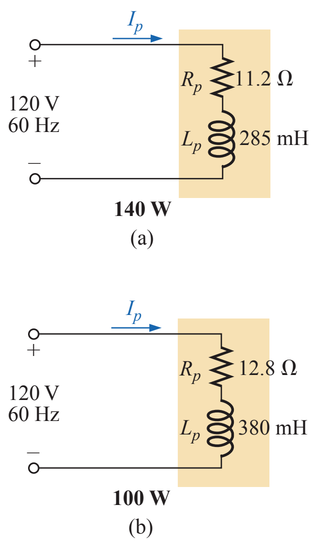


FIG. 18.90
Equivalent circuits for the soldering iron of Fig. 18.89(a): (a) at 140-W setting; (b) at 100-W setting.

Removing the tip and measuring the primary and secondary voltages resulted in 120 V/0.38 V for the 140-W setting and 120 V/0.31 V for the 100-W setting, respectively. Since the voltages of a transformer are directly related to the turns ratio, the number of turns in the primary (N_p) to that of the secondary (N_s) can be estimated by the following for each setting:

For 140 W,

$$\frac{N_p}{N_s} = \frac{120\text{ V}}{0.38\text{ V}} \cong \mathbf{316}$$

For 100 W,

$$\frac{N_p}{N_s} = \frac{120\text{ V}}{0.31\text{ V}} \cong \mathbf{387}$$

Looking at the photograph of Fig. 18.89(b), it would certainly appear that there are 300 or more turns in the primary winding.

The currents of a transformer are related by the turns ratio in the following manner, permitting a calculation of the secondary currents for each setting:

For 140 W,

$$I_s = \frac{N_p}{N_s} I_p = 316(1.17\text{ A}) \cong \mathbf{370\text{ A}}$$

For 100 W,

$$I_s = \frac{N_p}{N_s} I_p = 387(0.83 \text{ A}) \cong \mathbf{321 \text{ A}}$$

Quite clearly, the secondary current is much higher for the 140-W setting. The resulting current levels are probably higher than you might have expected, but keep in mind that the above analysis does not include the effect of the reflected impedance from the secondary to the primary that will reduce the primary current level (to be discussed in Chapter 21). In addition, as the soldering tip heats up, its resistance increases, further reducing the resulting current levels. Using an Amp-Clamp®, the current in the secondary was found to exceed 300 A when the power was first applied and the soldering tip was cold. However, as the tip heated up because of the high current levels, the current through the primary dropped to about 215 A for the 140-W setting and to 180 A for the 100-W setting. These high currents are part of the reason that the lifetime of most soldering tips on soldering guns is about 20 hours. Eventually, the tip will simply begin to melt. Using these levels of current and the given power rating, the resistance of the secondary can be approximated as follows:

For 140 W,

$$R = \frac{P}{I^2} = \frac{140 \text{ W}}{(215 \text{ A})^2} \cong \mathbf{3 \text{ m}\Omega}$$

For 100 W,

$$R = \frac{P}{I^2} = \frac{100 \text{ W}}{(180 \text{ A})^2} \cong \mathbf{3 \text{ m}\Omega}$$

which is as low as expected when you consider the cross-sectional area of the secondary and the fact that the tip is a short section of low-resistance, tin-plated copper.

One of the obvious advantages of the soldering gun versus the iron is that the iron is off when you release the trigger, thus reducing energy costs and extending the life of the tip. Applying dc current rather than ac to develop a constant current would be impractical because the high current demand would require a series of large batteries in parallel.

The above investigation was particularly interesting because of the manner in which the constant current characteristic was established, the levels of current established, and the excellent manner in which some of the theory introduced in the text was verified.

Electronic Systems

One of the blessings in the analysis of electronic systems is that the superposition theorem can be applied so that the dc analysis and ac analysis can be performed separately. The analysis of the dc system will affect the ac response, but the analysis of each is a distinct, separate process. Even though electronic systems have not been investigated in this text, a number of important points can be made in the description to follow that support some of the theory presented in this and recent chapters, so inclusion of this description is totally valid at this point. Consider the network of Fig. 18.91 with a transistor power amplifier, an 8-Ω speaker as the load, and a source with an internal resistance of 800 Ω. Note that each component of the design was isolated by a color box

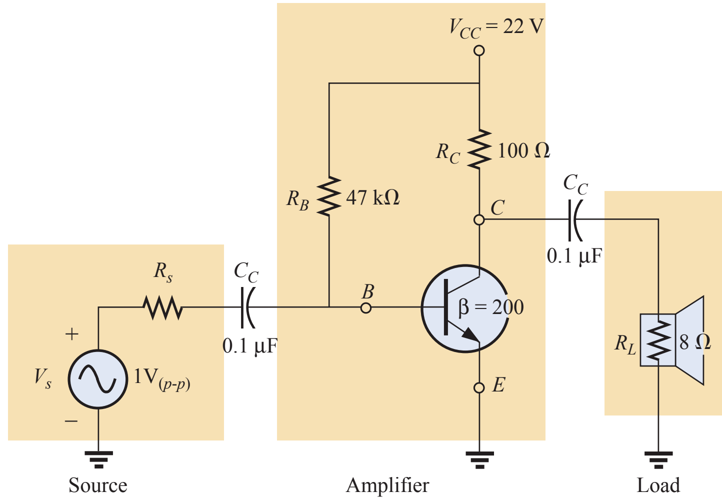


FIG. 18.91
Transistor amplifier.

to emphasize the fact that each component must be carefully weighed in any good design.

As mentioned above, the analysis can be separated into a dc and an ac component. For the dc analysis the two capacitors can be replaced by an open-circuit equivalent (Chapter 10), resulting in an isolation of the amplifier network as shown in Fig. 18.92. Given the fact that V_{BE} will be about 0.7 V dc for any operating transistor, the base current I_B can be found as follows using Kirchhoff's voltage law:

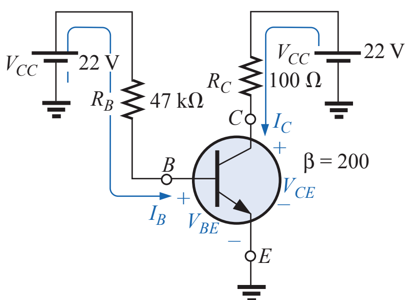


FIG. 18.92
dc equivalent of the transistor network of Fig. 18.91.

$$I_B = \frac{V_{R_B}}{R_B} = \frac{V_{CC} - V_{BE}}{R_B} = \frac{22\text{ V} - 0.7\text{ V}}{47\text{ k}\Omega} = \mathbf{453.2\ \mu A}$$

For transistors, the collector current I_C is related to the base current by $I_C = \beta I_B$, and

$$I_C = \beta I_B = (200)(453.2\ \mu\text{A}) = \mathbf{90.64\ mA}$$

Finally, through Kirchhoff's voltage law, the collector voltage (also the collector-to-emitter voltage since the emitter is grounded) can be determined as follows:

$$V_C = V_{CE} = V_{CC} - I_C R_C = 22\text{ V} - (90.64\text{ mA})(100\ \Omega) = \mathbf{12.94\ V}$$

For the dc analysis, therefore,

$$I_B = \mathbf{453.2\ \mu A} \qquad I_C = \mathbf{90.64\ mA} \qquad V_{CE} = \mathbf{12.94\ V}$$

which will define a point of dc operation for the transistor. This is an important aspect of electronic design since the dc operating point will have an effect on the ac gain of the network.

Now, using superposition, we can analyze the network from an ac viewpoint by setting all dc sources to zero (replaced by ground connections) and replacing both capacitors by short circuits as shown in Fig. 18.93. Substituting the short-circuit equivalent for the capacitors is valid because at 10 kHz (the midrange for human hearing response), the reactance of the capacitor is determined by $X_C = 1/2\pi fC = 15.92\ \Omega$ which can be ignored when compared to the series resistors at the source and load. In other words, the capacitor has played the important role of isolating the amplifier for the dc response and completing the network for the ac response.

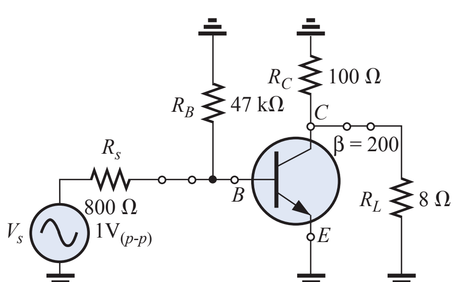


FIG. 18.93
ac equivalent of the transistor network of Fig. 18.91.

Redrawing the network as shown in Fig. 18.94(a) will permit an ac investigation of its reponse. The transistor has now been replaced by an equivalent network that will represent the behavior of the device. This process will be covered in detail in your basic electronics courses. This transistor configuration has an input impedance of 200 Ω and a current source whose magnitude is sensitive to the base current in the input circuit and to the amplifying factor for this transistor of 200. The 47-kΩ resistor in parallel with the 200-Ω input impedance of the transistor can be ignored, so the input current I_i and base current I_b are determined by

$$I_i \cong I_b = \frac{V_s}{R_s + R_i} = \frac{1\text{ V }(p\text{-}p)}{800\ \Omega + 200\ \Omega} = \frac{1\text{ V }(p\text{-}p)}{1\text{ k}\Omega} = 1\text{ mA }(p\text{-}p)$$

The collector current I_C is then

$$I_C = \beta I_b = (200)(1\text{ mA }(p\text{-}p)) = 200\text{ mA }(p\text{-}p)$$

and the current to the speaker is determined by the current divider rule as follows:

$$I_L = \frac{100\ \Omega(I_C)}{100\ \Omega + 8\ \Omega} = 0.926 I_C = 0.926(200\text{ mA }(p\text{-}p))$$
$$= 185.2\text{ mA }(p\text{-}p)$$

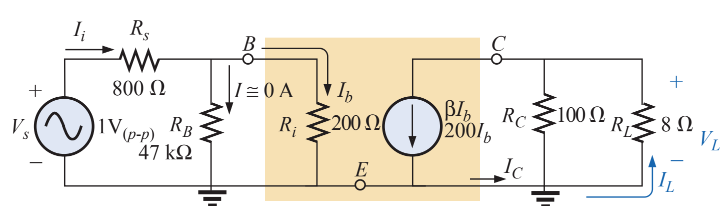
with the voltage across the speaker being

$$V_L = -I_L R_L = -(185.2\text{ mA }(p\text{-}p))(8\ \Omega) = -1.48\text{ V}$$

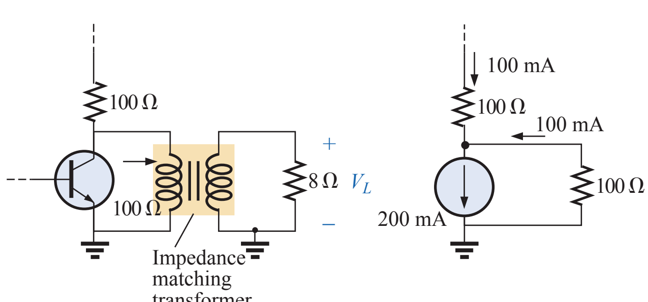
The power to the speaker is then determined as follows:

$$P_{\text{speaker}} = V_{L_{\text{rms}}} \cdot I_{L_{\text{rms}}} = \frac{(V_{L(p\text{-}p)})(I_{L(p\text{-}p)})}{8} = \frac{(1.48\text{ V})(185.2\text{ mA }(p\text{-}p))}{8}$$
$$= \mathbf{34.26\ mW}$$

which is relatively low. It would initially appear that the above was a good design for distribution of power to the speaker because a majority



(a)



(b)

FIG. 18.94

(a) Network of Fig. 18.93 following the substitution of the transistor equivalent network; (b) effect of the matching transformer.

of the collector current went to the speaker. However, one must always keep in mind that power is the product of voltage and current. A high current with a very low voltage will result in a lower power level. In this case, the voltage level is too low. However, if we introduce a matching transformer that makes the 8-Ω resistive load "look like" 100 Ω as shown in Fig. 18.94(b), establishing maximum power conditions, the current to the load will drop to half of the previous amount because current splits through equal resistors. But the voltage across the load will increase to

$$V_L = I_L R_L = (100 \text{ mA } (p\text{-}p))(100\ \Omega) = 10 \text{ V } (p\text{-}p)$$

and the power level to

$$P_{\text{speaker}} = \frac{(V_{L(p\text{-}p)})(I_{L(p\text{-}p)})}{8} = \frac{(10 \text{ V})(100 \text{ mA})}{8} = \mathbf{125\ mW}$$

which is 3.6 times the gain without the matching transformer.

For the 100-Ω load, the dc conditions are unaffected due to the isolation of the capacitor C_C, and the voltage at the collector is 12.94 V as shown in Fig. 18.95(a). For the ac response with a 100-Ω load, the output voltage as determined above will be 10 V peak-to-peak (5 V peak) as shown in Fig. 18.95(b). Note the out-of-phase relationship with the input due to the opposite polarity of V_L. The full response at the collector terminal of the transistor can then be drawn by superimposing the ac response on the dc response as shown in Fig. 18.95(c) (another application of the superposition theorem). In other words, the dc level simply shifts the ac waveform up or down and does not disturb its shape. The peak-to-peak value remains the same, and the phase relationship is unaltered. The total waveform at the load will include only the ac response of Fig. 18.95(b) since the dc component has been blocked out by the capacitor.

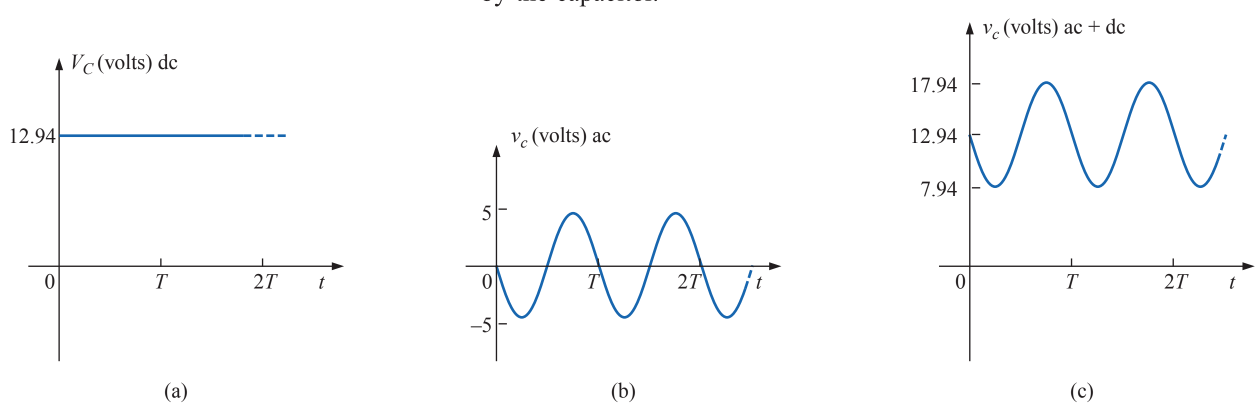


FIG. 18.95
Collector voltage for the network of Fig. 18.91: (a) dc; (b) ac; (c) dc and ac.

The voltage at the source will appear as shown in Fig. 18.96(a), while the voltage at the base of the transistor will appear as shown in Fig. 18.96(b) because of the presence of the dc component.

A number of important concepts were presented in the above example, with some probably leaving a question or two because of your lack of experience with transistors. However, if nothing else is evident from the above example, it should be the power of the superposition theorem to permit an isolation of the dc and ac responses and the ability to combine both if the total response is desired.

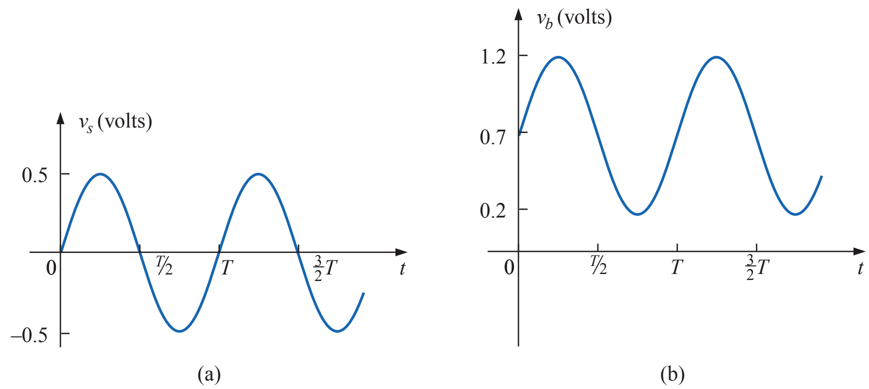


FIG. 18.96
Applied signal: (a) at the source; (b) at the base of the transistor.

18.8 COMPUTER ANALYSIS

PSpice

Superposition The analysis will begin with the network of Fig. 18.12 from Example 18.4 because it has both an ac and a dc source. You will find that it is not necessary to specify an analysis for each, even though one is essentially an ac sweep and the other is a bias point calculation. When **AC Sweep** is selected, the program will automatically perform the bias calculations and display the results in the output file.

The resulting schematic appears in Fig. 18.97 with **VSIN** and **VDC** as the **SOURCE** voltages. The placement of all the *R*-*L*-*C* elements and

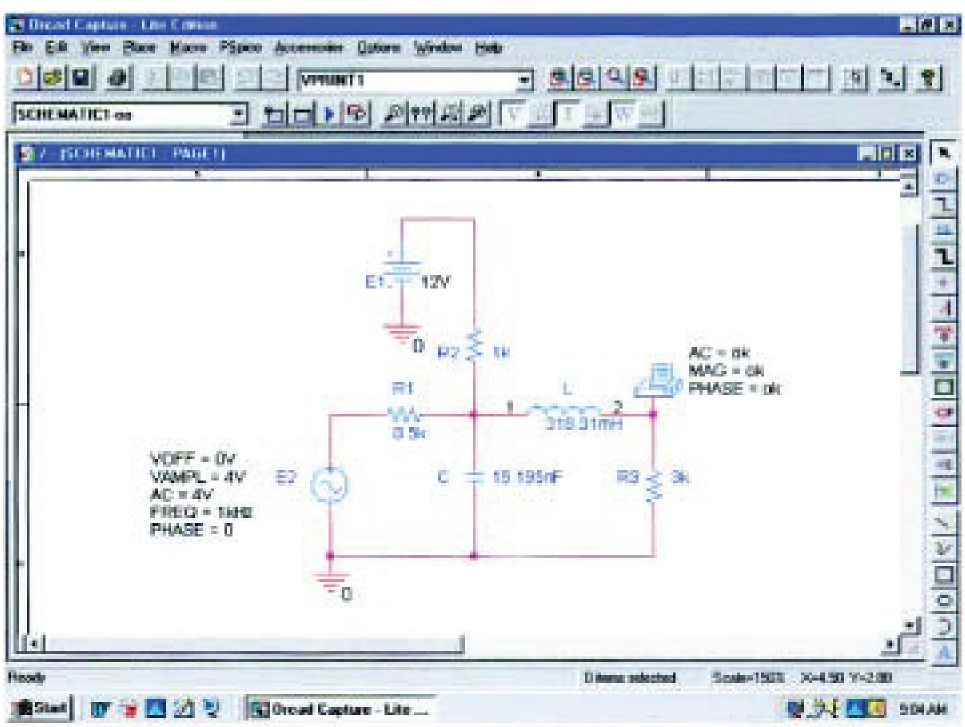


FIG. 18.97
Using PSpice to apply superposition to the network of Fig. 18.12.

the dc source should be quite straightforward at this point. For the ac source, be sure to double-click on the source symbol to obtain the **Property Editor** dialog box. Then set **AC** to 4 V, **FREQ** to 1 kHz, **PHASE** to 0°, **VAMPL** to 4 V, and **VOFF** to 0 V. In each case choose **Name and Value** under the **Display** heading so that we have a review of the parameters on the screen. Also, be sure to **Apply** before exiting the dialog box. Obtain the **VPRINT1** option from the **SPECIAL** library, place it as shown, and then double-click to obtain its **Property Editor**. The parameters **AC, MAG,** and **PHASE** must then recieve the **OK** listing, and **Name and Value** must be applied to each under **Display** before you choose **Apply** and **OK**. The network is then ready for simulation.

After you have selected the **New Simulation Profile** icon, the **New Simulation** dialog box will appear in which **SuperpositionAC** is entered as the **Name**. Following the selection of **Create**, the **Simulation Settings** dialog box will appear in which **AC Sweep/Noise** is selected. The **Start** and **End Frequencies** are both set at 1 kHz, and 1 is entered for the **Points/Decade** request. Click **OK**, and then select the **Run PSpice** key; the **SCHEMATIC1** screen will result with an axis extending from 0.5 kHz to 1.5 kHz. Through the sequence **Trace-Add Trace-V(R3:1)-OK**, the plot point appearing in the bottom of Fig. 18.98 will result. Its value is slightly above the 2-V level and could be read as 2.05 V which compares very nicely with the hand-calculated solution of 2.06 V. The phase angle can be obtained from **Plot-Add Plot to Window-Trace-Add Trace-P(V(R3:1))** to obtain a phase angle close to $-33°$. Additional accuracy can be added to the phase plot through the sequence **Plot-Axis Settings-Y Axis-User Defined $-$40d to $-$30d-OK**, resulting in the $-32.5°$ reading of Fig. 18.98— again, very close to the hand cal-

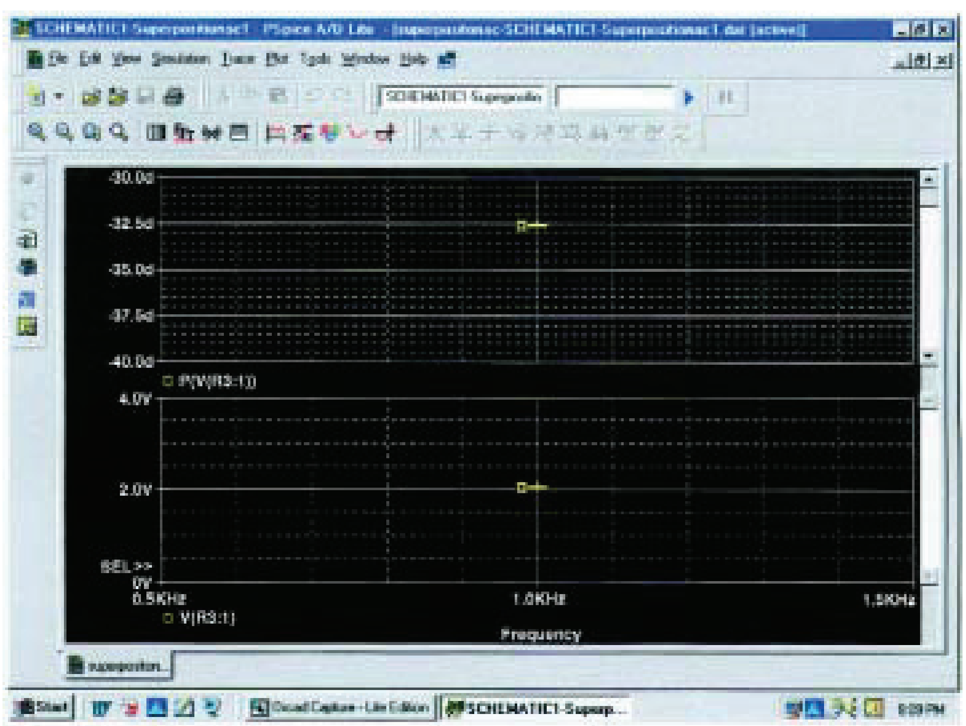


FIG. 18.98
The output results from the simulation of the network of Fig. 18.97.

culation of −32.74° of Example 18.4. Now this solution is fine for the ac signal, but it tells us nothing about the dc component.

By exiting the **SCHEMATIC1** screen, we obtain the **Orcad Capture** window on which **PSpice** can be selected followed by **View Output File**. The result is the printout of Fig. 18.99 which has both the dc and the ac solutions. The **SMALL SIGNAL BIAS SOLUTION** includes the nodes of the network and their dc levels. The node numbers are defined under the netlist starting on line 30. In particular, note the dc level of 3.6 V at node **N00676** which is at the top of resistor R_3 in Fig. 18.97. Also note that the dc level of both ends of the inductor is the same value because of the substitution of a short-circuit equivalent for the inductor for dc analysis. The ac solution appears under the **AC ANALYSIS** heading as 2.06 V at −32.66°, which again is a great verification of the results of Example 18.4.

```
49:
50:  ** Profile: "SCHEMATIC1-Superpositionac1"  [ C:\PSpice\superpositonac-SCHEMATIC
    1-Superpositionac1.sim ]
51:
52:
53:  ****     SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000 DEG C
54:
55:
56: ******************************************************************************
57:
58:
59:
60:  NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE
61:
62:
63: (N00530)    0.0000 (N00596)    3.6000 (N00623)   12.0000 (N00676)    3.6000
64:
65:
66:
67:
68:     VOLTAGE SOURCE CURRENTS
69:     NAME          CURRENT
70:
71:     V_E1        -8.400E-03
72:     V_E2         7.200E-03
73:
74:     TOTAL POWER DISSIPATION   1.01E-01  WATTS
75:
76:
77: **** 07/01/01 20:07:27 ************** PSpice Lite (Mar 2000) *****************
78:
79:  ** Profile: "SCHEMATIC1-Superpositionac1"  [ C:\PSpice\superpositonac-SCHEMATIC
    1-Superpositionac1.sim ]
80:
81:
82:  ****     AC ANALYSIS                      TEMPERATURE =   27.000 DEG C
83:
84:
85: ******************************************************************************
86:
87:
88:
89:   FREQ        VM(N00676)  VP(N00676)
90:
91:
92:    1.000E+03   2.060E+00  -3.266E+01
93:
```

FIG. 18.99

*The output file for the dc **(SMALL SIGNAL BIAS SOLUTION)** and **AC ANALYSIS** for the network of Fig. 18.97.*

Finally, if a plot of the voltage across resistor R_3 is desired, we must return to the **New Simulation Profile** and enter a new **Name** such as **SuperpositionAC1** followed by **Create fill** in the **Simulation Profile** dialog box. This time, however, we will choose the **Time Domain(Transient)** option so that we can obtain a plot against time. The fact that the source has a defined frequency of 1 kHz will tell the program which frequency to apply. The **Run to time** will be 5 ms, resulting in a five-cycle display of the 1-kHz signal. The **Start saving data after** will remain at 0 s, and the **Maximum step size** will be 5 ms/1000 = 5 μs. Click **OK**, and select the **Run PSpice** icon; the **SCHEMATIC1** screen will result again. This time **Trace-Add Trace-V(R3:1)-OK** will result in the plot of Fig. 18.100 which clearly shows a dc level of 3.6 V. Setting a cursor at $t = 0$ s (**A1**) will result in 3.6 V in the **Probe Cursor** display box. Placing the other cursor at the peak value at 2.34 ms **(A2)** will result in a peak value of about 5.66 V. The difference between the peak and the dc level provides the peak value of the ac signal and is listed as 2.06 V in the same **Probe Cursor** display box. A variety of options have now been introduced to find a particular voltage or current in a network with both dc and ac sources. It is certainly satisfying that they all verify our theoretical solution.

Thévenin's Theorem The next application will parallel the methods employed to determine the Thévenin equivalent circuit for dc circuits. The network of Fig. 18.28 will appear as shown in Fig. 18.101 when the open-circuit Thévenin voltage is to be determined. The open circuit is simulated by using a resistor of 1 T (1 million MΩ). The resistor is necessary to establish a connection between the right side of inductor L_2 and ground—nodes cannot be left floating for Orcad simulations.

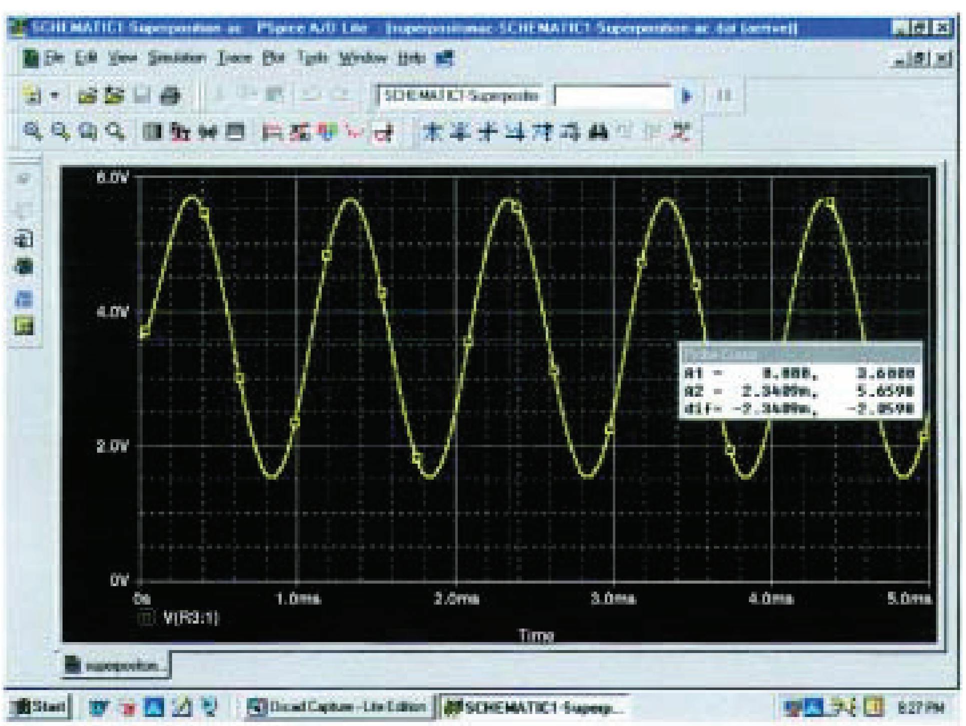


FIG. 18.100
Using PSpice to display the voltage across R_3 for the network of Fig. 18.97.

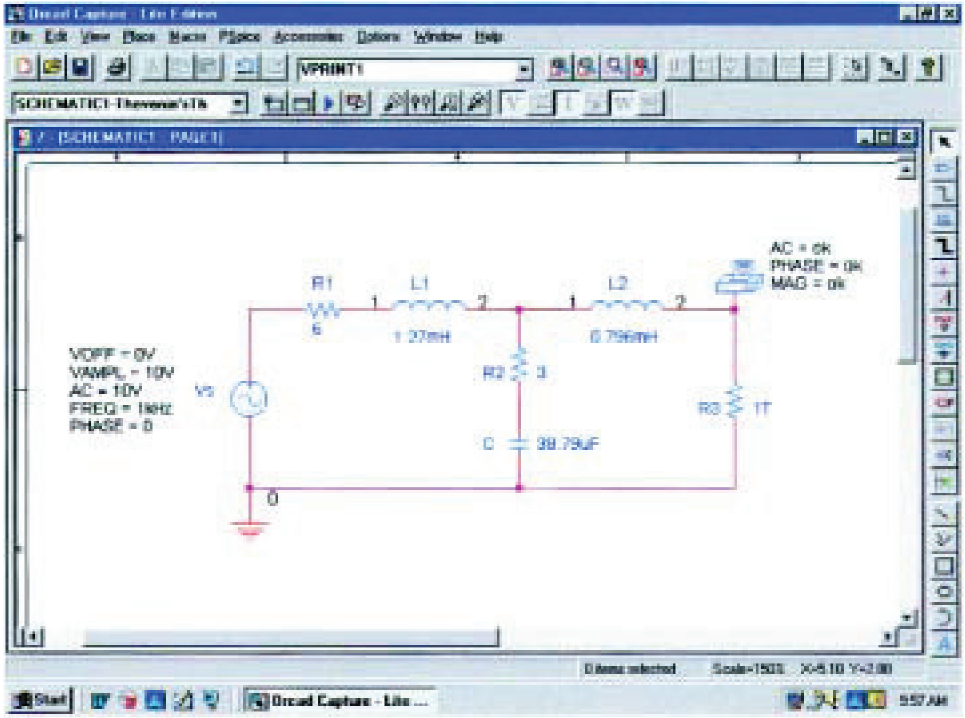


FIG. 18.101
Using PSpice to determine the open-circuit Thévenin voltage.

Since the magnitude and the angle of the voltage are required, **VPRINT1** is introduced as shown in Fig 18.101. The simulation was an **AC Sweep** simulation at 1 kHz, and when the **Orcad Capture** window was obtained, the results appearing in Fig. 18.102 were taken from the listing resulting from the **PSpice-View Output File**. The magnitude of the Thévenin voltage is 5.187 V to compare with the 5.08 V of Example 18.8, while the phase angle is −77.13° to compare with the −77.09° of the same example—excellent results.

```
79:
80:  ** Profile: "SCHEMATIC1-Thevenin'sTh"  [ C:\PSpice\thevenin'sth-SCHEMATIC1-Thevenin'sTh.si
   m ]
81:
82:
83:  ****     AC ANALYSIS                      TEMPERATURE =   27.000 DEG C
84:
85:
86: ******************************************************************************
87:
88:
89:
90:   FREQ        VM(N00433)   VP(N00433)
91:
92:
93:    1.000E+03   5.187E+00   -7.713E+01
94:
```

FIG. 18.102
The output file for the open-circuit Thévenin voltage for the network of Fig. 18.101.

Next, the short-circuit current will be determined using **IPRINT** as shown in Fig. 18.103, to permit a determination of the Thévenin impedance. The resistance R_{coil} of 1 $\mu\Omega$ had to be introduced because inductors cannot be treated as ideal elements when using PSpice; they must all show some series internal resistance. Note that the short-circuit current will pass directly through the printer symbol for **IPRINT.** Incidentally, there is no need to exit the **SCHEMATIC1** developed above to determine the Thévenin voltage. Simply delete **VPRINT** and **R3**, and insert **IPRINT.** Then run a new simulation to obtain the results of Fig. 18.104. The magnitude of the short-circuit current is 0.936 A at an angle of $-108°$. The Thévenin impedance is then defined by

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{Th}}{\mathbf{I}_{sc}} = \frac{5.187\text{ V }\angle -77.13°}{0.936\text{ A }\angle -108.0°} = 5.54\ \Omega\ \angle 30.87°$$

which is an excellent match with 5.49 Ω $\angle 32.36°$ obtained in Example 18.8.

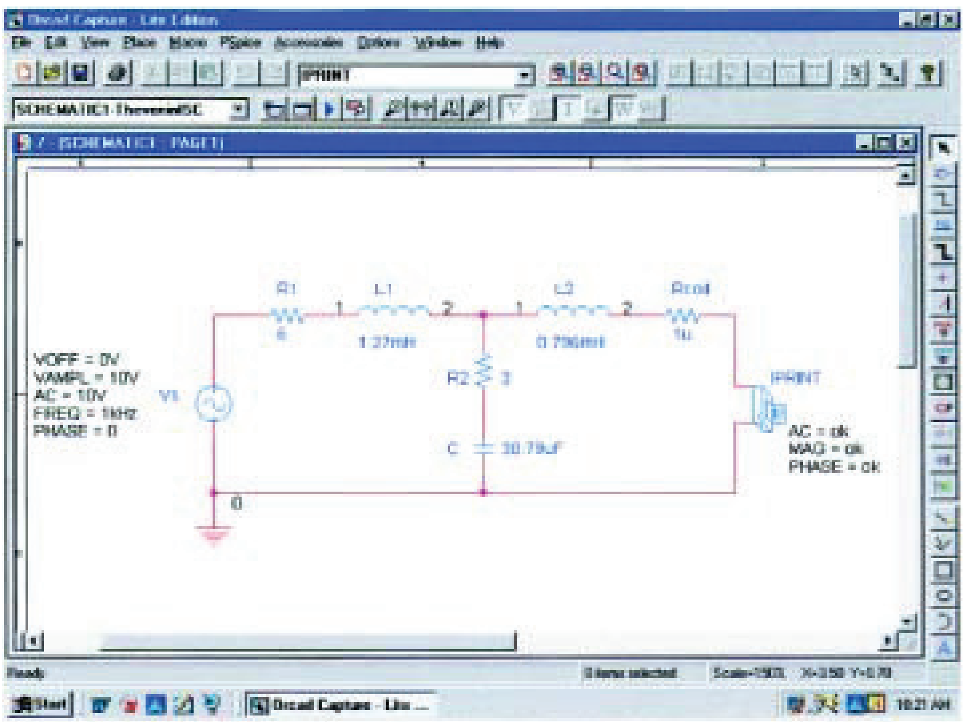


FIG. 18.103
Using PSpice to determine the short-circuit current.

VCVS The last application of this section will be to verify the results of Example 18.12 and to gain some practice using controlled (dependent) sources. The network of Fig. 18.50, with its voltage-controlled voltage source (VCVS), will have the schematic appearance of Fig. 18.105. The VCVS appears as **E** in the **ANALOG** library, with the voltage **E1** as the controlling voltage and **E** as the controlled voltage. In the **Property Editor** dialog box, the **GAIN** must be changed to 20 while

```
 ** Profile: "SCHEMATIC1-TheveninISC"  [ C:\PSpice\thevenin'sth-SCHEMATIC1-Thev
eninISC.sim ]


 ****     AC ANALYSIS                      TEMPERATURE =   27.000 DEG C


****************************************************************************



  FREQ        IM(V_PRINT2)IP(V_PRINT2)


   1.000E+03   9.361E-01  -1.086E+02

```

FIG. 18.104
The output file for the short-circuit current for the network of Fig. 18.103.

the rest of the columns can be left as is. After **Display-Name and Value, Apply** can be selected and the dialog box exited to result in **GAIN = 20** near the controlled source. Take particular note of the second ground inserted near **E** to avoid a long wire to ground that might overlap other elements. For this exercise the current source **ISRC** will be used because it has an arrow in its symbol, and frequency is not

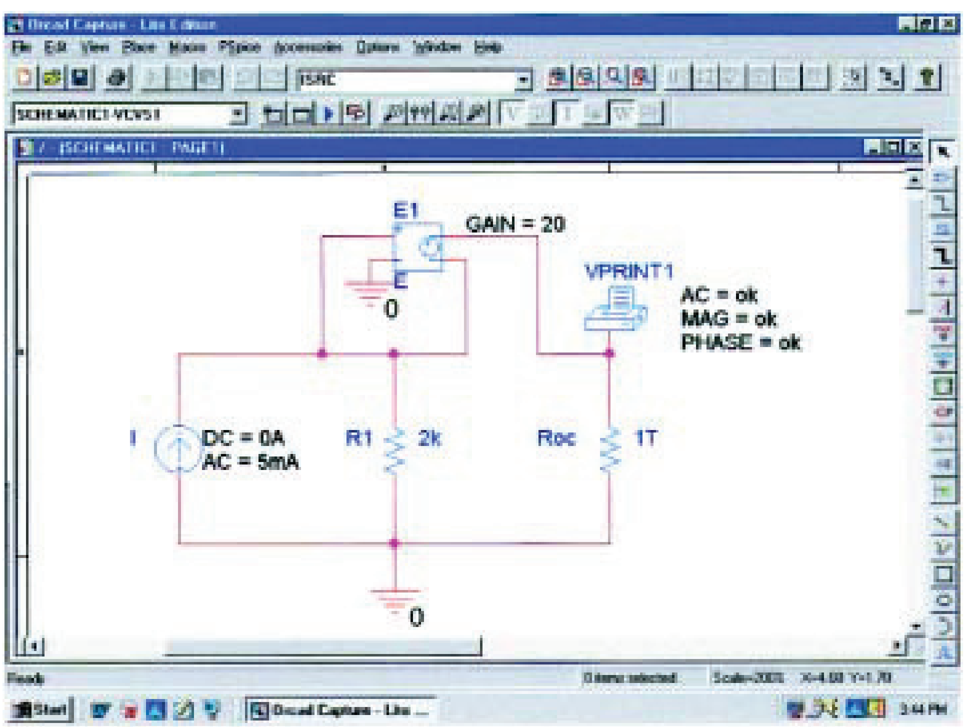


FIG. 18.105
Using PSpice to determine the open-circuit Thévenin voltage for the network of Fig. 18.50.

important for this analysis since there are only resistive elements present. In the **Property Editor** dialog box, the **AC** level is set to 5 mA, and the **DC** level to 0 A; both were displayed using **Display-Name and value. VPRINT1** is set up as in past exercises. The resistor **Roc** (open circuit) was given a very large value so that it would appear as an open circuit to the rest of the network. **VPRINT1** will provide the open circuit Thévenin voltage between the points of interest. Running the simulation in the **AC Sweep** mode at 1 kHz will result in the output file appearing in Fig. 18.106, revealing that the Thévenin voltage is 210 V $\angle 0°$. Substituting the numerical values of this example into the equation obtained in Example 18.12 confirms the result:

$$\mathbf{E}_{Th} = (1 + \mu)\mathbf{I}R_1 = (1 + 20)\,(5\text{ mA}\angle 0°)(2\text{ k}\Omega)$$
$$= \mathbf{210\ V\angle 0°}$$

```

** Profile: "SCHEMATIC1-VCVS1"  [ C:\PSpice\vcvs-SCHEMATIC1-VCVS1.sim ]


****     AC ANALYSIS                          TEMPERATURE =   27.000 DEG C


******************************************************************************



 FREQ         VM(N01658)  VP(N01658)


  1.000E+03   2.100E+02   0.000E+00

```

FIG. 18.106
The output file for the open-circuit Thévenin voltage for the network of Fig. 18.105.

Next, the short-circuit current must be determined using the **IPRINT** option. Note in Fig. 18.107 that the only difference between this network and that of Fig. 18.106 is the replacement of **Roc** with **IPRINT** and the removal of **VPRINT1**. There is therefore no need to completely "redraw" the network. Just make the changes and run a new simulation. The result of the new simulation as shown in Fig. 18.108 is a current of 5 mA at an angle of 0°.

The ratio of the two measured quantities will result in the Thévenin impedance:

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{\mathbf{E}_{Th}}{\mathbf{I}_{sc}} = \frac{210\text{ V}\angle 0°}{5\text{ mA}\angle 0°} = \mathbf{42\ k\Omega}$$

which also matches the longhand solution of Example 18.12:

$$\mathbf{Z}_{Th} = (1 + \mu)R_1 = (1 + 20)2\text{ k}\Omega = (21)2\text{ k}\Omega = \mathbf{42\ k\Omega}$$

The analysis of the full transistor equivalent network of Fig. 18.56 with two controlled sources can be found in the PSpice section of Chapter 26.

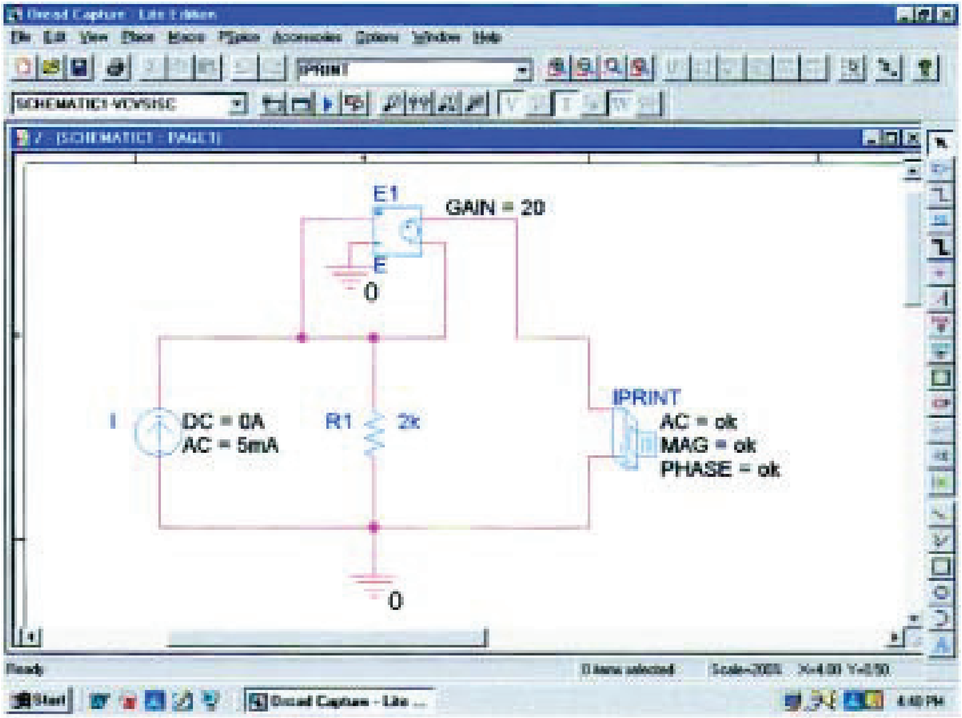


FIG. 18.107
Using PSpice to determine the short-circuit current for the network of Fig. 18.50.

```

** Profile: "SCHEMATIC1-VCVSISC"  [ C:\PSpice\vcvs-SCHEMATIC1-VCVSISC.sim ]


****     AC ANALYSIS                          TEMPERATURE =   27.000 DEG C


******************************************************************************



 FREQ        IM(V_PRINT3)IP(V_PRINT3)


  1.000E+03   5.000E-03   0.000E+00

```

FIG. 18.108
The output file for the short-circuit current for the network of Fig. 18.107.

PROBLEMS

SECTION 18.2 Superposition Theorem

1. Using superposition, determine the current through the inductance X_L for each network of Fig. 18.109.

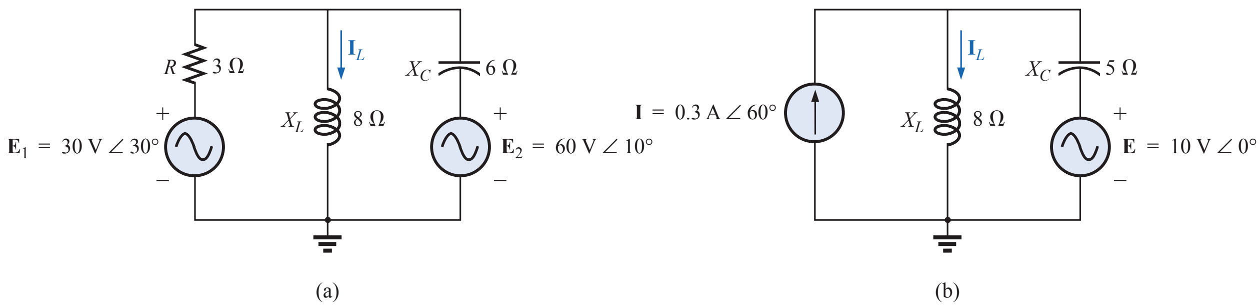


FIG. 18.109
Problem 1.

***2.** Using superposition, determine the current $\mathbf{I}_L$ for each network of Fig. 18.110.

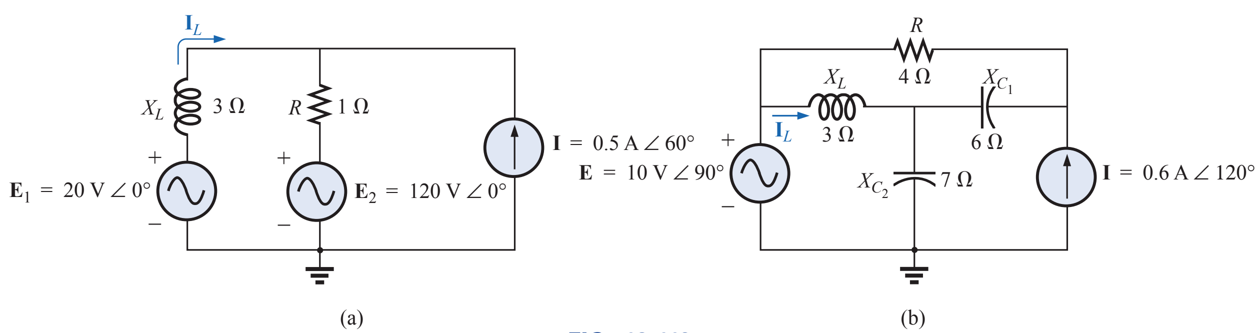


FIG. 18.110
Problem 2.

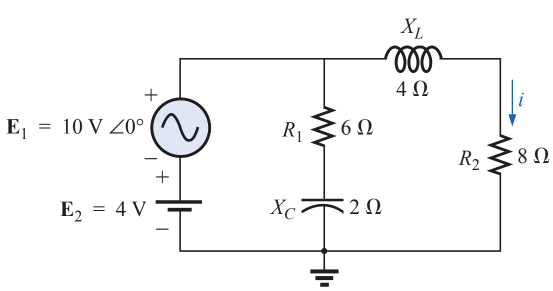


FIG. 18.111
Problems 3, 15, 30, and 42.

***3.** Using superposition, find the sinusoidal expression for the current i for the network of Fig. 18.111.

4. Using superposition, find the sinusoidal expression for the voltage v_C for the network of Fig. 18.112.

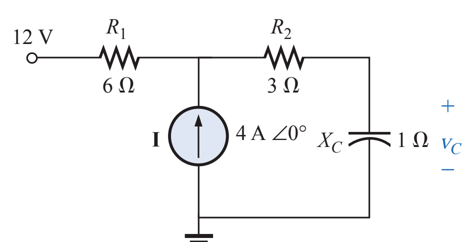


FIG. 18.112
Problems 4, 16, 31, and 43.

*5. Using superposition, find the current **I** for the network of Fig. 18.113.

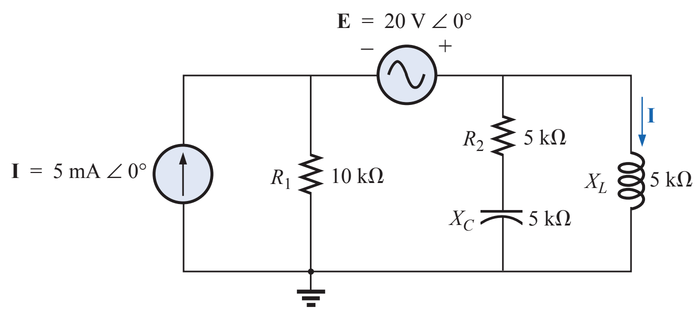


FIG. 18.113
Problems 5, 17, 32, and 44.

6. Using superposition, determine the current $\mathbf{I}_L$ ($h = 100$) for the network of Fig. 18.114.

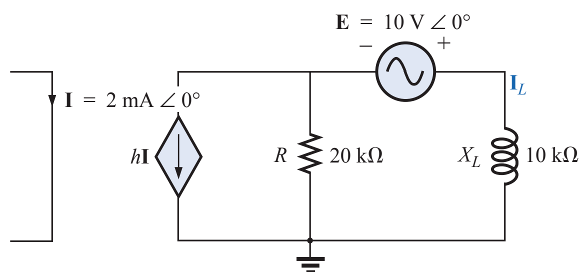


FIG. 18.114
Problems 6 and 20.

7. Using superposition, for the network of Fig. 18.115, determine the voltage $\mathbf{V}_L$ ($\mu = 20$).

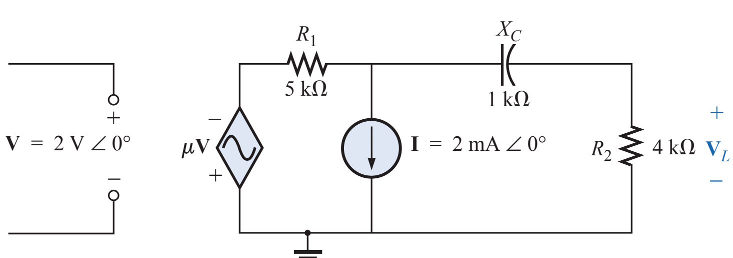


FIG. 18.115
Problems 7, 21, and 35.

***8.** Using superposition, determine the current $\mathbf{I}_L$ for the network of Fig. 18.116 ($\mu = 20$; $h = 100$).

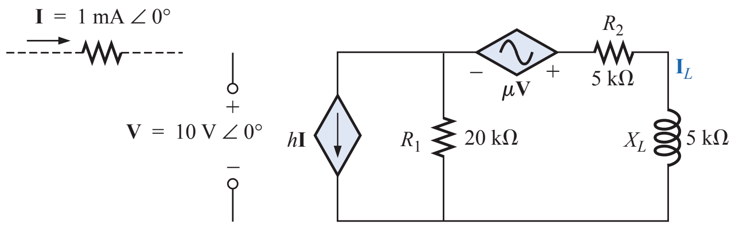


FIG. 18.116
Problems 8, 22, and 36.

***9.** Determine $\mathbf{V}_L$ for the network of Fig. 18.117 ($h = 50$).

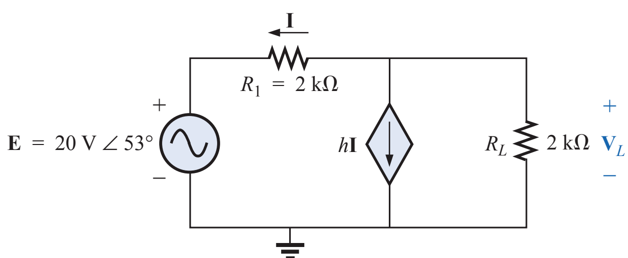


FIG. 18.117
Problems 9 and 23.

***10.** Calculate the current $\mathbf{I}$ for the network of Fig. 18.118.

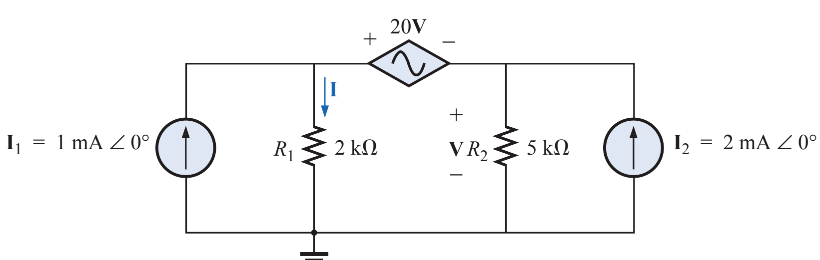


FIG. 18.118
Problems 10, 24, and 38.

11. Find the voltage $\mathbf{V}_s$ for the network of Fig. 18.119.

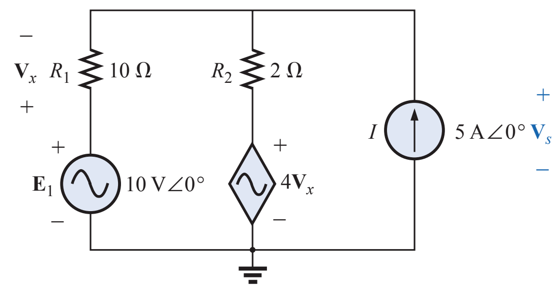


FIG. 18.119
Problem 11.

SECTION 18.3 Thévenin's Theorem

12. Find the Thévenin equivalent circuit for the portions of the networks of Fig. 18.120 external to the elements between points *a* and *b*.

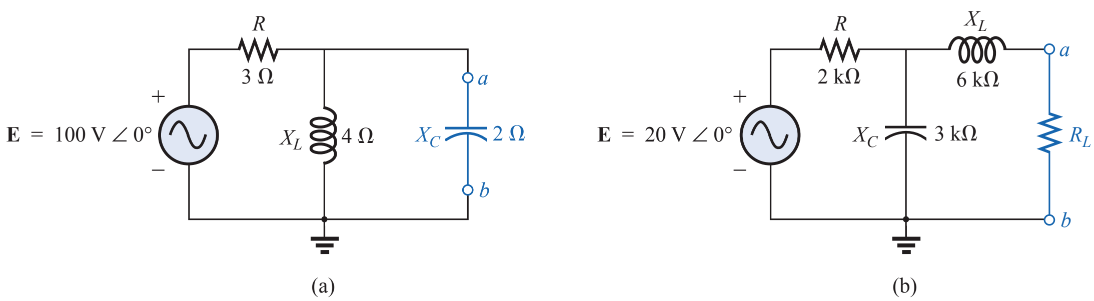


FIG. 18.120
Problems 12 and 26.

***13.** Find the Thévenin equivalent circuit for the portions of the networks of Fig. 18.121 external to the elements between points *a* and *b*.

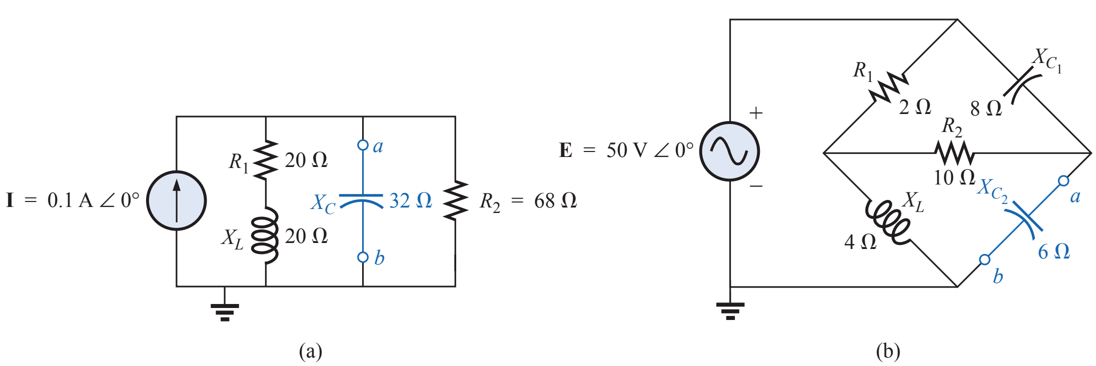


FIG. 18.121
Problems 13 and 27.

***14.** Find the Thévenin equivalent circuit for the portions of the networks of Fig. 18.122 external to the elements between points *a* and *b*.

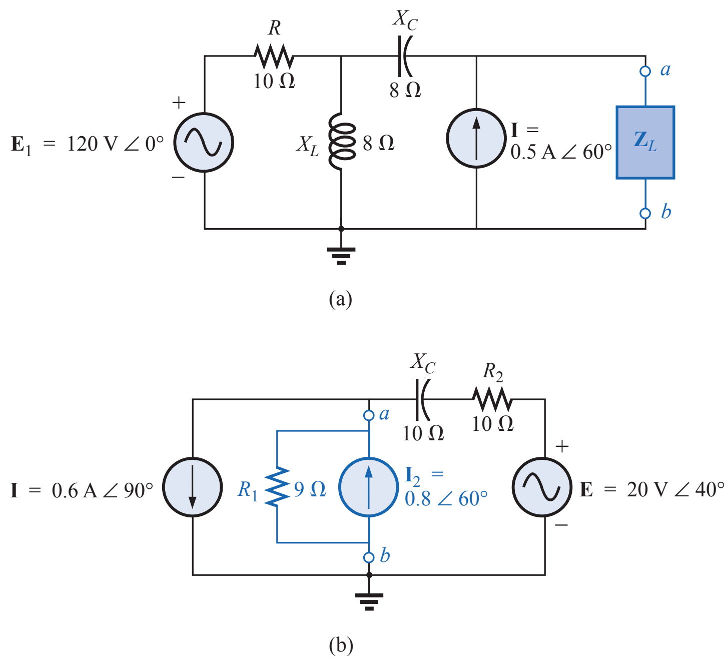


FIG. 18.122
Problems 14 and 28.

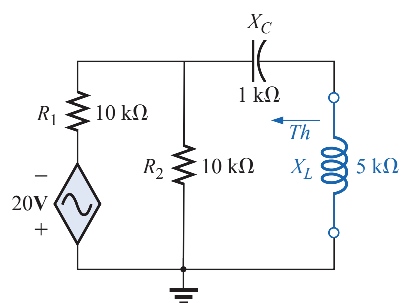


FIG. 18.123
Problems 18 and 33.

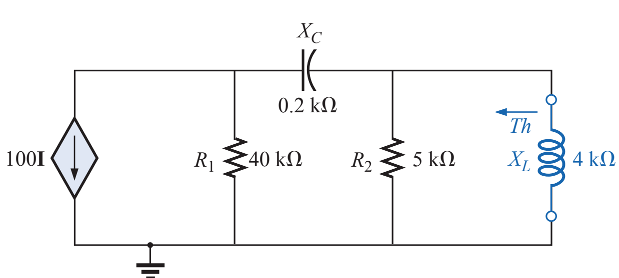


FIG. 18.124
Problems 19 and 34.

***15. a.** Find the Thévenin equivalent circuit for the network external to the resistor R_2 in Fig. 18.111.

b. Using the results of part (a), determine the current *i* of the same figure.

16. a. Find the Thévenin equivalent circuit for the network external to the capacitor of Fig. 18.112.

b. Using the results of part (a), determine the voltage $\mathbf{V}_C$ for the same figure.

***17. a.** Find the Thévenin equivalent circuit for the network external to the inductor of Fig. 18.113.

b. Using the results of part (a), determine the current **I** of the same figure.

18. Determine the Thévenin equivalent circuit for the network external to the 5-kΩ inductive reactance of Fig. 18.123 (in terms of **V**).

19. Determine the Thévenin equivalent circuit for the network external to the 4-kΩ inductive reactance of Fig. 18.124 (in terms of **I**).

20. Find the Thévenin equivalent circuit for the network external to the 10-kΩ inductive reactance of Fig. 18.114.

21. Determine the Thévenin equivalent circuit for the network external to the 4-kΩ resistor of Fig. 18.115.

***22.** Find the Thévenin equivalent circuit for the network external to the 5-kΩ inductive reactance of Fig. 18.116.

***23.** Determine the Thévenin equivalent circuit for the network external to the 2-kΩ resistor of Fig. 18.117.

***24.** Find the Thévenin equivalent circuit for the network external to the resistor R_1 of Fig. 18.118.

***25.** Find the Thévenin equivalent circuit for the network to the left of terminals *a-a′* of Fig. 18.125.

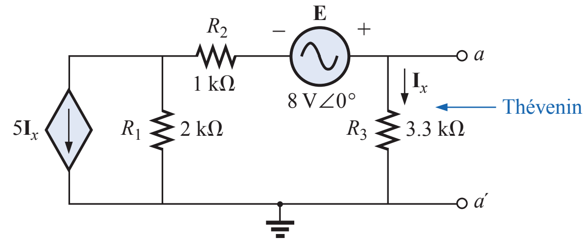


FIG. 18.125
Problem 25.

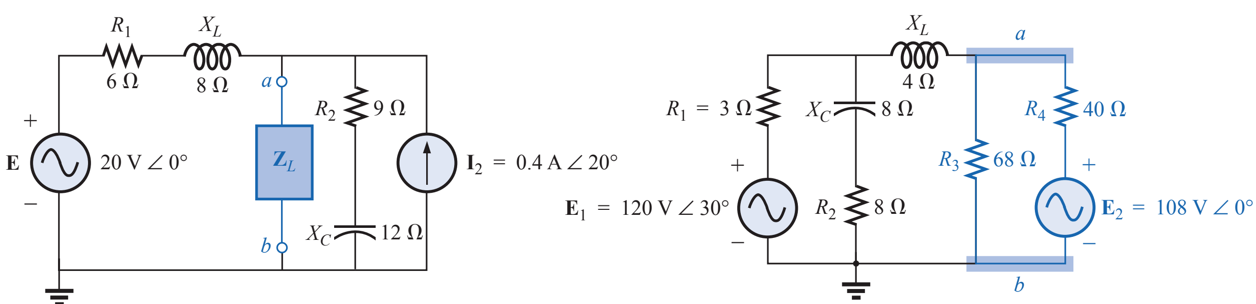
SECTION 18.4 Norton's Theorem

26. Find the Norton equivalent circuit for the network external to the elements between *a* and *b* for the networks of Fig. 18.120.

27. Find the Norton equivalent circuit for the network external to the elements between *a* and *b* for the networks of Fig. 18.121.

28. Find the Norton equivalent circuit for the network external to the elements between *a* and *b* for the networks of Fig. 18.122.

***29.** Find the Norton equivalent circuit for the portions of the networks of Fig. 18.126 external to the elements between points *a* and *b*.



(a) (b)

FIG. 18.126
Problem 29.

***30.** **a.** Find the Norton equivalent circuit for the network external to the resistor R_2 in Fig. 18.111.
b. Using the results of part (a), determine the current **I** of the same figure.

***31.** **a.** Find the Norton equivalent circuit for the network external to the capacitor of Fig. 18.112.
b. Using the results of part (a), determine the voltage V_C for the same figure.

***32.** **a.** Find the Norton equivalent circuit for the network external to the inductor of Fig. 18.113.
b. Using the results of part (a), determine the current **I** of the same figure.

33. Determine the Norton equivalent circuit for the network external to the 5-kΩ inductive reactance of Fig. 18.123.

34. Determine the Norton equivalent circuit for the network external to the 4-kΩ inductive reactance of Fig. 18.124.

35. Find the Norton equivalent circuit for the network external to the 4-kΩ resistor of Fig. 18.115.

***36.** Find the Norton equivalent circuit for the network external to the 5-kΩ inductive reactance of Fig. 18.116.

***37.** For the network of Fig. 18.127, find the Norton equivalent circuit for the network external to the 2-kΩ resistor.

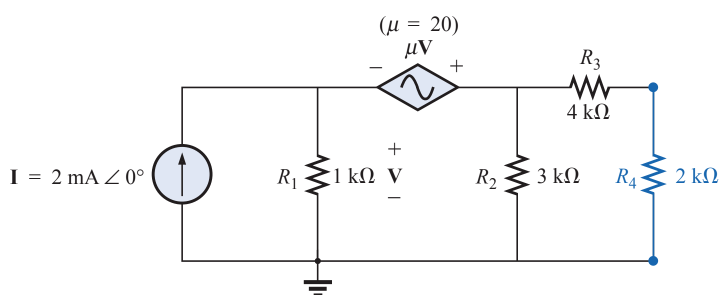


FIG. 18.127
Problem 37.

***38.** Find the Norton equivalent circuit for the network external to the $\mathbf{I}_1$ current source of Fig. 18.118.

SECTION 18.5 Maximum Power Transfer Theorem

39. Find the load impedance $\mathbf{Z}_L$ for the networks of Fig. 18.128 for maximum power to the load, and find the maximum power to the load.

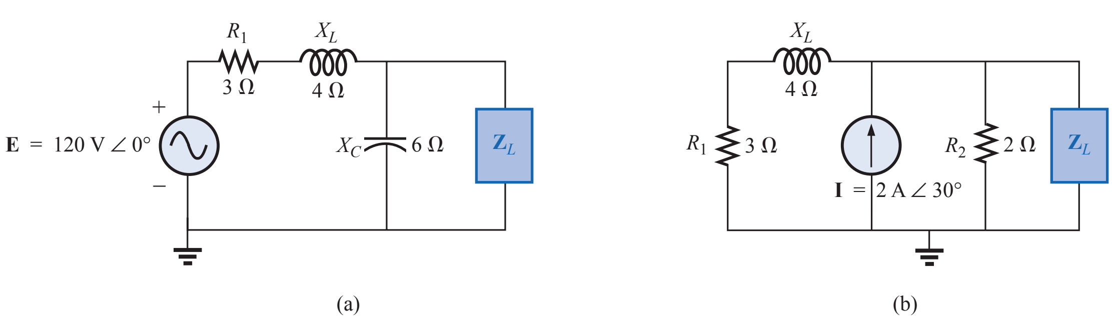


FIG. 18.128
Problem 39.

***40.** Find the load impedance $\mathbf{Z}_L$ for the networks of Fig. 18.129 for maximum power to the load, and find the maximum power to the load.

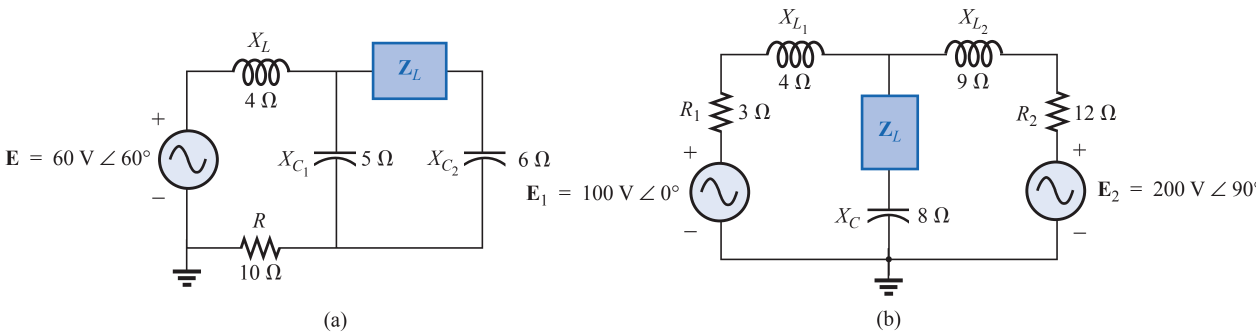


FIG. 18.129
Problem 40.

41. Find the load impedance R_L for the network of Fig. 18.130 for maximum power to the load, and find the maximum power to the load.

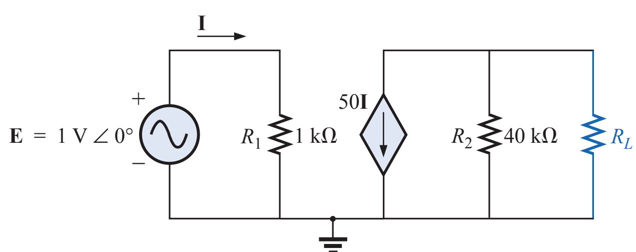


FIG. 18.130
Problem 41.

***42. a.** Determine the load impedance to replace the resistor R_2 of Fig. 18.111 to ensure maximum power to the load.

b. Using the results of part (a), determine the maximum power to the load.

***43. a.** Determine the load impedance to replace the capacitor X_C of Fig. 18.112 to ensure maximum power to the load.

b. Using the results of part (a), determine the maximum power to the load.

***44. a.** Determine the load impedance to replace the inductor X_L of Fig. 18.113 to ensure maximum power to the load.

b. Using the results of part (a), determine the maximum power to the load.

45. a. For the network of Fig. 18.131, determine the value of R_L that will result in maximum power to the load.

b. Using the results of part (a), determine the maximum power delivered.

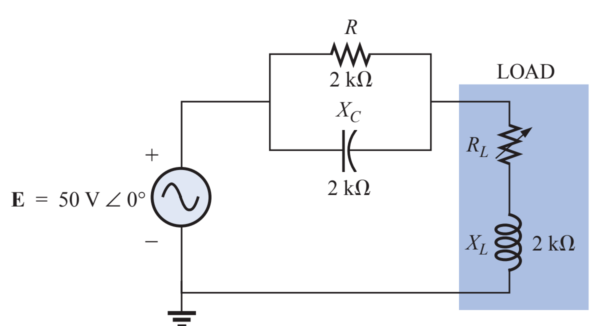


FIG. 18.131
Problem 45.

*46. **a.** For the network of Fig. 18.132, determine the level of capacitance that will ensure maximum power to the load if the range of capacitance is limited to 1 nF to 5 nF.
b. Using the results of part (a), determine the value of R_L that will ensure maximum power to the load.
c. Using the results of parts (a) and (b), determine the maximum power to the load.

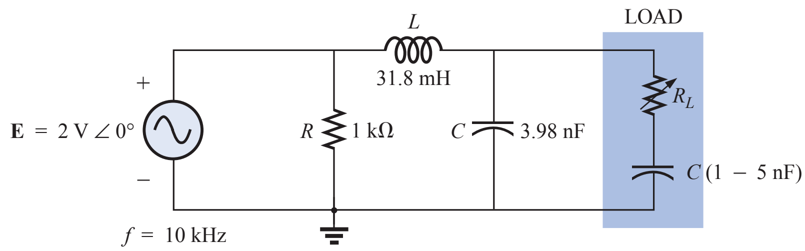


FIG. 18.132
Problem 46.

SECTION 18.6 Substitution, Reciprocity, and Millman's Theorems

47. For the network of Fig. 18.133, determine two equivalent branches through the substitution theorem for the branch *a-b*.

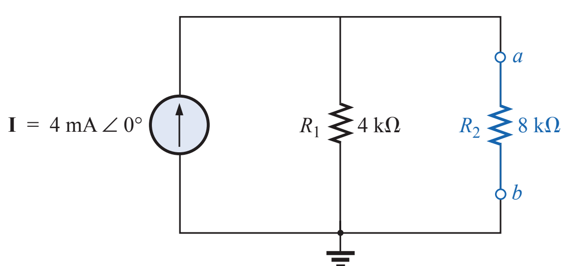


FIG. 18.133
Problem 47.

48. **a.** For the network of Fig. 18.134(a), find the current **I**.
b. Repeat part (a) for the network of Fig. 18.134(b).
c. Do the results of parts (a) and (b) compare?

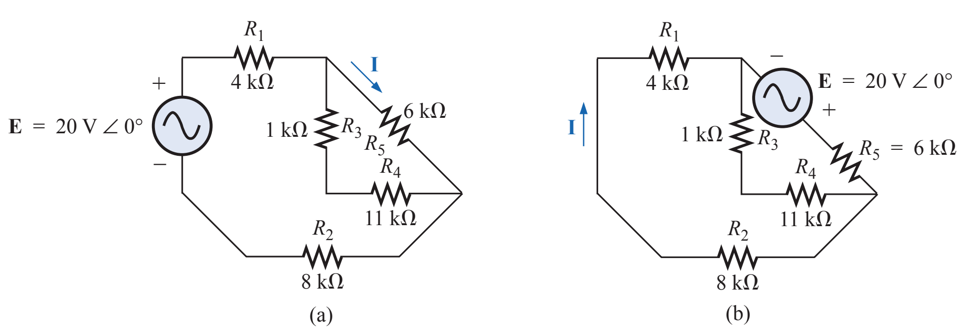


FIG. 18.134
Problem 48.

49. Using Millman's theorem, determine the current through the 4-kΩ capacitive reactance of Fig. 18.135.

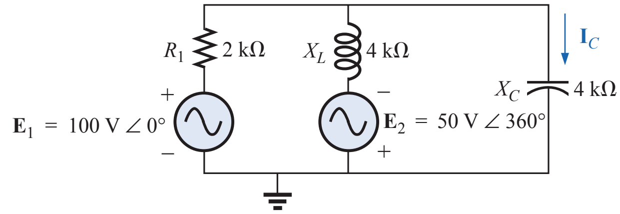


FIG. 18.135
Problem 49.

SECTION 18.8 Computer Analysis

PSpice or Electronics Workbench

50. Apply superposition to the network of Fig. 18.6. That is, determine the current **I** due to each source, and then find the resultant current.

***51.** Determine the current $\mathbf{I}_L$ for the network of Fig. 18.21 using schematics.

***52.** Using schematics, determine $\mathbf{V}_2$ for the network of Fig. 18.56 if $\mathbf{V}_i = 1\text{ V}\angle 0°$, $R_1 = 0.5\text{ k}\Omega$, $k_1 = 3 \times 10^{-4}$, $k_2 = 50$, and $R_2 = 20\text{ k}\Omega$.

***53.** Find the Norton equivalent circuit for the network of Fig. 18.75 using schematics.

***54.** Using schematics, plot the power to the *R-C* load of Fig. 18.88 for values of R_L from 1 Ω to 10 Ω.

Programming Language (C++, QBASIC, Pascal, etc.)

55. Given the network of Fig. 18.1, write a program to determine a general solution for the current **I** using superposition. That is, given the reactance of the same network elements, determine **I** for voltage sources of any magnitude but the same angle.

56. Given the network of Fig. 18.23, write a program to determine the Thévenin voltage and impedance for any level of reactance for each element and any magnitude of voltage for the voltage source. The angle of the voltage source should remain at zero degrees.

57. Given the configuration of Fig. 18.136, demonstrate that maximum power is delivered to the load when $X_C = X_L$ by tabulating the power to the load for X_C varying from 0.1 kΩ to 2 kΩ in increments of 0.1 kΩ.

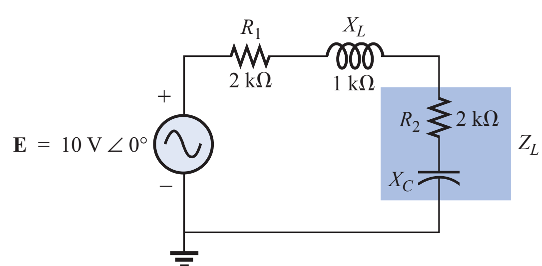


FIG. 18.136
Problem 57.

GLOSSARY

Maximum power transfer theorem A theorem used to determine the load impedance necessary to ensure maximum power to the load.

Millman's theorem A method employing voltage-to-current source conversions that will permit the determination of unknown variables in a multiloop network.

Norton's theorem A theorem that permits the reduction of any two-terminal linear ac network to one having a single current source and parallel impedance. The resulting configuration can then be employed to determine a particular current or voltage in the original network or to examine the effects of a specific portion of the network on a particular variable.

Reciprocity theorem A theorem stating that for single-source networks, the magnitude of the current in any branch of a network, due to a single voltage source anywhere else in the network, will equal the magnitude of the current through the branch in which the source was originally located if the source is placed in the branch in which the current was originally measured.

Substitution theorem A theorem stating that if the voltage across and current through any branch of an ac bilateral net-